

SYMMETRIC SPACE CONTEST

The goal of this contest is given some information to find as many symmetric spaces with these data as possible.

GIVEN A SYMMETRIC SPACE - FIND A GEOMETRIC MODEL

Recall that for the hyperbolic plane we have many different geometric models - the hyperboloid model (discussed in class), the Poincaré disc, the upper half plane and the projective Klein model (discussed on Problem set 2).

Consider the Riemannian symmetric pair $(G, K) = (U(2, 2), U(2) \times U(2))$. The associated symmetric space is an open subset of the complex Grassmannian of 2-planes in \mathbf{C}^4 . Describe this realization, it is a generalization of the hyperboloid model. Try to find many other geometric models of the symmetric space $U(2, 2)/U(2) \times U(2)$ and explicit maps between them.

GIVEN A GROUP - FIND A SYMMETRIC SPACE

Try to find many Riemannian symmetric pairs (G, K) with $G = U(n)$ or $G = SU(n)$ and describe the corresponding symmetric spaces What are the dual symmetric pairs/symmetric spaces?