MAT 520: Introduction to Geometry (Symmetric Spaces) – Spring 2009

Basic Information

- Time: M F 9:30 a.m. - 11:00 a.m.
- Classroom: Fine 1001
- Start Date: Monday, February 9, 2009
- My Office: Fine 1007, My Email: wienhard@math.princeton.edu

Course Description

Symmetric spaces (and locally symmetric spaces) play a crucial role in Algebraic Geometry, Differential Geometry, Mathematical Physics, Number Theory, and Representation Theory. They arise as moduli spaces (parameter spaces) for variations of geometric and arithmetic objects, e.g. the space of all k-dimensional subspaces of an n-dimensional vector spaces is a symmetric space, and the moduli space of elliptic curves is a locally symmetric space. Any symmetric space can be decomposed into irreducible symmetric spaces. There are three types of irreducible symmetric spaces: Euclidean, compact and noncompact, where the latter two are related by an interesting duality. An example of such dual spaces are the two-sphere and the hyperbolic plane. Due to their rich symmetry group symmetric spaces can be described both differential geometrically as well as algebraically. We will start from the differential geometric definition of (locally) symmetric spaces and establish their relation with certain real Lie algebras with involution. We study the structure theory of these Lie algebras and of the corresponding Lie groups as well as the geometric properties of compact and noncompact symmetric spaces. If time permits we will also discuss the special class of Hermitian symmetric spaces which are related to bounded symmetric domains. Throughout the course we will see many examples of symmetric spaces and various geometric models for them.

For undergraduates who would like to get credit for this course there will be bi-weekly problem sets and a take-home final exam.

Literature

Background on Riemannian Geometry.

- Gallot-Hulin-Lafontaine [3]
- do Carmo [2]
- Kobayashi-Nomizu [5, 6]

- Sakai [9]

Symmetric Spaces.

- Borel [1]
- Helgason [4] (”the bible”)
- Kobayashi-Nomizu [5, 6]
- Sakai [9]
- Wolf [10]
- Loos [7, 8] (another approach)

References

1. Armand Borel, *Semisimple groups and Riemannian symmetric spaces*, Texts and Readings in Mathematics, vol. 16, Hindustan Book Agency, New Delhi, 1998. MR MR1661166 (2000e:53063)
2. Manfredo Perdigão do Carmo, *Riemannian geometry*, Mathematics: Theory & Applications, Birkhäuser Boston Inc., Boston, MA, 1992, Translated from the second Portuguese edition by Francis Flaherty. MR MR1138207 (92i:53001)
3. Sylvestre Gallot, Dominique Hulin, and Jacques Lafontaine, *Riemannian geometry*, third ed., Universitext, Springer-Verlag, Berlin, 2004. MR MR2088027 (2005e:53001)
4. Sigurdur Helgason, *Differential geometry, Lie groups, and symmetric spaces*, Graduate Studies in Mathematics, vol. 34, American Mathematical Society, Providence, RI, 2001, Corrected reprint of the 1978 original. MR MR1834454 (2002b:53081)
5. Shoshichi Kobayashi and Katsumi Nomizu, *Foundations of differential geometry. Vol. I*, Wiley Classics Library, John Wiley & Sons Inc., New York, 1996, Reprint of the 1963 original, A Wiley-Interscience Publication. MR MR1393940 (97c:53001a)
6. ———, *Foundations of differential geometry. Vol. II*, Wiley Classics Library, John Wiley & Sons Inc., New York, 1996, Reprint of the 1969 original, A Wiley-Interscience Publication. MR MR1393941 (97c:53001b)
7. Ottmar Loos, *Symmetric spaces. I: General theory*, W. A. Benjamin, Inc., New York-Amsterdam, 1969. MR MR0239005 (39 #365a)
8. ———, *Symmetric spaces. II: Compact spaces and classification*, W. A. Benjamin, Inc., New York-Amsterdam, 1969. MR MR0239006 (39 #365b)
9. Takashi Sakai, *Riemannian geometry*, Translations of Mathematical Monographs, vol. 149, American Mathematical Society, Providence, RI, 1996, Translated from the 1992 Japanese original by the author. MR MR1390760 (97f:53001)
10. Joseph A. Wolf, *Spaces of constant curvature*, fifth ed., Publish or Perish Inc., Houston, TX, 1984. MR MR928600 (88k:53002)