

MAT455 Advanced Topics in Geometry: Lie algebras and Lie groups

Fall 2011

W F 11:00 am - 12:20 pm in McCosh 64

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Office Hours: Wednesdays 12:30 in my office 1007. If you cannot come during my office hours please send me an email to make an appointment.

Grader: TBD

Lie algebras and Lie groups are important in many areas of mathematics as well as theoretical physics. The course gives an introduction to the topic focussing on:

(1) **Lie groups, Lie algebras and their relation**

Definition of Lie groups, Examples

Actions of Lie groups, Representations, Homogeneous Spaces

Exponential Map, Logarithm, construction of the Lie algebra structure

Campbell Hausdorff Formula

Fundamental Theorems of Lie Theory

Complexifications and Real Forms

(2) **Lie algebras**

Definition of Lie algebras, general structure theory

Solvable and Nilpotent Lie algebras

Engel's Theorem, Lie's Theorem

Radical, semisimple Lie algebras

Jordan decomposition

Cartan Theorem

Representation theory of $\mathfrak{sl}(2, \mathbb{C})$

(3) **Structure Theory of semisimple Lie algebras**

Cartan subalgebras

Root decomposition, Root systems

Cartan Matrix, Dynkin Diagram, Weyl group

Classification of simple Lie algebras

Examples, classical groups

Compact forms, Non-compact forms

(4) Representation Theory

Universal Enveloping Algebra

Weights, highest weights

Classification of irreducible representations

Character formulas

General Information: Official prerequisites of the course are MAT 217 and MAT 328. But not having taken these courses should not refrain you from taking this one. Please email you with a short description of your background and I will let you know if I think that the course is suitable for you . You will definitely need a solid background in linear algebra. For the first part of the course, when we will consider Lie groups, knowing what a differentiable manifold, its tangent space, its fundamental group and its universal cover are would be helpful. If many people do not know what these are, I might these basic notions shortly

You are encouraged to work together on problem sets, but everyone has to write up the solutions independently. Please order the pages and staple them. Unreadable homework will not be corrected. No late homework will be accepted.

Problems sets: 50%**Take Home Final Exam:** 50%**Sample Literature:**

A. Kirillov: An Introduction to Lie Groups and Lie Algebras

J. E. Humphreys: Introduction to Lie Algebras and Representation Theory

V.S.Varadarajan: Lie groups, Lie Algebras, and Their Representations

J.-P. Serre: Lie algebras and Lie groups

J.-P. Serre: Complex semisimple Lie algebras