M327 Introduction to Differential Geometry
Fall 2007
MW 1:30 p.m.-2:50 p.m. in Fine 1001

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Office Hours: Potentially Monday, 3 p.m. - 4 p.m. If you cannot come during my office hours please send me an email to make an appointment.

Grader: Ye Li, yeli@math.princeton.edu, Fine 311

Textbook: *Differential Geometry of Curves and Surfaces*, by Manfredo P. do Carmo.

Contents of the Course: This course is an introduction to the area of Differential Geometry, a classical subject of modern mathematics. We will be primarily concerned with curves and surfaces in three dimensional space. To study the geometry of curves and surfaces in $\mathbb{R}^3$ we will use multi-variable calculus, linear algebra and also some ordinary differential equations. One goal of this course is to find invariants for curves and surface. We will prove several "Fundamental Theorems" stating that the set of invariants we found is complete and describing their relations. We will come across very nice and interesting theorems, for example the "Theorema Egregium" which explains why there cannot be a correct map of the world.

More concretely we will (hopefully) cover the following topics:

(1) *Curves in* $\mathbb{R}^3$: arclength, curvature, torsion, Frenet trihedron, Fundamental Theorem for space curves.
(2) *Surfaces in* $\mathbb{R}^3$: regular and parametrized surfaces, first fundamental form, Gauß map, second fundamental form, principal curvatures, Gauß curvature, mean curvature.
(3) *Instrinsic Geometry of Surface*: (local) isometries, vector fields, covariant derivative, Christoffel symbols, Theorema egregium, Fundamental Theorems for surfaces, geodesics
(4) *Gauß-Bonnet-Theorem*: geodesic curvature, local Gauß-Bonnet-Theorem, topology and triangulations of surfaces, global Gauß-Bonnet-Theorem.
(5) *Excursion to Spherical and Hyperbolic geometry*

General Information: Official prerequisite for the course is M218 Analysis in Several Variables. We will frequently use the *Inverse Function Theorem* and the *Implicit Function Theorem*, we might use some results about the existence and uniqueness of solutions of *Ordinary Differential Equations*. Do not worry if you do not yet know these results, I will provide you with a handout or reference to learn them.

There will be one **take home midterm exam** and a **take home final exam**. The **weekly homework assigments** will be posted on the course webpage and are usually due on Monday at the beginning of class.

You are encouraged to work together on homework problems, but everyone has to write up the solutions independently. Please order the pages and staple the pages. Unreadable homework will not be corrected. No late homework will be accepted.

In addition every student has to choose and hand in a **project** by December 5th. This project allows you to further explore a specific topic. I will provide you with a list of possible topics (see below) but you are also very welcome to choose a topic yourself after consultation with me.

Midterm Exam: 20%
Final Exam: 30%
Homework and Project: 50%

Possible Projects:

(1) Isoperimetric Inequality
(2) Four-Vertex Theorem
(3) Cauchy-Crofton-Formula
(4) Fary-Milnor Theorem
(5) Minimal Surfaces
(6) Non-Euclidean Geometry
(7) The Hyperbolic Plane
(8) Maps of the World (different projections of the sphere to the plane)
(9) Ruled Surfaces
(10) Rigidity of the Sphere
(11) Hilbert's Theorem
(12) Visualization of Curves and Surfaces

Further Literature (just a random selection, you are not required to get any of them):

Modern Differential Geometry of Curves and Surfaces, by Alfred Gray.
Elementary Differential Geometry, by Andrew Pressley.
Curves and Surfaces, by Sebastián Montiel and Antonio Ros.
Differential Geometry: Manifolds, Curves and Surfaces, by Marcel Berger and Bernard Gostiaux, Chapters 8–11.
Curves and Surfaces in Euclidean Space, by S.S. Chern, in: Studies in Global Geometry and Analysis, edited by S.S. Chern, Studies in Mathematics, Volume 4, pp.16–56. (This is not a textbook, it just discusses some nice theorems.)
General Investigations of Curved Surfaces of 1827 and 1825, by Carl F. Gauß. (If you are very brave!! This is a historical text. It is not written in modern mathematical language, so it is very difficult to read.)