HOMEWORK, DUE OCTOBER 15, 2007

Problem 1: Surfaces of Revolution

Let S be a surface of revolution and C its generating curve. (See the previous homework assigment for definition).

(1) Show that one can find local parametrizations $\mathbf{x}(u,v)$ of S such that in this local parametrizations the first fundamental form is given by $E = E(v)$, $F = 0$, $G = 1$, where $E(v)$ is a function depending on v, but not on u. (Note: in the previous homework assignment the variables (u,v) where denoted by (ϕ,t).)

(2) Let s be the arc length of the regular curve C and $\rho = \rho(s)$ the distance to the rotation axis of the point of C corresponding to s. Show *Pappus' Theorem*: The area of S is

$$2\pi \int_0^l \rho(s)ds,$$

where l is the length of C.

(3) Compute the area of the torus which is obtained by rotating a circle $\alpha(v) = (R + r\cos(v), 0, r\sin(v))$ with $0 \le v < 2\pi$ and $0 < r < R$, around the z-axis.

Problem 2: Tubular Surfaces

Let $I \subset \mathbb{R}$ be an open interval and $\alpha : I \to \mathbb{R}^3$ a regular parametrized curve, parametrized by arc length, with nonzero curvature $\kappa(s)$ everywhere. Let $K = \sup_{s\in I} \kappa(s)$. Let $n(s)$ be the normal vector to α in s and $b(s)$ the bi-normal vector of α at s. Fix some constant $\frac{1}{K} > r > 0$. The parametrized surface

$$\mathbf{x}(s,v) = \alpha(s) + r(n(s)\cos(v) + b(s)\sin(v))$$

with $s \in I$, $0 < v < 2\pi$ is called the *tube* of radius r around α.

(1) Show that the parametrized surface $\mathbf{x}$ is regular.

(2) Show that a unit normal vector to the surface at $p = \mathbf{x}(s,v)$ is given by $N(s,v) = -(n(s)\cos(v) + b(s)\sin(v))$.

(3) Show that the area of the *tube* is $2\pi r$ times the length of the curve $\alpha(I)$.

Problem 3: Generalized Helicoids

Read Example 3 on page 94. Solve Problem 13 on page 101.

Problem 4

Problem 15 on page 90.

Problem 5

Problem 3 on page 99

Problem 6

Problem 2 on page 99.

Not required

I will probably not cover this by Wednesday - so keep it in mind for the next homework!
For every $k > 0$ let

$$P_k = \{(x, y, z) \in \mathbb{R}^3 \mid z = x^2 + ky^2\}$$

be the paraboloid in $\mathbb{R}^3$. This is a regular surface. Study the Gauss map and its differential at the point $p = (0, 0, 0) \in P_k$.