Integrable Systems

Please let me (wienhard@math.princeton.edu) know soon, which talk you would like to give. Even if it says that one topic consists of 1 talk, a group of two people can joined up to prepare and present the topic. Talks are supposed to be around 60 minutes, some might be a bit longer, but should not exceed 90 minutes. If you have a question regarding one of the topics or would like to have further references, please contact me.

1. Definition, Examples, Arnold-Liouville Theorem – 2 talks (Will Cavendish, Cotton Seed, Sam Lewallen)

The talk should start by recalling some background from symplectic geometry (symplectic manifolds, Hamiltonian system, Poisson manifolds, symplectic leaves) and then define what an integrable system is. A definition of infinite dimensional integrable systems should be given as well. The Arnold-Liouville Theorem should be proved, and its significance (and geometric consequences) should be explained in some examples.

Several examples of integrable systems should be given, some of them should be discussed in some detail, but without proofs or excessive calculations. Particular examples include:

(1) Some spinning tops [Detailed Examples are discussed in [3]]
(2) Geodesics on an ellipsoid [A detailed discussion of this can be found e.g in Moser's book [21], see also Donagi-Markman [8]]

References. Good basic references are Chapter I and II in Audin's book [4] and the article by Dubrovin, Krichever and Novikov [10] (particularly for the infinite dimensional case, see the end of section 1). Other good references are Arnold's book [2] and the beginning of Segal's article in [15]. For the examples look at [3], [21] and [8].

2. Methods for construction of commuting flows – 2 talks (Bohua Zhan, Ilya Vinagradov, Heather Macbeth)

The classical Arnold-Liouville is just an existence theorem. It leaves open the problem of finding or construction commuting flows. This problem is addressed by the modern, more algebraic theory.

The talk should explain the method of Hamiltonian reduction as a way to construct integrable systems. In particular, present the Calogero-Moser system (See Chapter 7 in [5]), and explain how it can be constructed by Hamiltonian reduction (see chapter 1 of Etingof's paper [11], or the original paper [18])

Explain the notion of Lax pairs and revisit some of the classical examples discussed in the previous talks, i.e. write them as Lax equations (discuss this in detail for some examples, and just write the equation for others).

References. For Hamiltonian reduction look at [18], Chapter 1 in [11] and Chapter 7 in [5].

Basic references for Lax pairs are Audin's book [4] and the article by Dubrovin, Krichever and Novikov [10]).

3. KdV theory – 1-2 talks (Jonathan Luk, Shiwu Yang)

Introduce the KdV equation, its Hamiltonian form (see [10], end of section 1, or the book by Dickey [7]) and its Lax form.

Discuss pseudo-differential operators and how they are used to obtain the KdV hierarchy of infinitely many commuting symmetries for the KdV equation (follow e.g. the beginning of Segals article in [15] or chapters $1, 2$ of [20]).

Explain the method of bi-Hamiltonian structures and its application to the KdV hierarchy (see e.g. [7], in particular Chapter 4 and 5). It's also briefly explained in ch. 10 of Babelon [5]).

References. Segal's article in [15], Minor Thesis of Karigiannis [17]. The book of Dickey [7] and the book of Miwa, Jimbo and Date [20]. Another reference is Fadeev [12].

4. Towards Hitchin systems – 2 talks (Chi Li, Guangbo Xu, Rodolfo Rios Zertuche, Runpu Zong)

Explain the connection of solutions of Lax equations to line bundles over Riemann surfaces (following Hitchin's article in [15])

Construct the Hitchin systems following [16]. (This requires some background about moduli of vector spaces. Do not care about stability issues, reduce to nice open subset.)

Explain how the Calogero-Moser system arises as a quotient of the Hitchin system [see Chapter 7 in Babelon-Bernard-Talon [5] or also the articles by Gorsky and Nekrasov [13, 14].]

References. For the relation of Lax Equations to Riemann surface follow Hitchin's article in [15], this is also discussed in Chapter IV in Audin's book [4]. Another related nice paper might be Dubrovin [9].

For the construction of Hitchin systems look at [16] or [8] for a more algebraic treatment. One might also consult the rather inaccessible Beilinson-Drinfeld article (2.4) [6].

For the Calogero-Moser system consult Chapter 7 in [5] and [13, 14]

5. Pole solutions – 1 talk (Michael McBreen)

Explain the relation between the elliptic Calogero-Moser system and the poles of elliptic solutions for the KP hierarchy. References are section 7.10 of Babelon's book [5] and the papers [1, 19].

References

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