Homework problem set 13

Submission deadline on 31 January 2022 at noon

Problem 1 (M-theory)**.** In the lecture you will soon learn about superstring theory, which by arguments similar to bosonic string theory should live in $D = 10$ dimension. There are actually 5 types of super string theories: Type I, Type IIA, Type IIB, and 2 types of heterotic string theories. Type IIA super string theory admits stable D0-branes that at small string coupling behave as massive particles.

(i) The mass m_0 of a single D0-brane depends on string coupling, g_s and string length, $l_s = \sqrt{\alpha'}$, in the same way as in the bosonic string. Write down this dependence.

(ii) A set of n D0-branes is able to form a bound state of mass exactly equal to nm_0. At strong coupling (large g_s), these bound states become light and are interpreted as Kaluza-Klein momentum states associated with a compactified **eleventh** dimension. Assuming this interpretation, write down the relation between the radius R_{11} of that additional circle, string coupling, and string length.

(iii) The 11-dimensional theory (of which Type IIA string theory is the Kaluza-Klein reduction) is known as M-theory. This theory includes gravity and its interactions are governed by an eleven-dimensional Newton constant $G^{(11)}$, or alternatively, the fundamental Planck length $l_P = l_P^{(11)}$. Using (ii), express g_s and l_s in terms of l_P and R^{11}.

Remark: You don't need any knowledge about supersymmetry to solve this problem.

Problem 2 (D-brane bound states)**.** (i) Consider a D2 brane on a rectangular 2-torus, (i.e. the brane fills the Torus). We know from the lecture, that upon performing T-duality in x_1-direction, we get a D1-brane along the x_2-direction. Now suppose there is a non-vanishing magnetic field $F_{12} = B$ on the D2-brane. Describe the boundary conditions in presence of the magnetic field. Show that under T-duality this corresponds to a D1 brane with some relative angle α to the x_2-cycle.

(ii) The mass of a static D-brane is given by it's mass and it's volume and in this situation the Lagragian is the negative of the rest brane rest energy. Consider a static D1-brane along the diagonal of a rectangular 2-Torus. Show that the Lagrangian for the T-dual D2-brane is

$$L = -V_p T_p(g)\sqrt{1+(2\pi\alpha' B)^2} \tag{1}$$

This is a special case of the Dirac-Born-Infeld Lagrangian density

$$\mathscr{L} = -T_p(g)\sqrt{-\det(\eta_{mn} + 2\pi\alpha'\mathscr{F}_{mn})}.$$

(iii) Consider a string with mixed (DN) boundary conditions in x_1-x_2-directions and (DD) boundary conditions in all other directions, i.e. the string is fixed in space at $\sigma = 0$, say at $x = 0$ while the end at $\sigma = \pi$ can move in the x_1-x_2-plane. Describe the D-brane configuration for this string.

(iv) Compactify the x_1-x_2-directions on squared T^2 and apply T-duality to the circle in x_1 direction. What D-brane configuration do we arrive at? Argue that the resulting state will form a bound state and describe the lowest energy state. Use T-duality once more to get back to the original description and describe the final state there, i.e. the D-brane configuration with .