Homework problem set 11

Submission deadline on 17 January 2022 at noon

Problem 1 (Orientifolds). In the lecture we have so far discussed only the so-called oriented string theory. This means that there is a clear distinction between left- and right-movers for the closed string and between the two endpoints at $\sigma = 0$ and $\sigma = \pi$ for the open string.

(i) Show that even after gauge fixing the classical worldsheet action is invariant under the parity transformation

$$\tau \to \tau, \quad \sigma \to \pi - \sigma$$

(ii) At the quantum level this so-called orientifold symmetry is implemented via a unitary operator Ω acting on the string field as

$$X^\mu(\tau,\sigma) \to \Omega X^\mu(\tau,\sigma)\Omega^\dagger = X^\mu(\tau,\pi-\sigma).$$

Determine the action on the modes for the closed string and the open string with all possible boundary conditions.

(iii) One can define the unoriented string theory by keeping only those states in the string spectrum which are invariant under the above orientifold action. By a subtle quantum constraint (tadpole cancellations) in the full theory, one can show that in the closed sector the consistent phase of Ω acting the vacuum is +1. In the open string it turns out that +1 and -1 are consistent.
Determine the spectrum at the first excited level for the the closed string and for the open string ending on a single D-brane in the NN and DD sector.

(iv) Now consider the open string for a stack of N coincident D-branes. Depending on the phase, the orientifold action acts on the vacuum as

$$\Omega|0,p,rs\rangle = \pm|0,p,sr\rangle$$

i.e. the Chan-Paton factors are exchanged. What does this mean for the states kept at the first excited level for the two signs? How many states are kept? Guess the corresponding gauge groups on the D-branes.

Problem 2 (Possion resummation). Let $f \in \mathscr{S}(\mathbb{R})$ be a Schwarz function on the real line $\mathbb{R}$. Recall that the space of Schwarz functions has the important property, that the Fourier transform F is an automorphism on $\mathscr{S}(\mathbb{R})$.

(i) Proof that

$$\sum_{n=-\infty}^{\infty} f(x+n) = \sum_{n=-\infty}^{\infty} \hat{f}(n) e^{2\pi i n x} \tag{1}$$

where $\hat{f} = F(f)$ is the Fourier transform.
Hint: Construct a periodic function from f and use that it can be written as a Fourier series.

(ii) Consider the theta function

$$\theta(\tau) = \sum_{n=-\infty}^{\infty} e^{-\pi n^2 \tau} \tag{2}$$

where $s > 0$. Prove that $\theta(\tau)$ satisfies the functional equation.

$$\theta(\tau) = \frac{1}{\sqrt{\tau}}\theta\left(\frac{1}{\tau}\right). \tag{3}$$