Lecture 25 Heterosis (goal: 5 consistent superstrings) 8.2.2022

Last time: basic ingredients for definition of (Hilbert space of) superstring theories.

- super Virasoro as gauge algebra
- GSO projection

To wit: • $c_{crit} = 15 = \frac{3}{2}\hat{c}$ $\rightsquigarrow$ $D = 10$ (in most standard construction)

- left- and right-movers are independent (except for level matching).

- In NS sector ground state $|p\rangle_{NS}$ and first excited state $b^{\mu}_{-1/2}|p\rangle_{NS}$ have opposite $(-1)^F$ fermion number $\rightsquigarrow$ It is possible (but very drastic) to project out spacetime tachyon, keep massless states, at the expense of introducing (b/c modular invariance)
- The R sector, whose ground states are massless and in representation of Clifford algebra

$$[b_0^{\mu}, b_0^{\nu}] = \eta^{\mu\nu}$$

<u>Fact.</u> · For $D=10$, the smallest irrep S is 32-dimensional over $\mathbb{R}$ and splits into $S = S^+ \oplus S^-$ by spacetime chirality $= (-1)^F$ as irreps of Lorentz gp

$$J^{\mu\nu} = [\gamma^\mu, \gamma^\nu]$$

· Namely wrt basis labels $u = 1, .. 16$ for S^+ and $v = 1, .. 16$ for S^-, we have block forms

$$\gamma^\mu = \begin{pmatrix} 0 & (\gamma^\mu)_u{}^v \\ (\gamma^\mu)_v{}^u & 0 \end{pmatrix} \qquad \gamma^{11} = \gamma^0 \cdots \gamma^9 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$J^{\mu\nu} = \begin{pmatrix} J^{\mu\nu}{}_u{}^u & 0 \\ 0 & J^{\mu\nu}{}_v{}^v \end{pmatrix}$$

and everything real though with $\exp(2\pi i J^{ij}) = -1$

· Namely, the Ramond ground states behave like spacetime fermions, with chirality depending on GSO projection

· These considerations are (always correct, and) relevant for finding (massless) spectrum in old covariant quantization. Assuming S^+ for definiteness, the physical state condition on the 16 Rgs $|p,u\rangle$ is

$$(G_0)_v{}^u |p,u\rangle = 0 \qquad p^2 = 0$$

where $$(G_0)_v{}^u = \left(\sum \alpha^{\mu}_{p}\, b_{-p\,\mu}\right)_v{}^u = \underbrace{p_\mu \gamma^\mu{}_v{}^u}_{\text{matrix}} + \underbrace{\cdots}_{\text{scalar}}$$

$p^2=0$ follows from $G_0^i = L_0 = H = 0$

In rest frame $p_\mu = (p_0, p_9 = p_0, 0, \ldots 0)$ this says

$$\Gamma^- = \gamma^0 + \gamma^1 \, |p, u\rangle = 0$$

i.e. only (an even number of, b/c GSO) $\Gamma^{2,3,4,5}$ are excited.

$\rightsquigarrow$ massless physical states in Ramond sector form 8-dim rep of massless little gp. $SO(8)$, denoted 8_{S^+}.

NB: · As irreps of $SO(8)$ $8_{S^+} \neq 8_{S^-} \neq 8_V$ are all distinct though related by so-called triality, fundamental for superstring

· In lightcone gauge, we obtain directly only 8 physical states. From now on, think in these terms.

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① Open string spectrum

$$\left.\begin{array}{lll} \underline{NS:} & 8_V: & \text{gauge boson} \\ \underline{R:} & 8_{S^+}: & \text{M-W fermion} \end{array}\right\} \text{physical states of 10d } \mathcal{N}=1 \text{ vector multiplet.}$$

Namely, we have equal number of and symmetry between bosonic and fermionic degrees of freedom in spacetime, no tachyon.

CELEBRATE!

② Oriented closed $\mathcal{N}=(1,1)$ superstring

- Through modular invariance, the GSO projection that removes the NS-NS tachyon and keeps

$$b^{\mu}_{-1/2}\, b^{\nu}_{-1/2}\, |p\rangle_{NS} \quad : \quad 64 = \underset{g_{\mu\nu}}{35} + \underset{B_{\mu\nu}}{28} + \underset{\phi}{1} \quad \text{dofs}$$

requires the presence in $\mathcal{H}_{\text{string}}$ of an NS-R and R-NS sector

- The physical massless $(-1)^F = 1$ and level matched NS-R states are of the form

$$b^{\mu}_{-1/2}\, |p, \tilde{u}\rangle_R \qquad p^2 = 0 \qquad \Gamma^{\mu}_{\ v}\, u^{v} = 0.$$

and decompose under $SO(8)$ as

$$8_V \cdot 8_{S^+} = 64 = 8_{S^-} + 56_{S^-}$$

where 8_{S^-} – Majorana Weyl fermion

while 56_{S^-} are physical states of a spin $\frac{3}{2}$ massless spin $\frac{3}{2}$ vector spinor or Rarita-Schwinger field.

Spacetime considerations (interactions!) show that this is consistent only when spacetime supersymmetry is local (gauged), much like spin 2 is only consistent with general covariance.

$\rightsquigarrow$ supergravity

- We are not in a position to describe such theories in any detail. Sketch of kinetic term:

$$\bar{\psi}_\mu{}^v \left(F^{\mu\nu\rho}\right)_v{}^u \nabla_\nu \underbrace{\psi_{\rho u}}$$

vector-spinor. "gravitino"

$$F^{\mu\nu\rho} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho]}{}_v{}^u$$

(cmp $\bar{\psi}^u \gamma^\mu \nabla_\mu \psi_u$ for ordinary fermion)

- turning to R-NS sector, we have a twofold choice: (A) opposite chirality / $(-1)^F$ assignment as NS-R
(B) same

<u>(A)</u>: $\tilde{b}^\mu_{-1/2} \, |p, v\rangle_R \rightsquigarrow 8_V 8_{S^-} = 8_{S^+} + 56_{S^+}$

$\rightarrow$ 2 gravitini of different chirality

$\rightsquigarrow$ Type IIA Supergravity

<u>(B)</u>: $\tilde{b}^\mu_{-1/2} \, |p, u\rangle_R \rightsquigarrow 8_{S^-} + 56_{S^-}$

$\rightarrow$ 2 gravitini of same chirality

$\rightsquigarrow$ Type IIB Supergravity

After that, no further choices in RR sector.

(A) $|p,v,\tilde{u}\rangle$ are 64 physical states that decompose under $SO(8)$ as

8_v $\quad \gamma^{\mu}{}_{v}{}^{\tilde{u}}|p,v,\tilde{u}\rangle \longrightarrow$ vector $C^{(1)}_{\mu}$.

$56_v = \frac{8\cdot7\cdot6}{6}$ $\quad \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho]}{}_{v}{}^{\tilde{u}}|p,v,\tilde{u}\rangle \longrightarrow$ rk 3 antisymmetric tensor $C^{(3)}_{\mu\nu\rho}$.

(B) $|p,v,u\rangle$ are 64 physical states that decompose as

1 $\quad \delta_u{}^v |p,v,u\rangle \rightsquigarrow$ scalar $C^{(0)}$

28 $\quad \gamma^{[\mu}\gamma^{\nu]}|p,v,u\rangle \longrightarrow$ 2-form $C^{(2)}_{\mu\nu}$

$35 = \frac{8\cdot7\cdot6\cdot5}{2\cdot24}$ $\quad \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma]}|p,v,u\rangle$ + self-duality

$\longrightarrow$ self-dual 4-form $C^{(4)}_{\mu\nu\rho\sigma}$

$$dC^{(4)} = * dC^{(4)},$$

$\rightarrow$ The massless (bosonic) fields from Ramond-Ramond sector are (spin 1, abelian) p-forms, each with their own gauge invariance

Type IIA $\quad p = 1,3 \quad$ (odd)

Type IIB $\quad p = 0,2,4$ (self-dual!) $\quad$ (even)

- In both cases, all massless fields (with same number 128 + 128 of bosonic + fermionic on-shell dofs) are part of a single irreducible supergravity multiplet.

- There is much more to say.

③ Type I ~~str~~ superstring

- We have stated before that open strings require closed ones for consistency, but not vice-versa. This was not quite correct:

  - oriented closed superstrings are fine by themselves
  - unoriented closed and open depend on each other.

- Viewing unoriented strings as result of "orientifold" projection, one may think of open strings as its twisted sector (left → right at boundary) and by this logic alone understand that their number (i.e. dimension of CP space) is very specifically fixed.

by worldsheet parity left = right, see homework

Fact: The orientifold of Type IIB requires open strings with $SO(32)$ (in fact $Spin(32)$) Chan-Paton space for consistency.

Massless fields

		on-shell dofs
NS-NS	$g_{\mu\nu}$ $\cancel{B_{\mu\nu}}$ ϕ	$35+1$
NS-R = R-NS	$\psi_{\mu\alpha} + \psi_{\alpha}$	$56+8$
R-R	$\cancel{C^{(0)}}$ $C^{(2)}_{\mu\nu}$ $\cancel{C^{(4)}}$	28

$$64 + 64$$

states of $\mathcal{N} = (0,1)$ supergravity multiplet

Open string $\rightarrow \frac{31 \cdot 32}{2} \cdot (8_v + 8_{s^+})$ gauge bosons + gauginos

There is much more to say.

4 The heterotic string

· There is, finally, one more type of construction of consistent string theories without tachyons and (local) spacetime supersymmetry.

<u>Idea:</u> Use closed string worldsheets only, but a different gauge algebra for
left-movers: super-Virasoro
right-movers: Virasoro

· At first sight, the difference in critical central charges

$$c_L \overset{!}{=} 15 \qquad c_R \overset{!}{=} 26$$

make this proposal sound rather dubious, but in fact it is just one of the miraculous features that make this

$\mathcal{N} = (0,1)$ superstring, a.k.a. heterotic string

(and all other superstring theories) not just possible, but well inevitable.

· Further intuition: the left-moving (Ramond) fermions will supply spacetime fermions (and susy) while the GSO projection together with level-matching will remove also the tachyon from right-movers.

For implementation, we use as <u>worldsheet fields</u>

· 10 bosons X^μ $\mu = 0,..9$ with oscillators $\alpha_n^\mu, \tilde{\alpha}_n^\mu$

· 10 left-moving real fermions ψ_+^μ $\mu=0,..9$ oscillators $\tilde{b}_r^\mu$ with Ramond and Neveu-Schwarz sector and GSO projection.

$\rightsquigarrow$ $\tilde{c} = 15$, 10D Poincaré invariance and modular invariance have a chance of working. To patch up $c \stackrel{!}{=} 26$ we add

· 16 "right-moving half-bosons", meaning only their α_n^I $I = 11,..26$ oscillators, and zero mode charges α_0^I in an even self-dual lattice in euclidean! $\mathbb{R}^{16}$. (quasi compactified) unitarity.

<u>Note:</u> · It's not really possible to write actions for thes chiral bosons X_R^I so we won't, but mention that by s.th. known as bosonization one can also work with 32 real right-moving fermions

· Even self-dual lattices ~~exist only for~~ in euclidean $\mathbb{R}^n$ exist only for $n = 0 \mod 8$, so

$$c - \left(\hat{c} = \frac{2}{3}\tilde{c} = 10\right) = 16$$

is a lucky coincidence indeed.

- For $n=16$, there are in fact precisely two even self-dual euclidean lattices, known from Lie theory as

$SO(32)$ and $E_8 \times E_8$ root lattices

where: E_8 : exceptional Lie group of dim 258

root lattice : contains charges of gauge bosons of associated Yang-Mills theory. Notation α, $(\alpha,\alpha)=2$, $\# = 496$.

So that one finds the spacetime massless spectrum

- $\alpha^\mu_{-1} \tilde{\psi}^\nu_{-1/2} |p ; \overset{\alpha_0^I}{0}\rangle_{NS}$ $\quad g,\ C^{(2)}_{\mu\nu},\ \phi$
- $\alpha^\mu_{-1} | p, u ; 0 \rangle_R$ $\quad \psi_{\mu a},\ \psi_a$

$\Bigg\}$ $\mathcal{N}=(0,1)$ SUGRA

$\tilde{\psi}^\mu_{-1/2} |p ; \overset{\text{root}}{\alpha} \rangle_{NS}$ $\quad$ gauge bosons

$|p, u ; \alpha \rangle_R$ $\quad$ gauginos

- $SO(32)$ het has the same massless spectrum as type I, but rather different interactions
- E_8 is interesting for grand unification

There is much more to say.