Lecture 20 D-branes

18.1.22

Purpose: Illustrate (two) further ingredients / techniques for constructing
- phenomenologically interesting string backgrounds
- new backgrounds from old

... albeit all based on free field theory

① Dirichlet

For an open string $(\tau,\sigma) \in \mathbb{R}\times[0,\pi]$ coordinate $X(\tau,\sigma)\in\mathbb{R}$, the possible combinations of free (Neumann) and fixed (Dirichlet) boundary conditions at $\sigma = 0,\pi$ in the variational principle lead to the following conditions on the mode expansion

$$X = X_L(\sigma^+) + X_R(\sigma^-)$$

$$X_{L,R} = \frac{x_0}{2} + \sqrt{\frac{\alpha'}{2}}\,\tilde{\alpha}_0\,\sigma^\mp + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n}{n}e^{-in\sigma^\pm}$$

and zero modes $\alpha_0, \tilde{\alpha}_0$:

Neumann-Neumann $\quad \partial_\sigma X\big|_{0,\pi} = 0 \quad \rightsquigarrow \quad \alpha_n = \tilde{\alpha}_n \quad \forall n\in\mathbb{Z}$

while α_0 related to electric charge / canonical conjugate of x_0:

$$p = \frac{1}{2\pi\alpha'}\int_0^\pi \dot{X} = \frac{1}{\sqrt{2\alpha'}}\,\alpha_0$$

In the full string construction, the conformal weight of scalar $p \in \mathbb{R}$

$$h = \frac{\alpha_0^2}{2} = \alpha' p^2$$

contributes on "left-hand side" of mass-shell condition

$$m^2 = \alpha'(N^\perp - 1).$$

=

Dirichlet-Dirichlet $\quad \partial_\tau X|_{0,\pi} = 0 \quad \rightsquigarrow \quad \tilde{\alpha}_n = -\alpha_n \quad \forall n \in \mathbb{Z}$

~~whilst~~

$$\left.\begin{aligned} X(\tau,0) &= x_1 \\ X(\tau,\pi) &= x_2 \end{aligned}\right\} \text{ part of variational principle!}$$

give

$$x_0 = x_1$$

$$x_2 - x_1 = \sqrt{2\alpha'}\,\alpha_0 \cdot \pi$$

while translational invariance is broken, so there is no Noether charge, and x_0 is non-dynamical.

- In ~~the~~ a full string construction, one will (desire to retain translational invariance at least in light-cone) choose e.g.

NN: $\quad \mu = 0, 1, \ldots p$

DD: $\quad i = p+1, \ldots D-1$

and decompose spectrum in $SO(1,p)$ Poincaré reps, the DD conformal weight

$$h^i = \frac{\alpha_0^{i\,2}}{2} = \frac{(x_2^i - x_1^i)^2}{4\pi^2\alpha'}$$

will contribute on R.H.S. to shift the mass

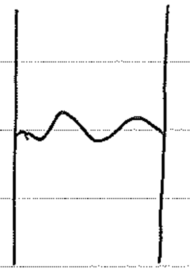
$$m_{p+1}^2 = \frac{|\vec{x}_2 - \vec{x}_1|^2}{4\pi^2\alpha'^2} + \alpha'(N^\perp - 1).$$

(compatible with $m \sim T \cdot L$ from first lecture)

Pictorially,

$x_{\parallel} = (x^0 \dots x^p)$

$x_{\perp} = (x^{p+1}, \dots x^{25})$



and we address "the locus $\{X^i = x^i\} \subset \mathbb{R}^{1,25}$ where open strings end" as a <u>Dp-brane</u>

($p+1$ - dim. subspace)

<u>Dirichlet-Neumann</u> $\partial_\sigma X|_0 = \partial_\tau X|_\pi = 0$

$$\rightsquigarrow \tilde{\alpha}_n = \alpha_n, \quad n \in \mathbb{Z} + \frac{1}{2} \; !!$$

There is no zero mode at all, but (conformal invariance is still O.K.) and Virasoro algebra in this sector has non-trivial conformal weight

$$h_{DN} = \frac{1}{16}$$

from normal ordering. (If one believes that $c=1$ like before, this can be figured from

$$\text{ground-state energy} = h - \frac{c}{24} = \frac{1}{2} \sum_{n \text{ odd}} \frac{n}{2}$$

and

$$\sum_{n\in\mathbb{Z}} n = \sum_{\text{odd}} n + \sum_{\text{even}} n = \sum_{\text{odd}} n + 2 \sum_{\text{all}} n$$

$$\sim \sum_{\text{odd}} n = - \sum_{\text{all}} n = \frac{1}{12} \ ; \quad \frac{1}{24} + \frac{1}{48} = \frac{1}{16} .)$$

In string theory, this arises when Dp-branes intersect Dq-branes in a d-dimensional subspace

	0	1	..	d	d+1	..	p	p+1	..	p+q−d	p+q−d+1	..	25
Dp	×	×	..				×	o	..	o	o	..	o
Dq	×	×	..	×	o	..	o	×	..	×	o		o

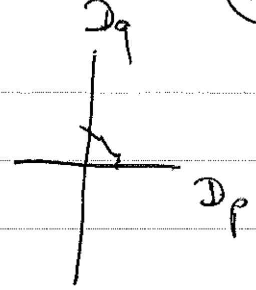
×: extended o: localized

$$m_{d+1}^2 = \sum_{i=p+q-d+1}^{25} \frac{(x_2^i - x_1^i)^2}{4\pi^2 \alpha'^2} + \frac{1}{\alpha'} (N^\perp + a)$$

$$N^\perp = \sum_{n=1}^{\infty} \Big(\sum_{\mu=2}^{d} n N_n^\mu + \sum_{i=p+q-d+1}^{25} n N_n^i$$

$$+ \sum_{i=d+1}^{p+q-d} \frac{n}{2} N^i_{n/2} \Big)$$

$$a = -1 + \frac{1}{16}\underbrace{(p+q-2d)}_{\#\text{ DN coordinates.}}$$



② Chan-Paton

- We have already pointed out that open strings alone will not be consistent because they will in general scatter into closed ones once interactions are included. Technically this arises because of divergences of open string one-loop amplitude.

- For similar reasons, if $\mathcal{H}_{\text{string}}$ contains stretching from Dp-brane at $\vec{x}_1$ to Dq-brane at $\vec{x}_2$, we will also require open strings from Dq to Dp to have any consistent interactions at all. (CPT symmetry.)
- A general open+closed string background ~~will~~ "containing" finite set $\{1, \ldots, N\} \ni I, J$ of D-branes of various dimensions has

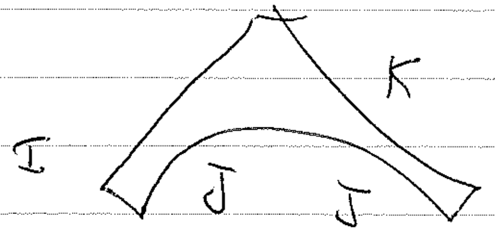
$$\mathcal{H}_{\text{string}} = \mathcal{H}_{\text{closed}} \oplus \bigoplus_{I,J} \mathcal{H}_{I,J}$$

(subject to constraints & further conditions).

- Usually, want branes to intersect ~~pl~~ in $d+1 \geq 4$ dimensions for ~~SO(1,3)~~ ~~Lorentz~~ invariance 4d Poincaré invariance

- Easiest to think of pieces of $\mathcal{H}$ as various "topological sectors", with consistent interactions deduced "by inspection".

E.g. a string diagram of the form



corresponds to scattering $\mathcal{H}_{IJ} \otimes \mathcal{H}_{JK} \to \mathcal{H}_{IK}$

- The massless spectrum for strings stretching from single Dp-brane to itself decomposes according to

$$SO(1,p) \times SO(25-p)$$

symmetry in "global symmetry".

$$\alpha^i_{-1} |p\rangle \qquad p^2_{p+1} = 0 \qquad i = 2, ..., p$$

- $U(1)$ gauge boson

and $\alpha^i_{-1}|p\rangle \qquad p^2_{p+1} = 0 \qquad i = p+1, ..., 25$

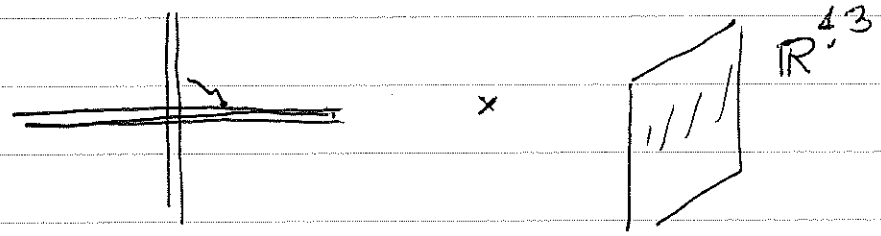
$25-p$ massless scalars.

In the special situation of N coincident Dp-branes

$$\bigoplus_{I,J=1}^{N} \mathcal{H}_{IJ} = \mathcal{H}_{open} \otimes \mathbb{C}^{N^2}$$

and we have N^2 massless vectors, which general field theory considerations require to form non-abelian gauge multiplet.

- In this context, the space $\mathbb{C}^N = \mathbb{C}\cdot\{1,2,\ldots,N\}$ is called Chan-Paton space. It is the simplest way to obtain non-abelian gauge symmetry in string theory.
- The fundamental charges are carried by open string endpoints.
- Typical constructions: "intersecting D-brane models".



extra dimensions

(see Zwiebach for glory details of [a] construction of standard model).

- Apart from this, D-branes also play an important role in string duality and in probing quantum geometry.

③ Polchinski 1

- T-duality between strings on circle $S^1_R = \mathbb{R}/2\pi R \ni X$ and circle $S^1_{R'} = \mathbb{R}/2\pi R'$, $R' = \frac{\alpha'}{R}$ involves exchange of

momentum $p = \frac{1}{2\pi\alpha'} \int \partial_\tau X \, d\sigma = \frac{n}{R}$

and winding $w' = \frac{1}{2\pi\alpha'} \int \partial_\sigma X' \, d\sigma = \frac{mR'}{\alpha'}$

$(w = p')$ and thereby corresponds at the level of worldsheet fields (path-integral variables) to the identification

$$\partial_\tau X = \partial_\sigma X' \qquad \partial_\sigma X = \partial_\tau X'$$

- Needless to say, this implies that at the level of boundary conditions, T-duality must exchange

Dirichlet b.c. $\partial_\tau X|_{\partial\Sigma} = 0$

with Neumann b.c. $\partial_\sigma X'|_{\partial\Sigma} = 0$.

- less immediate is the fate / mapping of background moduli. Solution:

- While Dirichlet boundary conditions include a parameter with values in S^1_R:

$$X|_{\sigma=0} = x_1 \qquad X|_{\sigma=\pi} = x_2 + 2\pi m R$$

which via $\tilde{\alpha}_0 = -\alpha_0$

$$\left(\tilde{\alpha}_0 - \alpha_0\right)\sqrt{\frac{\alpha'}{2}}\pi = x_2 - x_1 + 2\pi m R$$

$$\tilde{\alpha}_0 = \sqrt{\frac{2}{\alpha'}}\frac{1}{2\pi}\left(x_2 - x_1 + 2\pi m R\right)$$

$$= \sqrt{2\alpha'}\left(\frac{mR}{\alpha'} + \frac{x_2 - x_1}{2\pi\alpha'}\right)$$

leads to a physical energy/mass shift

$$\Delta m^2 = h = \frac{\tilde{\alpha}_0^2}{2\alpha'} = \left(\frac{mR}{\alpha'} + \frac{x_2 - x_1}{2\pi\alpha'}\right)^2$$

(changing representative $x_I \in \mathbb{R}$ ~~changes~~ mod $2\pi R$ ~~which~~ is compensated by winding number).

· Neumann boundary conditions allow boundary coupling to flat background gauge fields

$$\int_{\sigma=0} A^{(1)}\, \partial_\tau X^a d\tau - \int_{\sigma=\pi} A^{(2)}\, \partial_\tau X^a d\tau$$

which (does not affect equations of motion, but) similarly to B-field, shifts canonical Noether momentum

$$p = \frac{1}{2\pi\alpha'}\int \dot{X}^a + A^{(1)} - A^{(2)}$$

$$A^{(I)} \to \frac{A^{(I)}}{2\pi\alpha'}$$

$$p = \frac{\alpha_0}{\sqrt{2\alpha'}} + A^{(1)} - A^{(2)} = \frac{n}{R}$$

(this is the quantity that's quantised)

and therefore also contributes to the energy/mass

$$\Delta m^2 = \left(\frac{n}{R} + A^{(2)} - A^{(1)}\right)^2$$

Therefore, the formulas match if together with exchanging

$$R \to R' = \frac{\alpha'}{R} \qquad n \to m'$$

we identify

$$A^{(I)} \to X'_I = 2\pi\alpha' A^{(I)}$$

the periodicity of $A^{(I)} \sim A^{(I)} + \frac{1}{R}$ being explained by the fact that a flat gauge field $A = \frac{1}{R}$ on a circle of radius R can be absorbed by a globally well-defined gauge transformation

$$e^{-i\chi(X)} \qquad \chi(X) = \int_0^X A\,dX$$

$$A = \partial_X \chi$$

$e^{\int A}$ is known as "Wilson line".

$$\chi(2\pi R) = 2\pi.$$

Lessons: The moduli space of flat connections = boundary interactions for Neumann boundary conditions on a circle of radius R is the dual circle of radius $R' = \frac{\alpha'}{R}$ = moduli space of Dirichlet boundary conditions on that circle.

- duality
  - inverts coupling
  - exchanges topological and Noether charges
  - free and fixed b.c.
  - boundary conditions and boundary interactions.

(4) Polchinski 2

- The spectrum of open strings from a D24-brane along 0,1,...24 - directions to itself contains states

$$\alpha_{-1}^{25} |p\rangle \qquad p_{25}^2 = 0$$

of a massless 25-dimensional scalar ϕ

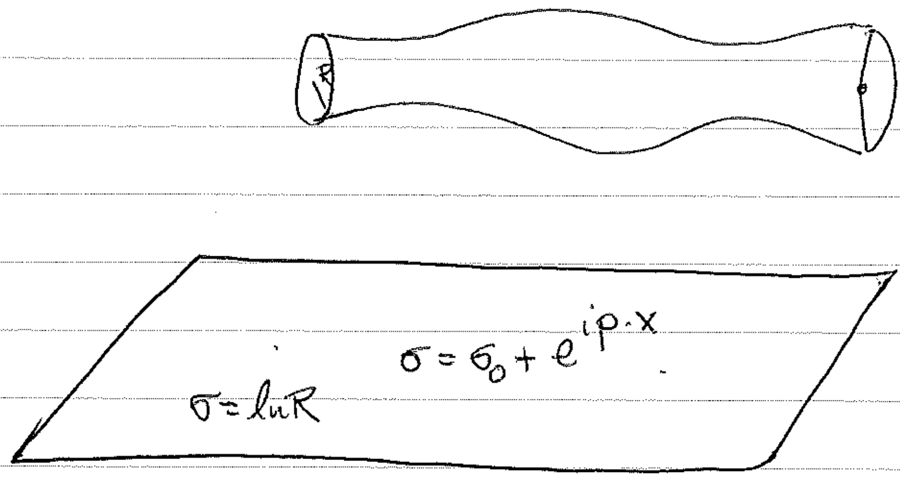
- When X^{25} is compactified on a circle, there is a dual description in which ϕ can be identified as a component

$$\phi = A_{25}$$

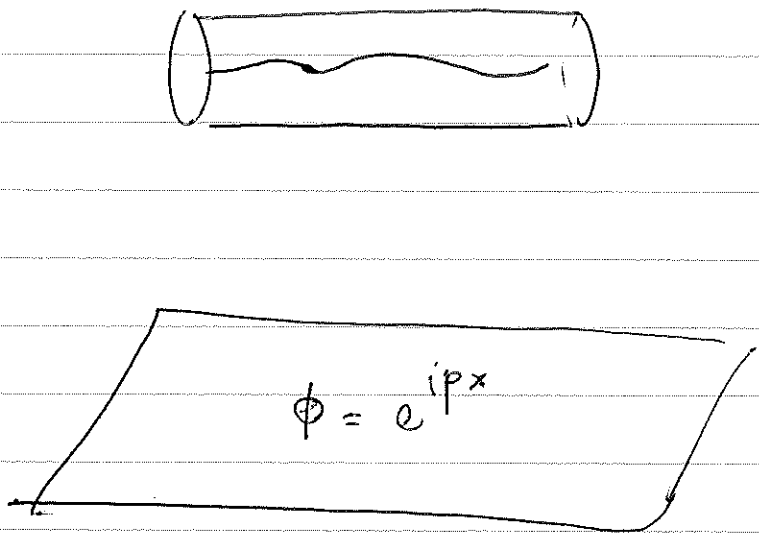
of the 26-dim. gauge field on a ~~D25 brane~~ D25-brane

- The background value of this field is the position of D24, and therefore the fluctuations of the scalar field must be fluctuations of the position of D24.

"KK radion"



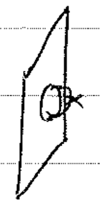
D-brane position



- Decompactifying and generalizing we learn: The $(25-p)$-dimensional vector of massless $(p+1)$-dimensional scalar fields describe transverse fluctuations of the position of a Dp-brane. Namely, a Dp-brane is a <u>dynamical object</u> in string theory.

- From the point of view of symmetry, the scalar fields can be thought of as Goldstone bosons associated with breaking of translational invariance.

- Of particular importance is the fact that the open string disk diagram with a single closed string (graviton) insertion is non-zero (because Conformal Killing group is compact, see end of lecture 17) in fact proportional to



$$\sim \underbrace{g_s}_{\text{normalization of closed string vertex operator}} \overbrace{g_s^{-1}}^{\chi(D)=1} = g_s^0$$

A comparison with $G_N \sim g_s^2$ then reveals that the gravitational tension (energy / volume dim $= p+1$) of a Dp-brane is proportional to g_s^{-1}. (infinitely heavy as $g_s \to 0$, but backreaction still small b/c $G_N \sim g_s^2$.

With ansatz $T_p = \tau_p \cdot h(g_s)$ where τ_p is g_s-independent constant of mass dimension $p+1$ (must be ℓ_s^{-p-1}, because that's the only dimensionful quantity around) and $h(g_s)$ is a p-independent function of string coupling, comparison of effective tension of Dp on circle of radius R with that of $D(p-1)$ on dual circle $R' = \frac{\alpha'}{R}$ on which $g_s' = g_s \frac{\sqrt{\alpha'}}{R}$ reveals

$$\tau_p \, h(g_s) \cdot 2\pi R = \tau_{p-1} \, h\left(\frac{g_s}{R}\sqrt{\alpha'}\right)$$

(with $R = \frac{\sqrt{\alpha'}}{g_s}$) $\qquad h(g_s) = \frac{\tau_{p-1}}{\tau_p} \cdot \frac{h(1)}{g_s \sqrt{\alpha'}}$

and thence

$$\frac{\tau_p}{\tau_{p-1}} = \frac{1}{2\pi\sqrt{\alpha'}}$$

which is conventionally normalized such that

$$T_p = \frac{1}{g_s} \frac{1}{(2\pi)^p \, \alpha'^{\frac{p+1}{2}}}$$

In particular

$$T_1 = \frac{1}{g_s \cdot 2\pi\alpha'} = \frac{1}{g_s} T$$

tension of fundamental string.

5 Tachyons

The fact that D-branes are dynamical has many intriguing consequences:

· D-branes can not only be "part of background data", but also have to be included in "Hilbert space of states".

· For example, a D1-brane wrapped on a circle of radius R should be thought of as a particle in 25-dimensions of mass $m_{25} = \frac{R}{g_s \alpha'}$. (necessarily not visible in perturbation theory)

· Classically, the dynamics of D-branes can be captured by action principles that generalize Nambu-Goto action

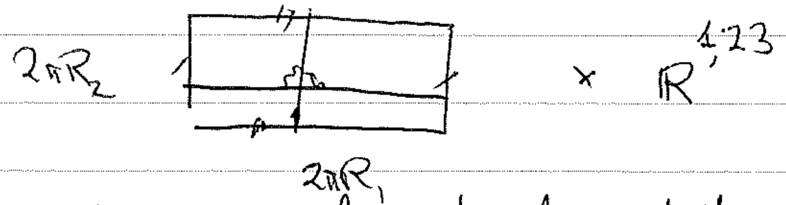
$$S_{Dp} = T_p \int_\Sigma d^{p+1}\sigma \sqrt{-\det \partial_\alpha X^\mu \cdot \partial_\beta X_\mu}$$

(+ other terms to account for gauge fields etc.)

but the full correct quantum-mechanical description is <u>not a field theory at all</u>, but rather <u>a theory of open strings itself</u>.

· What this means mathematically is hard to say, but in practice a combination of heuristic arguments, geometric pictures and consistency checks (dualities) is sufficient to convince any person of good faith.

· As an example, consider 2 D1-branes wrapped along the two sides of a rectangular 2-dimensional torus



and ask for dynamics of interactions between 24-dim. particles of mass

$$m^{(I)}_{24d} = \frac{1}{g_s} \frac{R_I}{\alpha'}$$

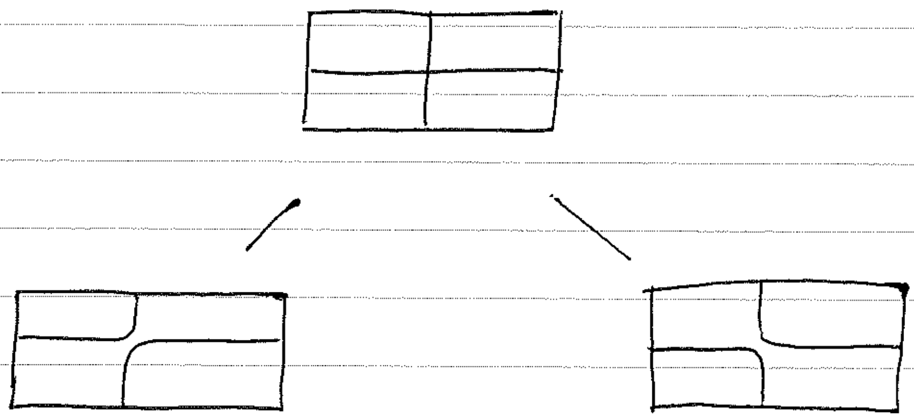
· According to previous discussion, the Hilbert space of open strings (which we argued to be the worldline theory of particles) stretching between the two D-branes contains a "scalar" of 0+1-dimensional mass

$$\mathcal{H}_{12}, \mathcal{H}_{21} \supset m_1^2 = \frac{1}{\alpha'}\left(\frac{1}{8} + \frac{|\vec{d}_{12}|^2}{4\pi^2\alpha'} - 1 \right)$$

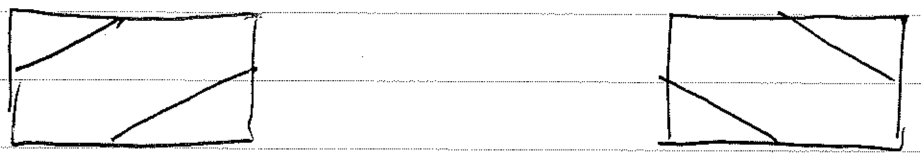
(~ 23-dim. distance)

i.e. tachyonic when the two particles come close.

· From the picture and the fact that scalars are localised at intersection pt. on T^2, one is led to identify the instability of the 0+1-dimensional theory signalled by the tachyon with the recombination.



which will further settle down into configuration of lower energy



corresponding to particles of mass $m^{new}_{24} = \frac{1}{g_s} \frac{\sqrt{R_1^2+R_2^2}}{\alpha'}$ presumably by radiating away energy in closed string modes.

Double lesson:

- D-branes can form bound states
- The (open string) tachyon (instability) can be given a geometric/physical meaning.

N.B.:

- Further corroboration from T-dual picture
- In superstring, D-branes are oriented, only one of the two recombinations is possible.