

Seminar (Homological) Mirror Symmetry

Wintersemester 2019/20

Description

CONTENT:

Mirror symmetry is one of the oldest and best understood dualities that have emerged from string theory and has had a profound impact in certain areas of pure mathematics. The original statements, motivated by Conformal Field Theory, concerned certain enumerative questions in algebraic geometry and their solution in terms of Hodge theory. Soon after, two mathematical formulations were put forward: Kontsevich's Homological Mirror Symmetry interprets the duality as an equivalence of symplectic and algebraic categories. The study of torus fibrations initiated by Strominger, Yau and Zaslow gives an explicit geometric correspondence. Today, mirror symmetry remains an extremely active research field, reaching in influence far beyond its original formulation as a duality between Calabi-Yau manifolds, to such subjects as representation theory, singularity theory, and knot theory.

This seminar will trace mirror symmetry from its origins to some modern developments. One of our main goals will be to understand Nick Sheridan's 2011 proof of homological mirror symmetry for the quintic threefold.

AUDIENCE:

The seminar is aimed at Masters students in mathematics and mathematical physics. Participants should have a solid background at least one of symplectic geometry, algebraic geometry or quantum field and string theory, with nodding acquaintance of the other two.

EVALUATION:

This seminar is listed in the LSF as "Masterpflichtseminar" in Physics and as "Master-Seminar" in Mathematics. To receive credit in the Physics program, you have to prepare a 6–10 page write-up of your talk, and the final grade will be based (in roughly equal amounts) on both the talk and the writeup. The Math program does not require a write-up, though of course you are welcome to provide one.

ORGANIZATION

The seminar will meet on Tuesdays at 14h30 (c.t.) in Mathematikon SR 2. If you were unable to attend the organizational meeting (on July 22) but would like to participate in the seminar, please send us an email.

Talks

General idea: Each session will be devoted to a fairly broad topic that should be split among two (or more) responsible speakers. The keywords listed below are indicative only, and should be adapted to the own previous knowledge and to the material covered by previous speakers. Each team will be tutored by one of the organizers that you should contact no later than one week in advance to cross-check content and scope.

T-DUALITY OF OPEN AND CLOSED STRINGS

The free boson in 2 dimensions on a circle, momentum and winding modes; Neumann and Dirichlet boundary conditions; path-integral point of view

References: [Polchinski], [BigBook]

Date: October 15

Speaker: Benjamin Haake, Fabian Hahner

Tutor: Ingmar Saberi

KÄHLER GEOMETRY

Compatible complex and symplectic structures; deformation of complex structures; Yau's theorem; moduli spaces.

References: [BigBook], [CoxKatz]

Date: October 22

Speaker: Levin Maier, Sebastian Nill

Tutor: self

MIRROR SYMMETRY FOR T^2

Sigma models, complexification of moduli, momentum and winding lattices; mirror symmetry

References: [BigBook]

Date: October 29

Speaker: Tristan Daus, Aarya Gaurav

Tutor: Lukas Hahn

$\mathcal{N} = (2, 2)$ SUPERSYMMETRY IN 2-DIMENSIONS

Supersymmetry, superfields, supersymmetric actions; sigma models; Landau-Ginzburg models; gauged linear sigma models

References:

Date: November 5

Speaker: Tobias Witt, David Korbany

Tutor: Johannes Walcher

TORIC GEOMETRY

Batyrev-Borisov mirror symmetry

References: [BigBook], [CoxKatz]

Date: November 12

Speaker: Niklas Jost, N.N.

Tutor: self/Johannes Walcher

HORI-VAFA PROOF OF MIRROR SYMMETRY

Gauged linear sigma model; deformations; generation of superpotential

References: [BigBook], [HoriVafa]

Date: November 19

Speaker: Michael Bleher, Lukas Hahn

Tutor: self

D-BRANES

Overview of D-branes in string theory and mirror symmetry; Chern-Simons gauge theory as a string theory; physics of categories

References: [Aspinwall], [Douglas], [Witten93]

Date: December 3

Speaker: Johannes Walcher, Ingmar Saberi

Tutor: self

DERIVED CATEGORIES

Review of homological algebra; derived categories of coherent sheaves; categories of matrix factorizations

References: [GelfandManin], [Thomas]

Date: December 10

Speaker: Fabio Schindwein, N.N.

Tutor: Ingmar Saberi

FUKAYA CATEGORIES

Strong and weak A_∞ structures; moduli of disks;

References: [FOOO], [Auroux1], [Auroux2], [Fukaya], [Aspinwall]

Date: December 17

Speaker: Menelaos Zikidis, N.N.

Tutor: Lukas Hahn

HOMOLOGICAL MIRROR SYMMETRY FOR THE ELLIPTIC CURVE

θ -functions; vector bundles on elliptic curve; Fukaya category of torus; equivalence between the two

References: [Polishchuk-Zaslow]

Date: January 7

Speaker: Johanna Bimmermann, Steffen Schmidt

HOMOLOGICAL MIRROR SYMMETRY FOR PROJECTIVE SPACE

References: Seidel, Auroux-Katzarkov-Orlov

Date: January 14

Speaker: Adnan Demirci, N.N.

HOMOLOGICAL MIRROR SYMMETRY FOR THE QUINTIC

References: [Seidel], [Sheridan]

Date: January 21

Speaker: N.N.

REFERENCES

- [BigBook] Hori et al., “Mirror Symmetry,”
- [CoxKatz] D. Cox, S. Katz, “Mirror Symmetry and Algebraic Geometry”
- [FOOO] K. Fukaya, Y.-G. Oh, H. Ohta, K. Ono “Lagrangian Intersection Floer Theory (Part 1)”
- [Fukaya] K. Fukaya “Floer homology and Mirror symmetry (Part 1)”
- [Auroux1] D. Auroux “A beginner’s introduction to Fukaya categories”
- [Auroux2] D. Auroux “Lecture Notes on Mirror Symmetry”
- [Aspinwall] P. S. Aspinwall “D-Branes on Calabi-Yau Manifolds”
- [Douglas] M. Douglas “D-branes, categories, and $N = 1$ supersymmetry”
- [Witten93] E. Witten “Chern-Simmons gauge theory as a string theory”
- [GelfandManin] S. Gelfand, Yu. Manin “Methods of Homological Algebra”
- [Thomas] R. P. Thomas “Derived categories for the working mathematician”
- [Polishchuk-Zaslow] A. Polishchuk, E. Zaslow “Categorical mirror symmetry: the elliptic curve”
- [Seidel] P. Seidel “Homological mirror symmetry for the quartic surface”
“Fukaya categories and Picard-Lefschetz theory”
- [Sheridan] N. Sheridan “Homological Mirror Symmetry for Calabi-Yau hypersurfaces in projective space”