SUPERGEOMETRY AND SUPERGRAVITY HW 3

HEIDELBERG UNIVERSITY 2017

***Problem* 1** (Central charges and BPS bound)**:** Consider supersymmetric quantum mechanics with Hamiltonian H and 2 complex supercharges Q_1, Q_2 with non-trivial commutators

$$\{Q_1, Q_1^\dagger\} = H \qquad \{Q_2, Q_2^\dagger\} = H \qquad \{Q_1, Q_2^\dagger\} = Z \tag{1}$$

where Z is a *complex* central charge. Show that

$$H \geq |Z| \tag{2}$$

and describe all irreducible representations of the supersymmetry algebra. *Hint:* Use that $Q_\alpha := Q_1 + e^{i\alpha} Q_2$ satisfies $\{Q_\alpha, Q_\alpha^\dagger\} \geq 0$ for all α.

***Problem* 2** (Supersymmetry algebras)**:**

(a) Give the structure constants describing the extension of $\mathfrak{p} = \mathrm{so}(1,3) \oplus V$ to the $4d$ $\mathcal{N} = 1$ supersymmetry algebra $\mathcal{S} = \mathfrak{p} \oplus S$ with respect to a suitable *real* basis of $S \cong \mathbb{R}^4$. *Hint:* You may either start with a real ("Majorana") representation of $Cl(V)$, or work backward from the presentation

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \tag{3}$$

of $\mathcal{S}$ in terms of the complex basis Q_α of S (and conjugate basis $\bar{Q}_{\dot\beta}$ of $\bar{S}$) given in class. Also rewrite the invariant $\epsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ in the real basis.

(b) Repeat the last part of the exercise in 5d $\mathcal{N} = 1$ to give an explicit description of the real central charge: By writing $so(1,4)$ as a subalgebra of $\mathrm{Mat}(2, \mathbb{H})$ acting on $S \cong \mathbb{H}^2$, identify the symmetric pairing $S \times S \to \mathbb{R}$ as the real part of a (quaternionic) hermitian form on $S \cong \mathbb{H}^2$. *Hint:* You may use the description of $\mathrm{so}(1,5) \cong \mathrm{sl}(2, \mathbb{H})$ given in class, or start from $\mathrm{so}(1,3) \cong \mathrm{sl}(2, \mathbb{C}) \subset \mathrm{Mat}(2, \mathbb{H})$ and work upwards. Gamma-matrices would be fine too.