Seminar im Sommersemester 2022

Die mathematische Unausweichlichkeit der

Quantenmechanik

Beschreibung

Die Quantenmechanik beherrscht seit bald 100 Jahren die Grundlagen unseres Verständnisses der Naturbeschreibung. Ziel dieser Veranstaltung ist die Offenlegung einiger *mathematischen* Resultate, die ihre *Alternativlosigkeit* als grundlegende physikalische Theorie beleuchten. Hierzu zählen Sätze aus der Analysis, zur Geometrie des Zustandsraums, der klassischen Alternative, und der Darstellung von Symmetrien.

Voraussetzungen

Dieses Seminar richtet sich an fortgeschrittene Studierende in den Bachelor-Studiengängen Physik und Mathematik. Kenntnisse der Quantenmechanik auf dem Niveau von PTP4 sind unabdingbar, eine Wertschätzung mathematischer Präzision ist erwünscht.

Anrechnung

Die Veranstaltung zählt als "Pflichtseminar" im Physik Bachelor und als "Proseminar" im Bachelor Mathematik. Die Unterrichtssprache ist Deutsch, der Vortrag auf Englisch wird nahegelegt.

Organisation

- Das Seminar trifft sich dienstags, von 16-18h im INF 227 / SR 3.404.
- Dozenten und Tutoren: Prof. Walcher, Dr. Noja, S. Schmidt
- Homepage: https://www.mathi.uni-heidelberg.de/˜walcher/teaching/sose22/qm/

AG Mathematische Physik

Background

The purpose of this class is to give the participants the opportunity to reflect on the formalism of quantum mechanics as it is taught in the introductory experimental and theoretical courses, specifically PTP4@Heidelberg, in order to prepare or complement its applications in various circumstances. This includes questions regarding

· what the formalism says and why it makes sense;

· how quantum mechanics differs fundamentally from classical mechanics;

· what mathematics is involved, and why;

· why, given all of the above, quantum mechanics is, in some sense, *inevitable*.

Therefore, in order to benefit from the course, one requires (in addition to PTP4 or equivalent)

· a certain interest in "foundational matters";

· an appreciation for mathematical precision, ideally developed in some sort of math courses that go beyond HöMa 2&3.

Foundational remarks

· One of my main contentions is that the foundations of quantum mechanics are, in a well-defined sense, not at all shaky as one is tried to make believe in certain corners of the internet, some popular press, and even some philosophy or physics departments.[1]

· In particular, what is often called the "measurement problem" is, with the right point of view, at worst a pseudo-problem, and at best a somewhat unfortunate name for the completely well-defined, if rather technical, problem of "cashing in" probabilities in the classical limit. In a similar vein, there are no "interpretative issues" that cannot be resolved by taking a reasonable attitude towards physics in general, and certainly no need for "alternative interpretations".

· These remarks are *not* to say that there is no deep message to quantum mechanics, but rather to point out that statements to the effect that "we do not understand quantum mechanics" do not attest to the incompetence or intellectual laziness of the elder (Heisenberg, Bohr, Born, Pauli, Schrödinger, even Einstein, Feynman, and others) but just of the author themselves.

· And whilst these remarks suffice for them to not take center stage in this course (lest it be about nothing), they are not meant to stifle the discussion about that deeper meaning in any way, because it can be damn confusing indeed.

[1] I cannot, in this context, refrain from carrying forward the sentiments expressed at the beginning of the recent review https://arxiv.org/abs/1905.06603.

Mathematical remarks

So, the idea of the course is to lay bare the ideas of quantum mechanics by collecting a series of mathematical statements and theorems, which is fun. Still, it is a physics course, and each talk should lead with the physics—why?—questions as much as possible, and aim for mathematical generality only insofar as it can be justified by physics. Yet again, if at any point you despair over that meaning, you are welcome to take refuge in the mathematics, and we will be happy to translate.

Practical matters

• There are, in my opinion, more ways to give good talks than bad. (Here is one of them.) So relax.

• Formally, the PSEM requires a 45 min. presentation, and no write-up. You should however expect that questions and other imponderables will slow you down, and that we will spend a good 60 to 90 mins. together in each session. (Some sessions might feature several talks, depending on the number of participants.)

• To receive credit as "Proseminar" in the Mathematics Bachelor, you need to provide a 5 to 10 pages write-up of your talk.

• As remarked above, each talk should include a physics part that gives the motivation, theoretical prerequisites, and possibly experimental input, and a math part that defines the relevant objects, gives a precise statement of the main theorem, and possibly a sketch of the proof.

• Each talk is assigned a tutor to help in the identification of relevant references, in the clarification of any technical questions, and in the organisation of the material for presentation. It is important that you start on the literature immediately, get in touch with your respective tutor early on, and be essentially ready no later than two weeks before your actual talk.

• It is a mandatory common sense curtesy that you attend all talks by other participants both before and after your own.

Structure

The topics are grouped around the following four themes:

I. Functional analysis (spectral theorem motivated by canonical commutation relations and related structures)

II. Representation of symmetries (Wigner's theorem etc.)

III. Geometry of state space (Bell inequalities, entanglement, etc.)

IV. The classical alternative (from the algebra of elementary observables to decoherence) and the talks will be organized more or less in this order. In essence however, each talk is basically self-contained and should depend only to some limited degree on the others.

Talks

To lighten the mood, we recommend the classic "Quantum mechanics in your face" by Sidney Coleman.

STATES&OPERATORS

General properties of quantum systems. Formalism of the finite-dimensional case. Observables as PVM's and spectral decomposition. Generalization to POVM's and measuring operators. (Give a physical example for the difference between PVM and POVM.) Formalism for the infinite-dimensional case, brief treatment of Schrödinger representation.

Guiding reference is [MorSC], Chapter 1 and 2. Compare with the associated treatment in [MorSpQM]. For observables as PVM's we refer to [MorSpQM], Chapter 7.5.1, and for the notion of POVM's we refer to [MorSpQM], Chapter 13.2.2.

References: [Mor], [MorSC], [MorSpQM]

Date: 26. April 2022

Speaker: Paula Heim

Tutor: S.S.

WIGNER'S THEOREM

Symmetries of a quantum mechanical system in the sense of Wigner. Unitary operators and symmetries: Wigner's Theorem and its proof. Dual action of symmetries on observables and their physical interpretation (with examples). Dan Freed's point of view: quantum mechanics, projective geometry and the Fubini-Study metric.

Guiding reference is [Mor], Chapter 7.1 and [Freed]. Alternatively, one may follow [Wein]. Further material, with a very detailed mathematical treatment can be found in [MorSpQM], Chapter 12.1: see in particular, the Sections 12.1.4 and 12.1.5 and 12.1.6. A very profound (and advanced) discussion can be found in [LanFQT], Chapter 5.

References: [Mor], [LanFQT], [MorSpQM], [Wein]

Date: 3. May

Speaker: Charles Praum

Tutor: S.N.

GALILEI-HEISENBERG-WEYL AND CCR

From Wigner's theorem to unitary projective representations. Representations of: the Galilei group, the Weyl-Heisenberg Lie group and Lie algebra, canonical commutation relations.

Our guiding references are [Fol], Chapter 1.1-1.3, and [MorSpQM], Chapter 11.4.1 and 11.5.1-11.5.3. See [Ros] or [Var2] for background information and further reading.

References: [Fol], [MorSpQM], [Ros], [Var2]

Date: 10. May

Speaker: Richard Pospich

Tutor: S.S., S.N.

STONE-VON NEUMANN THEOREM

Stone-von Neumann theorem with a heuristic argument. Uniqueness of the canonical commutation relations. The problem of a time operator and Pauli's principle. Ambiguities for an infinite number of degrees of freedom.

An overview and introduction with selective history can be found in [Ros]. Our guiding references are [Fol], Chapter 1.5, and [MorSpQM], Chapter 11.5.4-11.5.5 and 11.5.7. A heuristic argument can be found in [Hall], Chapter 14.1. For Pauli's principle we refer to [MorSpQM], Chapter 13.2.1.

References: [Fol], [Hall], [MorSpQM], [Ros]

Date: 17. May

Speaker: Nawder Stokes

Tutor: S.S.

SYMMETRY CLASSES

Idea: Instead of describing symmetries of a given system, we classify Hamiltonians compatible with a given symmetry group. Main task: Describe Dyson's threefold way (with physical examples of each class), following [Zirn] or other reviews by the same author. Extension to ten-fold way conceivable but not at all necessary at this stage. See e.g. [Moore] for a comprehensive treatment.

References: [Zirn], [Moore]

Date: 24. May

Speaker: Julius Viol

Tutor: S.N.

PURE&MIXED STATES

The geometry of state space for "elementary systems". The Bloch sphere and the existence of non-commuting observables. Basic measurement theory and irreducibility of statistical interpretation.

This talk might be technically the simplest, and conceptually the deepest. Suggested preliminary readings include [Banks], [Suss]. As main reference one may follow [Zyc], Chapters 1&2, Chapter 5.2, and especially Chapter 8.

References: [Banks], [Suss], [Zyc]

Date: 31. May

Speaker: Marieke Steinfatt

Tutor: J.W.

JAUCH-PIRON

Lattices in classical and quantum mechanics. Loomis-Sikorski theorem. The non-boolean logic of quantum mechanics and recovery of Hilbert space structure.

A classic source is [Var1], but all necessary material should be covered in [Mor] Chapter 4. A recent (but otherwise random) ref. is https://arxiv.org/abs/2109.07418v2

References: [Mor], [Var1]

Date: 7. June

Speaker: Moritz Merz

Tutor: J.W.

GLEASON'S THEOREM

Quantum states as probability measures and Gleason's Theorem, following [Mor] Chapter 4.4. Relation to Kochen-Specker theorem following [Mor] Chapter 5.1 . See also [Zyc], Chapter 5.7.

References: [Mor], [Zyc]

Date: 14. June

Speaker: Ruth Kaiser

Tutor: S.S.

Bell and CHSH inequalities

The geometry of state space for "composite systems". Quick review of EPR. Entanglement, locality and contextuality. Geometry of BCHSH and the convex hull problem.

There is more material here than could possibly be covered even in a semester-long course on quantum information theory. One might start with [Bell], and then quickly work one's way up to [Mor], Chapter 5. Do consult [WeWo] and do not get distracted by [Maud].

References: [Bell], [Mor], [WeWo], [Maud]

Date: 21. June

Speaker: Konrad Kockler

Tutor: J.W.

Entropy and entanglement measures

Entanglement measures. Werner states. Entropy and information. Distillability. Ditto.

References: [Mor], [Zyc], [WeWo]

Date: 28. June

Speaker: Johannes Dreckhoff

Tutor: S.N.

Decoherence

Review of von Neumann measurements and "the problem". Decoherence and the recovery of classical probabilities. Following [Schloss] and related references will suffice for a coherent presentation.

Alternatively, one may embark on "consistent histories" following [Griffiths] and [Omnes]. See also [LanFQT].

References: [Schloss], [Griffiths], [Omnes], [LanFQT]

Date: 5. July

Speaker: Siying Zheng

Tutor: J.W.

DECOHERENCE MODELS

Time evolution of reduced density matrix, the Lindblad equation. Explicit models of decoherence and non-demolition measurements. Alternatively, one may revisit issues related to decoherence and emergence of classical probabilities not covered in the previous talk.

References: [Schloss], []

Date: 12. July

Speaker: Luis Yague

Tutor: S.S.

References

(It goes without saying that virtually all references are available online, and that you are welcome to dig deeper on your own.)

[Banks] T. Banks, *The interpretation of quantum mechanics*, Guest post at Preposterous Universe, pdf available here.

[Bell] J. S. Bell, *On the Einstein Podolsky Rosen Paradox* Physics 1 (3): 195–200.

[Fol] G.B. Folland, *Harmonic Analysis in Phase Space*, AM-122, Princeton university press (2016)

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[Griffiths] R. B. Griffiths, *Consistent histories and the interpretation of quantum mechanics* J. Stat. Phys. 36 (1984) 219

[Hall] B. Hall, *Quantum theory for mathematicians*, Springer Verlag (2013)

[LanFQT] K. Landsman, *Foundations of Quantum Theory - From Classical Concepts to Operator Algebras*, Springer Verlag (2017)

[Maud] T. Maudlin, *What Bell did*, J. Phys. A: Math. Theor. 47 (2014)

[Moore] G. Moore, *Quantum Symmetries and Compatible Hamiltonians*, lecture notes available at author's homepage

[Mor] V. Moretti, *Fundamental Mathematical Structures of Quantum Theory*, Springer Verlag (2019)

[MorSC] V. Moretti, *Mathematical foundations of quantum mechanics: An advanced short course*, International Journal of Geometric Methods in Modern Physics, volume 13 (2016)

[MorSpQM] V. Moretti, *Spectral Theory and Quantum Mechanics - With Introduction to Algebraic Formulation*, Springer Verlag (2018)

[Omnes] R. Omnes, *Consistent interpretations of quantum mechancis*, Rev. Mod. Phys. 64, 339

[Ros] J. Rosenberg, *A selective history of the Stone-von Neumann theorem*, Providence, RI: American Mathematical Society, Contemporary Mathematics, volume 365, (2004) 331–354

[Schloss] M. Schlosshauer, *Quantum decoherence*, Phys. Rep. 831, 1-57 (2019), see also *The quantum-to-classical transition and decoherence* https://arxiv.org/abs/1404.2635 for a shorter version, or the more wordy *Decoherence and the Quantum-to-classical transition* (Springer, 2007)

[Suss] L. Susskind and A. Friedman, *Quantum Mechanics: The theoretical minimum* (2012) Also available as online course at https://theoreticalminimum.com/courses

[Var1] V. S. Varadarajan, *Geometry of Quantum Theory*, Vol. I. D. van Nostrand Company, Princeton, 1968

[Var2] V. S. Varadarajan, *Geometry of Quantum Theory*, Vol. II. D. van Nostrand Company, Princeton, 1970

[Wein] S. Weinberg, *The Quantum Theory of Fields*, Appendix A to Chapter 2, Vol. 1

[WeWo] R. F. Werner, M. M. Wolf, *Bell inequalities and entanglement*, Quantum Information&Computation Vol. 1, 1 (2001)

[Zirn] M. R. Zirnbauer, *Symmetry classes*, in The Oxford Handbook of Random Matrix Theory, available at https://arxiv.org/abs/1001.0722

[Zyc] I. Bengtsson, K. Zyczkowski, *Geometry of Quantum States*, 2nd edition Cambridge University Press 2017