Def A *Lie algebroid* over M is a vector bundle $E \to M$ together with:

- a morphism of vector bundles

$$\begin{array}{ccc} E & \xrightarrow{\rho} & TM \\ & \searrow \quad \swarrow & \\ & M & \end{array}$$

("anchor map")

- a Lie bracket on its sections

$$[\cdot,\cdot]_E : \Gamma(E) \times \Gamma(E) \longrightarrow \Gamma(E)$$

satisfying the Leibniz formula $\boxed{[u, fw] = \rho(u)[f]w + f \cdot [u,w]}$ $\forall f \in C^\infty(M)$, $u,w \in \Gamma(E)$

(where $\rho(u) := \rho \circ u \in \Gamma(TM)$ is a vector field on M)

Basic examples

① $E = TM$, $\rho = id_{TM}$

$[\cdot,\cdot] =$ Lie bracket of vector fields

② $M = \{pt.\}$

$E = \mathfrak{g}$ Lie algebra

$\rho = 0$

Main example

③ To a $\quad M \to M$ we can associate:

action Lie groupoid $E = \mathfrak{g} \times M \longrightarrow M$ (trivial bundle)

bracket on $\Gamma(E)$: $[s^a T_a, t^b T_b]_E = s$