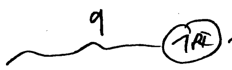
Conformal invariance of $\mathcal{N}=4$ SYM

1. Renormalization in YM ($\to$ β-fct, RG-eq, flow)
2. β-functions for $\mathcal{N}=1,2$ and 4 SYM
3. Conformal invariance of $\mathcal{N}=4$ SYM
4. Anomalies in SYM ($\to$ see e.g. $\mathcal{N}=1$ SYM not c.i.)

I Renormalization

$$(q^2\eta^{\mu\nu}-q^\mu q^\nu)\,\Pi(q)$$
$$\shortparallel$$

I.1. Motivation


$= i\Pi^{\mu\nu}(q)$

- Remember electron selfenergy?

If you calculate e^-, e^- scattering with intermed photon, then process Amplitude dependent on q^2 — bare coupling in $\mathcal{L}$

$\rightsquigarrow$ coupling $e = e(q^2) := \dfrac{e_0}{\sqrt{1-\Pi^2(q)}}$ "renorm charge" physical coupling. $\quad \dfrac{\text{"}\infty\text{"}}{\text{"}\infty\text{"}} = \text{finite}$

- Want to keep track of behaviour of coupling.

I.2. Yang-Mills-Theory

- General setting: Take a Principal G-bundle over mfd M and take a connection A. To A we associate the curvature 2-Form $F := dA + g\, A\wedge A$
- The action:

$$S_{YM} := \overbrace{\frac{1}{4}}^{\text{convention}}\int_M \mathrm{Tr}(\underbrace{*F\wedge F}_{\text{top-Form}}) \quad , \text{where } * \text{ is the Hodge-dual operator}$$

defines a gauge theory with gauge-grp G

We call g the coupling of the theory. We now want to know how g behaves in a Quantum theory, depending on the energy-scale.

Def:

Let μ be an energy-scale for our theory. We call

$$\beta(g) := \frac{dg}{d\ln\mu}$$ the Beta-fct associated to g.

I.3 Callan-Symanzyk-eq.

- We want to calculate the β-fct.
- Consider the bare (unrenorm) theory and the bare n-point fct.
- Bare theory is independent on the energy scale, so let $G_n^{(0)}(g)$ be bare the n-point fct, then

$$\frac{d}{d\mu} G_n^{(0)}(g) = 0$$

c.f. electron: $Z = 1 - \Pi(q^2)$

- Let Z be the wf renorm for our gauge field A, then we have

$$A = Z^{-\frac{1}{2}} A_0 \quad \Rightarrow \quad G_n(g) = Z^{-\frac{n}{2}} G_n^{(0)}(g)$$

$$\Rightarrow \mu \frac{d}{d\mu} \left(Z^{\frac{n}{2}} G_n \right) = 0$$

$$\Leftrightarrow \left(\mu \frac{\partial}{\partial\mu} + \beta(g) \frac{\partial}{\partial g} + \frac{n}{2} \frac{\mu}{Z} \cdot \frac{dZ}{d\mu} \right) G_n(g) = 0 \qquad \text{"Callan-Symanzyk-eq"}$$

- By calculating $G_n(g)$ order by order, we can insert this in the CS-eq and calculate $\beta(g)$ order by order

Expl: QED:

Take 3pt-fct + $\underbrace{\text{tree level counter terms}}_{\Rightarrow Z}$

$$\Rightarrow \beta^{1\text{-loop}}(e) = \frac{e^3}{12\pi^2}$$

1st Order D.E.

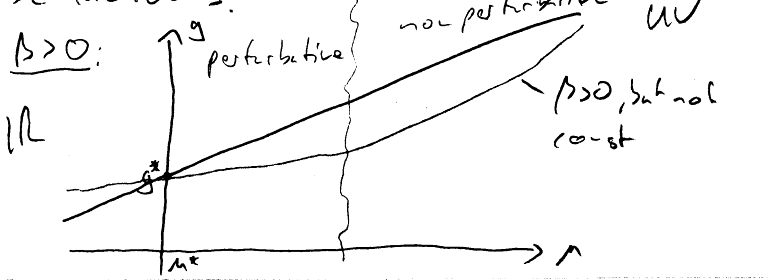
Remark:

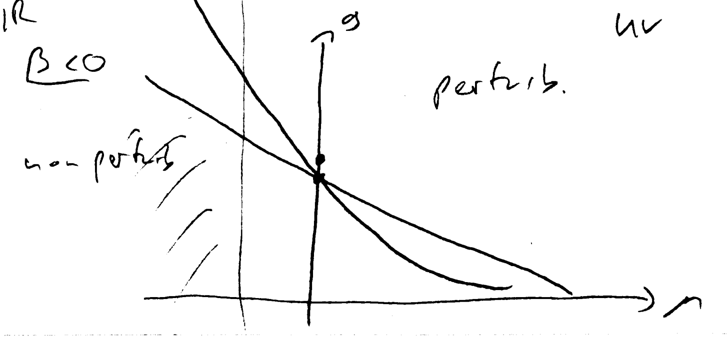
If we know the β-fct of the theory, we can Integrate the β-fct. From the definition, we then can predict the coupling at any given scale, provided we can measure a reference value g^* at μ^*.

This change of g depending on μ is interpreted as RG-flow.

I.4 Asymptotic behaviours

a) Assume first β is constant. then depending on the sign, we have different behaviours.



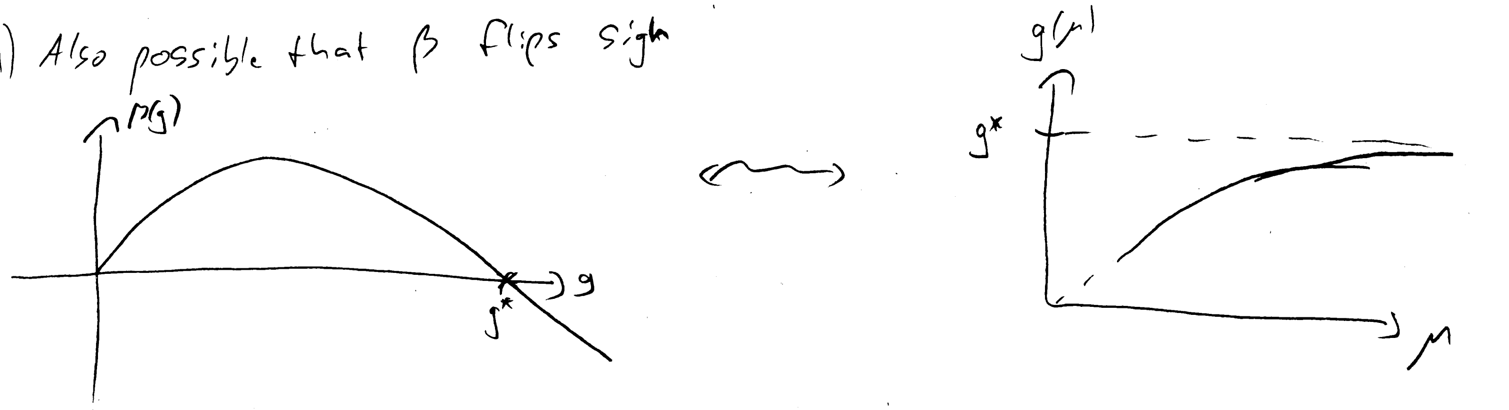


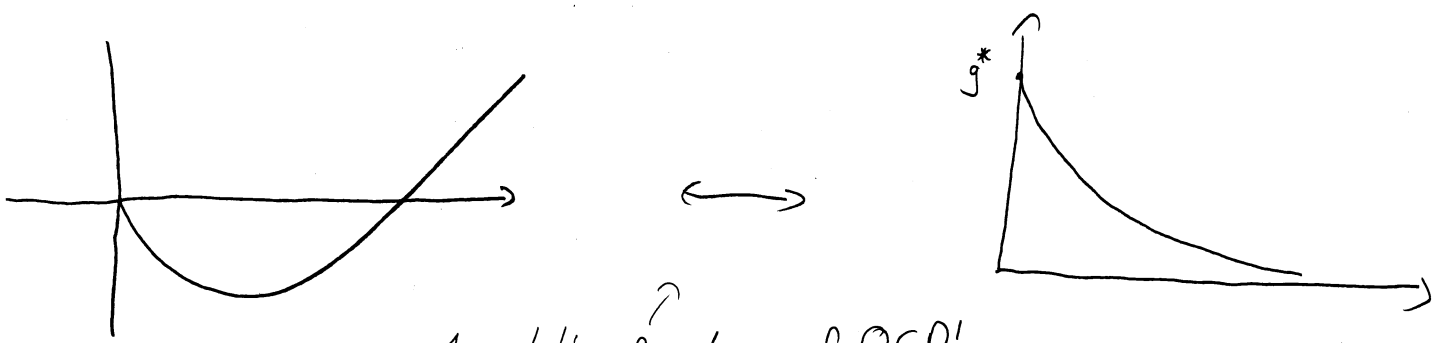
b) $\beta \to 0$ for $g \to 0$ $\Rightarrow$ get free theory in the limit

Depending on sign we either get an IR or an UV fixed point ($g=0$) of our theory

c) If $\beta \equiv 0$, then g is indep of μ, so our theory is scale invariant. If all β-fcts of a theory vanishes, this implies that the theory is conformal invariant.

d) Also possible that β flips sign





Asymptotic freedom of QCD!

I.V Wilsonian picture:

Usual procedure: $\to$ introduce cutoff Λ so divergent integrals become finite (by this get the renorm scale μ.)

$\to$ show independence of cutoff $\Rightarrow$ cutoff $\to \infty$

$\to$ Not much physical meaning, loose predictability of the theory

Wilsons: If theory is UV-divergent, view it as an effective field theory of a more fundamental theory, that is valid only up to a certain scale

$\leadsto$ have with the RG-flow a space of field theories and the β-fct determines to which theory our theory flows.

Instead of taking the limit ~~on~~ $\Lambda \to \infty$, study what the effective action has to be, st a change of the cutoff $\Lambda \to \Lambda'$ doesn't change the low energy physics

$\Rightarrow$ effective action for our effective field theory.

$$\boxed{\begin{aligned} &\mathcal{L} = \frac{1}{16}\int d^2\theta \frac{1}{g_h^2} W^a(V_h)W^a(V_h) \quad + h.c. \\ &1/g_h^{2*} = 1/g^2 \end{aligned}}$$

II β- fct:

Idea: Change $\Lambda \longrightarrow \Lambda'$ and see how this affects the coupling. Derive β-fct according to RGE

II 1. $\mathcal{N}=1$

a) Holomorphic coupling g_h

Holomorphy arguments (hol coupling from VEV's of cplx superfields in F-Term) consider

holomorphic coupling $\frac{1}{g_h^2} = \frac{1}{g^2} + i\frac{\Theta}{8\pi^2}$

Now change cutoff $\Lambda \longrightarrow \Lambda'$, set $\gamma = \frac{8\pi}{g_h^2}$, $t = \ln\frac{\Lambda}{\Lambda'}$

We get a change in γ:

$$\gamma \longrightarrow \gamma' = \gamma + f(\gamma, t) \quad \text{, where}$$

f is holomorphic in the coupling and in t (the W. eff action can be shown to also be holomorphic). We know $f(\gamma, 0) = 0$ and as a shift in the Θ, by 2π doesn't change S, we get

$$f(\gamma + 2\pi i, t) = f(\gamma, t) + 2\pi i\, n(t) \quad \text{with } n(t) \in \mathbb{Z},\ n(0)=0.$$

As f is continous (assume Spacetime to be connected) $\Rightarrow n(t) = 0 \ \forall t$

$\Rightarrow f$ is periodic in γ and as $\frac{d}{dt} f = \beta$, so is the β-fct

$\Rightarrow$ Can F-expand β: $\beta(\gamma) = \sum_{n\in\mathbb{Z}} b_n e^{-n\gamma}$.

As the Theory has to make sense ~~...~~ in weak compl. regime $\rightsquigarrow n \geq 0$

Anomaly considerations (Non-renorm-theorems show that $n \geq 1$ cannot arrise

$\Rightarrow \beta(\gamma) = b_0 = \text{const.}$

special for $\beta(g)$ in $\mathcal{N}=1$ pure SYM

b) canonical coupling

Going from $g_h \to g$ we have to transform our vector multiplet $V_h \to gV$ (for normalization reasons). Thus we get a Jacobian in our Path Integral

[Calculation: see hep-th/9707133, A.4 (~3 pages..)]

$$\mathcal{D}(gV) = \mathcal{D}(V) \exp\left(\frac{1}{16}\int d^4y \int d^2\Theta \, \frac{2t_2(A)}{8\pi^2} \ln g \, W^a(gV) W^a(gV) + h.c\right),$$

where $\delta^{ab} t_2(R) := Tr_R(\underbrace{T^a T^b}_{\text{generators of Rep}})$ is the dynkin-index

and A stands for the adjoint Rep.

Remark:

One could possibly include higher Order R terms in the Jacobian but the power on non-renorm. thm tells us, that such terms can never produce an F term as $W.W$.

Further there are no IR-singular D-terms generated, so we don't have to worry about such subtleties.

So modulo gauge fixing terms, we have for the gen. functional at fixed Λ

$$Z = \int \mathcal{D}V_h \exp\left(-\frac{1}{16}\int d^4y \int d^2\Theta \, \frac{1}{g_h^2} W^a(V_h) W^a(V_h) + h.c\right)$$

$$= \int \mathcal{D}V \exp\left(-\frac{1}{16}\int d^4y \int d^2\Theta \left(\frac{1}{g_h^2} - \frac{2t_2(A)}{8\pi^2}\ln g\right) W^a(gV) W^a(gV) + h.c\right)$$

Due to normalization reasons, it must hold, that

$$\frac{1}{g^2} = Re\left(\frac{1}{g_h^2}\right) - \frac{2t_2(A)}{8\pi} \ln g$$

From a), we know, that the difference in g_h and g_h' is 1-loop effect

$$\overset{?}{\Rightarrow} \quad \beta(g_{..}) = -\frac{g^3}{16\pi^2} \frac{3t_2(A)}{1 - \frac{t_2(A)}{8\pi^2}g^2}$$

II.2 NSVZ β-fct

$\Rightarrow$ Novikov-Shifman-Vainshtein-Zakharov (1983)

The above β-fct actually is the β-fct for $\mathcal{N} = 1, 2, 4$ SYM and can also be generalized for SYM with matter (remember matter is massless in SYM)

$$\Rightarrow \boxed{\beta(g) = -\frac{g^3}{16\pi^2} \frac{3t_2(A) - \sum t_2(R_i)(1-\gamma_i)}{1 - \frac{t_2(A)}{8\pi^2}g^2}},$$

where $\gamma_i := -\frac{d \ln Z}{d \ln \mu}$ are the anom. dim of the matter fields, R_i Reps of them.

Facts:

- The NSVZ-β-fct is "exact", meaning it is valid in all orders in Perturbation theory.
- It was originally derived from an instanton calculation (non-perturb)
- Non renorm-theorems show that there are no corrections beyond 1-loop, not even non-perturb.
- For $\mathcal{N}=4$, the NSVZ-β-fct vanishes!!

III Conformal invariance of $\mathcal{N}=4$ SYM

- In a famous paper Sohnius and West showed 1981, that the β-fct for $\mathcal{N}=4$ SYM vanishes (under certain assumptions).
- 1982 AVDEEV / TARASOV, verified this "numerically" (using PC prog) up to 3-loops
- later it was also shown without the assumptions and in 1988 by Seiberg with non-perturbative methods.

We want to understand the proof of Sohnius and West:

$\longrightarrow$ It was shown... that the trace anomaly Θ^μ_μ is proportional to the β-fct

III 1) Assumptions

a) SUSY + O(4) invariance remain unbroken order by order in P.T.

b) The currents of the quantized theory should act as sources for SUGRA.

From this one can follow;

i) The trace of the e.m.t and the divergence of an axial vector lie in the same irred multiplet of $\mathcal{N}=1$ SUSY (so under one of the 4 SUSYs)

ii) At least on axial current is conserved.

How can we see this?

~~If b) is done, this implies that we can assume that our complete~~

II 2. Argument for the Assumptions:

The complete multiplet of currents in $N=4$ SUSY has $N=2$ SUSY as a subset. If (b) is true, then we can assume two things:

1) We have at most an additional 2nd central charge
2) The multiplet of currents for $N=2$ has a special form (as the one in Sohnius (The multiple currents for $N=2$ ext. SUSY)

The special form includes that we have an explicit $SU(2)$ invariance and two types of currents: T_μ for $SU(2)$ and $j_\mu^{(5)}$ for chiral $U(1)$.

Calculating the susy variation for those, one see's that is is unchanged by the introduction of the 2nd central charge and that from Supersymmetry one gets $\partial^\mu T_\mu = 0$, but $\partial^\mu j_\mu^{(5)} \neq 0$.

To avoid reality conditions involving γ_5 for spinors, go to largest non-chiral subgroup $O(2)$.

There T_μ reduces to one polar vector and two axial vector

$\Rightarrow$ have at least one conserved axial vector!

III 3 Proof of C.I:

We now want to deduce, that $N=4$ SYM is conformal.

$N=4$ SYM has $SU(4)$ as R-symmetry group. This gives rise to a conserved current T_μ transforming in the adjoint (15 dim) of $SU(4)$. $(T_\mu)_i{}^j$

(i) $\Rightarrow$ Θ^μ_μ and ∂^μ (axial curent) lie in same multiplet and at tree level $\Theta^\mu_\mu = 0$. $\Rightarrow$ at least one axial vector is conserved at tree level, so it has to be one of the $(T_\mu)_i{}^j$ $\leftarrow$ possible broken by chiral anomaly $\to$ later.

Pass to largest non-chiral subgroup $O(4) \subset SU(4)$

$$\Rightarrow (T_\mu)_i{}^j \text{ splits } \begin{cases} \to (t_\mu)_{ij} \text{ antisym} \rightsquigarrow 6 \\ \to (j_\mu^{(5)})_{ij} \text{ symmetric traceless} \rightsquigarrow 9 \end{cases} \left.\begin{matrix} \text{polar} \\ \text{of } O(4) \\ \text{axial} \end{matrix}\right.$$

a) $\Rightarrow$ $O(4)$ inv holds order by order $\Rightarrow$ $\partial^\mu t_\mu = 0$ to all orders 8

(ii) $\Rightarrow$ at least one axial vector is conserved, so

$\partial^\mu (j^{(5)}_\mu)_{ij}$ for at least one of them.

But they form a multiplet under $O(4)$, so all of them are conserved and we get $\Theta^\mu_\mu = 0$ order by order.

Hence $\mathcal{N}=4$ SYM is conformal invariant for all orders in PT!!

IV Anomalies

Anomalies $\hat{=}$ failure of classical symm to be a quantum symm.

IV.1) Motivation

- Global symmetries: not so bad; doesn't make the theory inconsistent, but maybe some classically forbidden stuff happens

- Gauge symmetries: game over; leads to inconsistencies

  $\hookrightarrow$ needed for unitarity and renormalizability

  $\Rightarrow$ absolute must have that gauge sym are anomaly free

Physical application: Anomaly cancellation:

- String-Theory: $\to$ critical dimension via vanishing of the Trace anomaly

  $\to$ Green-Schwarz-Mech: Consistent superstring theory must have a gauge group of certain dimension.

- SUSY: MSSM $\to$ Need to have 2 higgsinos for consistent anomaly cancellation $\Rightarrow$ Somewhere between today's Energy scale and the planck scale has to be another Higgs or else SUSY is not the answer to extent the SM.

Physicists loove calculating stuff

IV 2) Computational facts:

a) Ward-Takahashi-Id

- Quantum analouge of Noethers theorem
- To symmetry ⇝ have current. If it is also a symmetry of the QT, then current must be conserved as an operator equation. anomalous WT-Id. $\partial^\mu j_\mu^5 = O(\neq 0)$
- If not conserved ⇝ anomaly, which manifests itself through the fact that the PI-measure is not invariant.

IV.2) b) Fujikawa-PI-Method

Due to symmetry transformation in Lagrangian, get a Jacobian in our Fermionic PI measure. If this isn't trivial, out gen. fct is not invariant under this symmetry ⇒ anomaly

To calculate the Jacobian ⊛ one has to introduce a regulator $f(s)$ with $f(0)=1$ and $f(s) \xrightarrow{s\to\infty} 0$ (so e.g e^{-s})

For s we insert an expresson $\sim \frac{1}{\mu^2}$ ← renorm. scale

[⊛ There are usually expressions of the form "$0\cdot\infty$" or other stupid stuff]

⇝ long and nasty calculations, but very important

Expl: Adler-Bell-Jackiv anomaly (ABJ) in even dim. For $d=2n$:

$$\partial^\mu j_\mu^{(5)} = (-1)^{n-1} \frac{2g^n}{n!(4\pi)^n} \varepsilon^{\mu_1\cdots\mu_{2n}} F_{\mu_1\mu_2}\cdots F_{\mu_{2n-1}\mu_{2n}}$$

Remark:

- It is not trivial that we can apply this to SUSY-Theories. We need to take into account, that the regulator is:

i) manifest supersymmetric ii) chiral iii) gauge covariant

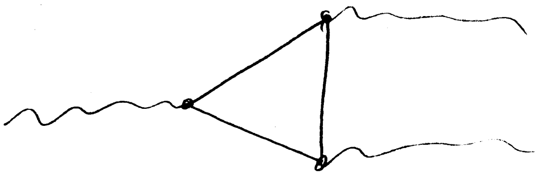
This was constructed by Konishi, Shizuya; 1985.

$$\frac{L}{M^2} = \frac{1}{16M^2}\bar{D}^2 e^{-V} D^2 e^{V}$$, M: energy-scale

eg axial/chiral anomaly: $\partial^\mu j^5_\mu = -\frac{e^2}{16\pi^2}$ "gauge anomaly"

c) Diagram method:

- By considering a background gauge field (e.g our gauge grp, or another internal symmetry), get for Fermions Diagrams:



Triangle-Diagrams

- By calculating such diagrams get value for anomalies
- For such diagrams, one finds an anomaly coefficient:

$$A = \underset{\substack{\uparrow\\ \text{Repr of field}}}{\mathrm{tr}_R}\,(T_a\{\underset{\substack{\uparrow\\ \text{generators}}}{T_b, T_c}\})$$

as each of the vertices carries one generator

- By considering other external fields, get other types aka gravitational / mixed anomalies.

IV 3 Anomalies in SYM:

a) R-symmetry in $N=4$ (Argument by Bilal/Chu | hep-th/0003129)

- R-sym given by $SU(4)$ $\rightsquigarrow$ consider external $SU(4)$ gauge field (background)
- Chiral fermions transform in the $\bar{4}$ cplx conjugate traffo
- By considering 1-loop $\rightsquigarrow$ anomaly is proportional to the number of fermions aka dim of adj of $SU(N)$ = gauge-grp of our SYM

$\Rightarrow A \sim N^2 - 1$ is not zero for non $U(1)$ SYM.

Rem: Brutal CFT computation gives exact result, that matches with the one we get from AdS/CFT!!

b) Trace-anomaly in $\mathcal{N}=1$ (only abelian case)

a) Chiral Multiplet:

Consider a rescaling of the chiral Multiplet:

$$\Phi \rightarrow e^{\alpha}\Phi \quad , \quad \Phi(x,\theta) = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

$\Rightarrow$ get Jacobian (in a certain choice of regularization) $t = \frac{1}{M^2}$

$$\ln J = \alpha \left(\underset{\substack{\uparrow \\ \langle\phi| e^{t D_\mu D^\mu} |\phi\rangle}}{\mathrm{Tr}_\phi\, e^{t(D_\mu D^\mu)}} - \mathrm{Tr}_\psi\, e^{t \cancel{D}^2} + \mathrm{Tr}_F\, e^{t(D_\mu D^\mu)} \right)$$

original source $\langle\phi|\cancel{D}^2|\phi\rangle$ compare remark

Remark: • The different exponent come from considerations of the chiral anomaly + Fujikawa method + no anomaly for no background gauge field (aka $D_\mu = \partial_\mu$)

• In a manifestly supersymmetric formulation the above formular is given by

$$\ln J = \alpha\, \mathrm{STr}\left(e^{tL}\, \frac{-\bar{D}^2}{4}\right), \quad \text{with}$$

$$L = \frac{1}{16}\bar{D}^2 e^{-V} D^2 e^{V}$$

This reduces to $(D_\mu)^2$ on ϕ and F component and to $\cancel{D}^2$ on spinor component.

b) Vector-Multiplet

Consider: $V \rightarrow e^{\alpha} V$

Wess-Zumino-gauge $\rightsquigarrow$ $V = V(V_\mu, \underbrace{a, \lambda, \bar{\lambda}, D})$

same procedure as for chiral multiplet

Need to take care of Fadeev-Popov procedure:

V_μ: decompose V^μ into V_L^μ, V_T^μ with

$$V_L^\mu = \frac{D^\mu D^\nu}{(D\mathfrak{g})^2} V_\nu \qquad V_T^\mu = V^\mu - V_L^\mu$$

SO(4) generators, parameter from gauge fixing

$$\Rightarrow \ln J_v = \alpha \left(\mathrm{Tr}_{V_T}\, e^{-t[(D_\mu)^2 \delta_{\mu\nu} - i F_{\rho\sigma}(M^{\rho\sigma})_{\mu\nu}]} + \mathrm{Tr}_{V_L}\, e^{-t\xi D_\mu D^\mu}\right.$$

Rem: • Manifestly susy formulation possible but super technical.

c) Combining for the result:

• We now rescale the fields according to their canonical dimension d by changing coords from $x \rightarrow e^{-\lambda}x \Rightarrow$ fields pick up factor $e^{d\lambda}$

- Infinitessimally one gets

$$\ln J_- = \lambda \operatorname{Tr}_-\left[(d-2) - \frac{1}{2}\{x_\mu, \partial_x^\wedge\}\right) e^{t(D_\mu)^2} \Big]$$

or $\cancel{D}^2$

- since $d=2$ for F in chiral multiplet drops out
- after long calculation:

$$\Rightarrow \ln J^c = \frac{\lambda}{16} \int d^4x\, d^2\Theta\, \frac{\sum t_2(R_i)}{8\pi^2} W^2 \quad \text{for chiral multiplet}$$

$$\ln J^v = \frac{\lambda}{16} \int d^4x\, d^2\Theta\, \frac{-3t_2(A)}{8\pi^2} W^2 \quad \text{for vector multiplet.}$$

combining both gives us the trace anomaly
This gives also the proportionality to the β-fct!

IV 4) Outlook

a) c-Theorem:

"QFT RG-flows are irreversible"

Thm: $\exists$ fct $c(M^2 z\bar{z})$ for RG-flow $CFT_{uv} \to CFT_{IR}$, with

i) $c(M^2 z\bar{z}) \to c_{uv}$ as $|z| \to 0$ and $c(M^2 \bar{z}z) \to c_{IR}$ as $|z| \to \infty$

ii) Monotonic condition $\Rightarrow$ $c_{uv} > c_{IR}$

iii) c's are measured by Weyl anomaly $\langle\Theta\rangle = -\frac{c}{12} R$.

Intuition: c measures effective # dof at scale $x = \sqrt{z\bar{z}}$

Have a: coeff in $\langle T^i{}_i\rangle$ in front of $\left(\frac{1}{2}\varepsilon_{ij}{}^{mn} R_{mnkl}\right)^2$

In SUSY CFT in 4d:

$$a_{uv} - a_{IR} > 0 \quad \text{modell indep}$$

b) t'Hooft Anomaly matching

- Anom glob symm G with A_{uv}: triangle anomaly. Weakly couple G by adding new gauge fields + add massless fermions st $A_\psi = -A_{uv}$
  $\rightsquigarrow$ non anomalous theory
- consider this new theory at lower scale $\mu' < \mu$. Since we can take the coupling g of the new fields to be arb. weak $g \to 0$ $\Rightarrow$ IR dynamics are same as old theory + arb weak coupled sector $\Rightarrow$ A_ψ shouldn't chang.
- Theory still anomaly free $\Rightarrow$ $A_{IR} + A_\psi = 0$ $\Leftrightarrow$ $A_{IR} = A_{uv}$ even in $g \to 0$ limit
- So global anomaly coeffs are scale indep

$\rightsquigarrow$ can be used to show that certain theories don't have a mass gap, or that certain symm are spont. broken.

In preparing the part on Renormalization and the Wilsonian Picture, I used: [1, 2, 3, 4, 5] In the derivation of the NSZV β-function I followed [6], the original derivation can be found in [7]. The statements on the conformal invariance of $\mathcal{N} = 4$ SYM can be found in [8, 9, 10]. The part on anomalies in Quantum Field Theories follows [1, 3, 11]. The superspace Fujikawa method is discussed in [12], the R-symmetry anomaly can be found in [13] and in the derivation of the trace anomaly I followed [6].
As general references over the whole talk I used [14, 15].

References

[1] Hebecker, Arthur. Lecture Notes on Advanced Quantum Field Theory.

[2] Bilal, Adel. Lecture Notes on Advanced Quantum Field Theory.

[3] Timo Weigand. Lecture Notes on Quantum Field Theory.

[4] Georgi, Howard. Review on Effective Field Theories.

[5] Steven Weinberg. What is quantum field theory, and what did we think it is? In *Conceptual foundations of quantum field theory. Proceedings, Symposium and Workshop, Boston, USA, March 1-3, 1996*, pages 241–251, 1996.

[6] Nima Arkani-Hamed and Hitoshi Murayama. Holomorphy, rescaling anomalies and exact beta functions in supersymmetric gauge theories. *JHEP*, 06:030, 2000. doi: 10.1088/1126-6708/2000/06/030.

[7] V. A. Novikov, Mikhail A. Shifman, A. I. Vainshtein, and Valentin I. Zakharov. Exact Gell-Mann-Low Function of Supersymmetric Yang-Mills Theories from Instanton Calculus. *Nucl. Phys.*, B229:381–393, 1983. doi: 10.1016/0550-3213(83)90338-3.

[8] Martin F. Sohnius and Peter C. West. Conformal Invariance in N=4 Supersymmetric Yang-Mills Theory. *Phys. Lett.*, 100B:245, 1981. doi: 10.1016/0370-2693(81)90326-9.

[9] L. V. Avdeev and O. V. Tarasov. The Three Loop Beta Function in the $N = 1$, $N = 2$, $N = 4$ Supersymmetric Yang-Mills Theories. *Phys. Lett.*, 112B:356–358, 1982. doi: 10.1016/0370-2693(82)91068-1.

[10] Nathan Seiberg. Supersymmetry and Nonperturbative beta Functions. *Phys. Lett.*, B206:75–80, 1988. doi: 10.1016/0370-2693(88)91265-8.

[11] Adel Bilal. Lectures on Anomalies. 2008.

[12] Ken-ichi Konishi and Ken-ichi Shizuya. Functional Integral Approach to Chiral Anomalies in Supersymmetric Gauge Theories. *Nuovo Cim.*, A90: 111, 1985. doi: 10.1007/BF02724227.

[13] Adel Bilal and Chong-Sun Chu. Testing the AdS / CFT correspondence beyond large N. 1999. [PoStmr99,009(1999)].

[14] Bertolini, Matteo . Lectures on Supersymmetry.

[15] Eric D'Hoker and Daniel Z. Freedman. Supersymmetric gauge theories and the AdS / CFT correspondence. In *Strings, Branes and Extra Dimensions: TASI 2001: Proceedings*, pages 3–158, 2002.