Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$		
	$Y^2 - 1$		

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
$p =$	$X^2Y + XY^2 + Y^2$	$XY - 1$		
		$Y^2 - 1$		

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
$p =$	$X^2Y + XY^2 + Y^2$	$XY - 1$		
		$Y^2 - 1$		

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
$p =$	$X^2Y + XY^2 + Y^2$	$XY - 1$		
		$Y^2 - 1$		

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
$p =$	$X^2Y + XY^2 + Y^2$	$XY - 1$	X	
		$Y^2 - 1$		

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
$p =$	$X^2Y + XY^2 + Y^2$	$XY - 1$	X	
		$Y^2 - 1$		
	$-X(XY - 1)$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
$p =$	$X^2Y + XY^2 + Y^2$	$XY - 1$	X	
		$Y^2 - 1$		
	$-X(XY - 1)$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	X	
	$Y^2 - 1$		

$$p = \frac{-X(XY-1)}{XY^2 + X + Y^2}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	X	
	$Y^2 - 1$		

$$p = \frac{-X(XY-1)}{XY^2 + X + Y^2}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	X	
	$Y^2 - 1$		

$$p = \frac{-X(XY-1)}{XY^2 + X + Y^2}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$		

$$p = \frac{-X(XY-1)}{XY^2 + X + Y^2}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$		

$$p = \begin{array}{r} -X(XY-1) \\ \hline XY^2 + X + Y^2 \\ -Y(XY-1) \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$		

$$p = \begin{array}{r} -X(XY-1) \\ XY^2 + X + Y^2 \\ -Y(XY-1) \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$		

$$p = \begin{matrix} -X(XY-1) \\ \overline{XY^2 + X + Y^2} \\ -Y(XY-1) \\ \overline{X + Y^2 + Y} \end{matrix}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$		

$$p = \begin{array}{r} -X(XY-1) \\ \hline XY^2 + X + Y^2 \\ -Y(XY-1) \\ \hline X + Y^2 + Y \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$		

$$p = \begin{array}{r} -X(XY-1) \\ \hline XY^2 + X + Y^2 \\ -Y(XY-1) \\ \hline X + Y^2 + Y \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$		

$$p = \begin{array}{r} -X(XY-1) \\ \hline XY^2 + X + Y^2 \\ -Y(XY-1) \\ \hline X + Y^2 + Y \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$$
\begin{array}{rrcrr}
 & f & & f_1, f_2 & a_1, a_2 \quad r \\
 & X^2Y + XY^2 + Y^2 & & XY - 1 & X + Y \\
 & & & Y^2 - 1 & \\
 & -X(XY-1) & & & \\
 & XY^2 + X + Y^2 & & & \\
 & -Y(XY-1) & & & \\
p = & X + Y^2 + Y & \longrightarrow & & X
\end{array}
$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$$
\begin{array}{rrcll}
 & f & & f_1, f_2 & a_1, a_2 \quad r \\
 & X^2Y + XY^2 + Y^2 & & XY - 1 & X + Y \\
 & & & Y^2 - 1 & \\
 & -X(XY-1) & & & \\
 & XY^2 + X + Y^2 & & & \\
 & -Y(XY-1) & & & \\
p = & X + Y^2 + Y & \longrightarrow & & X
\end{array}
$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$p =$

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$		
$\underline{-X(XY-1)}$			
$XY^2 + X + Y^2$			
$\underline{-Y(XY-1)}$			
$X + Y^2 + Y$	$\longrightarrow$		X
$Y^2 + Y$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$$p = \begin{array}{r|l|l|l} f & f_1, f_2 & a_1, a_2 & r \\ X^2Y + XY^2 + Y^2 & XY - 1 & X + Y & \\ & Y^2 - 1 & & \\ \hline -X(XY-1) & & & \\ XY^2 + X + Y^2 & & & \\ \hline -Y(XY-1) & & & \\ X + Y^2 + Y & \rightarrow & & X \\ \hline Y^2 + Y & & & \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$$p = \begin{array}{r|l|l|l} f & f_1, f_2 & a_1, a_2 & r \\ X^2Y + XY^2 + Y^2 & XY - 1 & X + Y & \\ & Y^2 - 1 & & \\ \hline -X(XY-1) & & & \\ XY^2 + X + Y^2 & & & \\ \hline -Y(XY-1) & & & \\ X + Y^2 + Y & \longrightarrow & & X \\ \hline Y^2 + Y & & & \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$$p = \begin{array}{r|l|l|l} f & f_1, f_2 & a_1, a_2 & r \\ X^2Y + XY^2 + Y^2 & XY - 1 & X + Y & \\ & Y^2 - 1 & & \\ \hline -X(XY-1) & & & \\ XY^2 + X + Y^2 & & & \\ \hline -Y(XY-1) & & & \\ X + Y^2 + Y & \rightarrow & & X \\ \hline Y^2 + Y & & & \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$p =$

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$	1	
$-X(XY-1)$			
$XY^2 + X + Y^2$			
$-Y(XY-1)$			
$X + Y^2 + Y$	$\rightarrow$		X
$Y^2 + Y$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$-X(XY - 1)$			
	$XY^2 + X + Y^2$			
	$-Y(XY - 1)$			
	$X + Y^2 + Y$	$\rightarrow$		X
$p =$	$Y^2 + Y$			
	$-(Y^2 - 1)$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$	1	

$$
\begin{array}{rr}
 & -X(XY-1) \\ \hline
 & XY^2 + X + Y^2 \\
 & -Y(XY-1) \\ \hline
 & \underline{X + Y^2 + Y} \quad \rightarrow \quad X \\
p = & Y^2 + Y \\
 & -(Y^2 - 1)
\end{array}
$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$$
\begin{array}{rrcrr}
 & f & & f_1, f_2 & a_1, a_2 \quad r \\
 & X^2Y + XY^2 + Y^2 & & XY - 1 & X + Y \\
 & & & Y^2 - 1 & 1 \\
 & -X(XY-1) & & & \\
 & \overline{XY^2 + X + Y^2} & & & \\
 & -Y(XY-1) & & & \\
 & \overline{X + Y^2 + Y} & \rightarrow & & X \\
 & \overline{Y^2 + Y} & & & \\
 & -(Y^2-1) & & & \\
 & \overline{Y + 1} & & & \\
p = & & & &
\end{array}
$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$	1	
$-X(XY - 1)$			
$XY^2 + X + Y^2$			
$-Y(XY - 1)$			
$X + Y^2 + Y$	$\rightarrow$		X
$Y^2 + Y$			
$-(Y^2 - 1)$			
$p = \quad Y + 1$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$$p = \begin{array}{r|l|l|l} f & f_1, f_2 & a_1, a_2 & r \\ \hline X^2Y + XY^2 + Y^2 & XY - 1 & X + Y & \\ & Y^2 - 1 & 1 & \\ \hline -X(XY-1) & & & \\ \hline XY^2 + X + Y^2 & & & \\ -Y(XY-1) & & & \\ \hline X + Y^2 + Y & \rightarrow & & X \\ \hline Y^2 + Y & & & \\ -(Y^2-1) & & & \\ \hline Y + 1 & & & \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$	1	
$-X(XY - 1)$			
$XY^2 + X + Y^2$			
$-Y(XY - 1)$			
$X + Y^2 + Y$	$\rightarrow$		X
$Y^2 + Y$			
$-(Y^2 - 1)$			
$p = \quad Y + 1$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$	1	
$-X(XY - 1)$			
$XY^2 + X + Y^2$			
$-Y(XY - 1)$			
$X + Y^2 + Y$	$\rightarrow$		X
$Y^2 + Y$			
$-(Y^2 - 1)$			
$p =$ $Y + 1$	$\rightarrow$		$X + Y$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$$
\begin{array}{rrcrr}
 & f & & f_1, f_2 & a_1, a_2 \quad r \\
 & X^2Y + XY^2 + Y^2 & & XY - 1 & X + Y \\
 & & & Y^2 - 1 & 1 \\
 & \underline{-X(XY-1)} & & & \\
 & XY^2 + X + Y^2 & & & \\
 & \underline{-Y(XY-1)} & & & \\
 & X + Y^2 + Y & \rightarrow & & X \\
 & \underline{-(Y^2-1)} & & & \\
p = & Y + 1 & \rightarrow & & X + Y
\end{array}
$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$	1	
$-X(XY - 1)$			
$XY^2 + X + Y^2$			
$-Y(XY - 1)$			
$X + Y^2 + Y$	$\rightarrow$		X
$Y^2 + Y$			
$-(Y^2 - 1)$			
$Y + 1$	$\rightarrow$		$X + Y$
$p = 1$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$\color{red}{XY} - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$-X(XY - 1)$			
	$XY^2 + X + Y^2$			
	$-Y(XY - 1)$			
	$X + Y^2 + Y$	$\to$		X
	$Y^2 + Y$			
	$-(Y^2 - 1)$			
	$Y + 1$	$\to$		$X + Y$
$p =$	$\color{red}{1}$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$p =$

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$	1	
$-X(XY-1)$			
$XY^2 + X + Y^2$			
$-Y(XY-1)$			
$X + Y^2 + Y$	$\rightarrow$		X
$Y^2 + Y$			
$-(Y^2-1)$			
$Y + 1$	$\rightarrow$		$X + Y$
1			

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$	1	
$-X(XY - 1)$			
$XY^2 + X + Y^2$			
$-Y(XY - 1)$			
$X + Y^2 + Y$	$\rightarrow$		X
$Y^2 + Y$			
$-(Y^2 - 1)$			
$Y + 1$	$\rightarrow$		$X + Y$
$p = 1$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$-X(XY-1)$			
	$XY^2 + X + Y^2$			
	$-Y(XY-1)$			
	$X + Y^2 + Y$	$\rightarrow$		X
	$Y^2 + Y$			
	$-(Y^2-1)$			
	$Y + 1$	$\rightarrow$		$X + Y$
$p =$	1	$\rightarrow$		$X + Y + 1$

$$
\begin{array}{rrrrr}
 & f & f_1, f_2 & a_1, a_2 & r \\
 & X^2Y + XY^2 + Y^2 & XY - 1 & X + Y & \\
 & & Y^2 - 1 & 1 & \\
 & \underline{-X(XY-1)} & & & \\
 & XY^2 + X + Y^2 & & & \\
 & \underline{-Y(XY-1)} & & & \\
 & \underline{X + Y^2 + Y} & \rightarrow & & X \\
 & Y^2 + Y & & & \\
 & \underline{-(Y^2-1)} & & & \\
 & \underline{Y + 1} & \rightarrow & & X + Y \\
p = & 1 & \rightarrow & & X + Y + 1
\end{array}
$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\underline{-X(XY-1)}$			
	$XY^2 + X + Y^2$			
	$\underline{-Y(XY-1)}$			
	$X + Y^2 + Y$	$\rightarrow$		X
	$\underline{-(Y^2-1)}$			
	$Y + 1$	$\rightarrow$		$X + Y$
	$\underline{1}$	$\rightarrow$		$X + Y + 1$
$p =$	0			$X + Y + 1$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\underline{-X(XY-1)}$			
	$XY^2 + X + Y^2$			
	$\underline{-Y(XY-1)}$			
	$X + Y^2 + Y$	$\rightarrow$		X
	$\underline{-(Y^2-1)}$			
	$Y + 1$	$\rightarrow$		$X + Y$
	$\underline{1}$	$\rightarrow$		$X + Y + 1$
$p =$	0			$X + Y + 1$

$$X^2Y + XY^2 + Y^2 = (X + Y)(XY - 1) + 1 \cdot (Y^2 - 1) + X + Y + 1$$