

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{array}$$

$$a_1, a_2$$

$$r$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$p = X^2Y + XY^2 + Y^2$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{array}$$

$$a_1, a_2$$

$$r$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$p = \textcolor{red}{X}^2 \textcolor{red}{Y} + XY^2 + Y^2$$

$$\begin{array}{l} f_1, f_2 \\ \textcolor{red}{X}Y - 1 \\ Y^2 - 1 \end{array}$$

$$a_1, a_2$$

$$r$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$p = X^2Y + XY^2 + Y^2$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{array}$$

$$a_1, a_2$$

$$r$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$p = X^2Y + XY^2 + Y^2 \quad \begin{matrix} f \\ f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{matrix} \quad \begin{matrix} a_1, a_2 \\ X \end{matrix} \quad r$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{cccc}
 & & f_1, f_2 & a_1, a_2 & r \\
 p = & X^2Y + XY^2 + Y^2 & XY - 1 & X & \\
 & -X(XY - 1) & Y^2 - 1 & &
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{cccc}
 & f & f_1, f_2 & a_1, a_2 & r \\
 p = & X^2Y + XY^2 + Y^2 & XY - 1 & X & \\
 & -X(XY - 1) & Y^2 - 1 & &
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{cccc}
 & f & f_1, f_2 & a_1, a_2 & r \\
 & X^2Y + XY^2 + Y^2 & XY - 1 & X & \\
 & & Y^2 - 1 & & \\
 p = & \frac{-X(XY - 1)}{XY^2 + X + Y^2} & & &
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{cccc}
 & f & f_1, f_2 & a_1, a_2 & r \\
 & X^2Y + XY^2 + Y^2 & \textcolor{red}{XY} - 1 & X & \\
 & & Y^2 - 1 & & \\
 p = & \frac{-X(XY - 1)}{\textcolor{red}{XY}^2 + X + Y^2} & & &
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{cccc}
 & f & f_1, f_2 & a_1, a_2 & r \\
 & X^2Y + XY^2 + Y^2 & XY - 1 & X & \\
 & & Y^2 - 1 & & \\
 p = & \frac{-X(XY - 1)}{XY^2 + X + Y^2} & & &
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \end{array}$$

r

$$p = \frac{-X(XY - 1)}{XY^2 + X + Y^2}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \end{array}$$

r

$$p = \frac{-X(XY - 1)}{XY^2 + X + Y^2} - Y(XY - 1)$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{cccc}
 & f & f_1, f_2 & a_1, a_2 & r \\
 & X^2Y + XY^2 + Y^2 & XY - 1 & X + Y & \\
 & & Y^2 - 1 & & \\
 p = & \frac{-X(XY - 1)}{XY^2 + X + Y^2} & & & \\
 & -Y(XY - 1) & & &
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \end{array}$$

r

$$p = \frac{-X(XY - 1)}{XY^2 + X + Y^2} - \frac{Y(XY - 1)}{X + Y^2 + Y}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ \textcolor{red}{XY} - 1 \\ Y^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \end{array}$$

r

$$p = \frac{-X(XY - 1)}{XY^2 + X + Y^2} - \frac{Y(XY - 1)}{\textcolor{red}{X} + Y^2 + Y}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$		
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$			
$p =$				

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \end{array}$$

r

$$p = \frac{-X(XY - 1)}{XY^2 + X + Y^2} - \frac{Y(XY - 1)}{X + Y^2 + Y}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{cccc}
 & f & f_1, f_2 & a_1, a_2 & r \\
 & X^2Y + XY^2 + Y^2 & XY - 1 & X + Y & \\
 & & Y^2 - 1 & & \\
 p = & \frac{-X(XY - 1)}{XY^2 + X + Y^2} & \rightarrow & & \\
 & \frac{-Y(XY - 1)}{\textcolor{red}{X} + Y^2 + Y} & & & \textcolor{red}{X}
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$		
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
$p =$	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$	$\rightarrow$		X

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{rclcl}
 & & f & & \\
 X^2Y + XY^2 + Y^2 & & & & \\
 & & f_1, f_2 & & \\
 & & XY - 1 & & \\
 & & Y^2 - 1 & & \\
 & & a_1, a_2 & & \\
 & & X + Y & & r \\
 & & & & \\
 p = & \frac{-X(XY - 1)}{XY^2 + X + Y^2} & & & \\
 & \frac{-Y(XY - 1)}{X + Y^2 + Y} & \rightarrow & & X \\
 & Y^2 + Y & & &
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{rcll}
 & f & f_1, f_2 & a_1, a_2 & r \\
 X^2Y + XY^2 + Y^2 & & \textcolor{red}{XY} - 1 & X + Y & \\
 & & Y^2 - 1 & & \\
 \\
 p = \begin{array}{r}
 -X(XY - 1) \\
 \hline
 XY^2 + X + Y^2 \\
 -Y(XY - 1) \\
 \hline
 X + Y^2 + Y \\
 \hline
 \textcolor{red}{Y}^2 + Y
 \end{array} & \rightarrow & & X
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{rclcl}
 & & f & & f_1, f_2 & & a_1, a_2 & & r \\
 X^2Y + XY^2 + Y^2 & & & & XY - 1 & & X + Y & & \\
 & & & & \textcolor{red}{Y}^2 - 1 & & & & \\
 & & & & & & & & \\
 \begin{array}{r}
 -X(XY - 1) \\
 \hline
 XY^2 + X + Y^2 \\
 -Y(XY - 1) \\
 \hline
 X + Y^2 + Y \\
 \hline
 \textcolor{red}{Y}^2 + Y
 \end{array} & \rightarrow & & & X
 \end{array}$$

$p =$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{rcll}
 & & f & \\
 X^2Y + XY^2 + Y^2 & & & \\
 & & f_1, f_2 & \\
 & & XY - 1 & \\
 & & Y^2 - 1 & \\
 & & a_1, a_2 & \\
 & & X + Y & \\
 & & & r \\
 \\
 p = & \begin{array}{r}
 -X(XY - 1) \\
 \hline
 XY^2 + X + Y^2 \\
 -Y(XY - 1) \\
 \hline
 X + Y^2 + Y \\
 \hline
 Y^2 + Y
 \end{array} & \rightarrow & X
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$\begin{array}{rclcl}
 & & f & & \\
 X^2Y + XY^2 + Y^2 & & & & \\
 \\
 \begin{array}{r}
 -X(XY - 1) \\
 \hline
 XY^2 + X + Y^2 \\
 -Y(XY - 1) \\
 \hline
 X + Y^2 + Y \\
 \hline
 Y^2 + Y
 \end{array} & \rightarrow & \begin{array}{r}
 f_1, f_2 \\
 XY - 1 \\
 Y^2 - 1
 \end{array} & & \begin{array}{r}
 a_1, a_2 \\
 X + Y \\
 1
 \end{array} & r \\
 p = & & & & & X
 \end{array}$$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$			
$p =$	$\frac{X + Y^2 + Y}{Y^2 + Y}$	$\rightarrow$		X
	$-(Y^2 - 1)$			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$			
$p =$	$\frac{Y^2 + Y}{-(Y^2 - 1)}$	$\rightarrow$		X

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$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \\ 1 \end{array}$$

r

$$\begin{array}{r} -X(XY - 1) \\ \hline XY^2 + X + Y^2 \end{array}$$

$$\begin{array}{r} -Y(XY - 1) \\ \hline X + Y^2 + Y \end{array}$$

$$Y^2 + Y$$

$$-(Y^2 - 1)$$

$$Y + 1$$

$p =$

$\rightarrow$

X

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ \textcolor{red}{XY} - 1 \\ Y^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \\ 1 \end{array}$$

r

$$\begin{array}{r} -X(XY - 1) \\ \hline XY^2 + X + Y^2 \end{array}$$

$$\begin{array}{r} -Y(XY - 1) \\ \hline X + Y^2 + Y \end{array}$$

$$Y^2 + Y$$

$$\begin{array}{r} -(Y^2 - 1) \\ \hline \end{array}$$

$p =$

$$\textcolor{red}{Y} + 1$$

$\rightarrow$

X

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ \textcolor{red}{Y}^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \\ 1 \end{array}$$

r

$$\begin{array}{r} -X(XY - 1) \\ \hline XY^2 + X + Y^2 \end{array}$$

$$\begin{array}{r} -Y(XY - 1) \\ \hline X + Y^2 + Y \end{array}$$

$$Y^2 + Y$$

$$\begin{array}{r} -(Y^2 - 1) \\ \hline \end{array}$$

$p =$

$$\textcolor{red}{Y} + 1$$

$\rightarrow$

X

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ XY - 1 \\ Y^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \\ 1 \end{array}$$

r

$$\begin{array}{r} -X(XY - 1) \\ \hline XY^2 + X + Y^2 \end{array}$$

$$\begin{array}{r} -Y(XY - 1) \\ \hline X + Y^2 + Y \end{array}$$

$$Y^2 + Y$$

$$\begin{array}{r} -(Y^2 - 1) \\ \hline \end{array}$$

$p =$

$$Y + 1$$

$\rightarrow$

X

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$	$\rightarrow$		X
	$\frac{Y^2 + Y}{-(Y^2 - 1)}$			
$p =$	$\frac{Y + 1}{Y + 1}$	$\rightarrow$		$X + Y$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$	$\rightarrow$		X
	$\frac{Y^2 + Y}{-(Y^2 - 1)}$			
$p =$	$\frac{Y + 1}{Y + 1}$	$\rightarrow$		$X + Y$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$	$\rightarrow$		X
	$\frac{Y^2 + Y}{-(Y^2 - 1)}$			
	$\frac{Y + 1}{1}$	$\rightarrow$		$X + Y$
$p =$				

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

$$X^2Y + XY^2 + Y^2 \quad f$$

$$\begin{array}{l} f_1, f_2 \\ \textcolor{red}{XY} - 1 \\ Y^2 - 1 \end{array}$$

$$\begin{array}{l} a_1, a_2 \\ X + Y \\ 1 \end{array}$$

r

$$\begin{array}{r} -X(XY - 1) \\ \hline XY^2 + X + Y^2 \end{array}$$

$$\begin{array}{r} -Y(XY - 1) \\ \hline X + Y^2 + Y \end{array}$$

$$Y^2 + Y$$

$$\begin{array}{r} -(Y^2 - 1) \\ \hline Y + 1 \end{array}$$

$$Y + 1$$

$p =$

$\textcolor{red}{1}$

$\rightarrow$

X

$\rightarrow$

$X + Y$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$	$\rightarrow$		X
	$\frac{Y^2 + Y}{-(Y^2 - 1)}$			
	$\frac{Y + 1}{1}$	$\rightarrow$		$X + Y$
$p =$	1			

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

f	f_1, f_2	a_1, a_2	r
$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
	$Y^2 - 1$	1	
$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
$\frac{-Y(XY - 1)}{X + Y^2 + Y}$	$\rightarrow$		X
$\frac{Y^2 + Y}{-(Y^2 - 1)}$			
$\frac{Y + 1}{1}$	$\rightarrow$		$X + Y$

$p =$

1

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$	$\rightarrow$		X
	$\frac{Y^2 + Y}{-(Y^2 - 1)}$			
	$\frac{Y + 1}{1}$	$\rightarrow$		$X + Y$
$p =$	1	$\rightarrow$		$X + Y + 1$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$	$\rightarrow$		X
	$\frac{Y^2 + Y}{Y^2 + Y}$			
	$\frac{-(Y^2 - 1)}{Y + 1}$	$\rightarrow$		$X + Y$
$p =$	1	$\rightarrow$		$X + Y + 1$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$	$\rightarrow$		X
	$\frac{Y^2 + Y}{Y^2 + Y}$			
	$\frac{-(Y^2 - 1)}{Y + 1}$	$\rightarrow$		$X + Y$
	$\frac{1}{1}$	$\rightarrow$		$X + Y + 1$
$p =$	0			$X + Y + 1$

Bsp. 6.3: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " $\leq_{lex}$ "

	f	f_1, f_2	a_1, a_2	r
	$X^2Y + XY^2 + Y^2$	$XY - 1$	$X + Y$	
		$Y^2 - 1$	1	
	$\frac{-X(XY - 1)}{XY^2 + X + Y^2}$			
	$\frac{-Y(XY - 1)}{X + Y^2 + Y}$			
	$\frac{Y^2 + Y}{Y^2 + Y}$	$\rightarrow$		X
	$\frac{-(Y^2 - 1)}{Y + 1}$			
	$\frac{1}{1}$	$\rightarrow$		$X + Y$
$p =$	0	$\rightarrow$		$X + Y + 1$
				$X + Y + 1$

$$X^2Y + XY^2 + Y^2 = (X + Y)(XY - 1) + 1 \cdot (Y^2 - 1) + X + Y + 1$$