# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|
| $XY^2 + 1$ | $XY + 1$ | | |
| | $Y + 1$ | | |

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| $p =$ | $XY^2 + 1$ | $XY + 1$ | | |
| | | $Y + 1$ | | |

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| $p =$ | $XY^2 + 1$ | $XY + 1$ | | |
| | | $Y + 1$ | | |

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| $p =$ | $XY^2 + 1$ | $XY + 1$ | | |
| | | $Y + 1$ | | |

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| $p =$ | $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | | $Y + 1$ | | |

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| $p =$ | $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | | $Y + 1$ | | |
| | $-Y(XY + 1)$ | | | |

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| $p =$ | $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | | $Y + 1$ | | |
| | $-Y(XY + 1)$ | | | |

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | | |

$$p = \begin{array}{r} -Y(XY+1) \\ \hline -Y+1 \end{array}$$

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
| --- | --- | --- | --- |
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | | |

$$p = \begin{array}{r} -Y(XY+1) \\ \hline -Y + 1 \end{array}$$

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
| --- | --- | --- | --- |
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | | |

$$p = \frac{-Y(XY+1)}{-Y+1}$$

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | | |

$$p = \begin{array}{r} -Y(XY+1) \\ \hline -Y + 1 \end{array}$$

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | $-1$ | |

$$
\begin{array}{rr}
 & -Y(XY+1) \\ \hline
p = & -Y + 1
\end{array}
$$

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| | $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | | $Y + 1$ | $-1$ | |
| $p =$ | $-Y(XY+1)$ | | | |
| | $-Y + 1$ | | | |
| | $-(-(Y+1))$ | | | |

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | $-1$ | |

$$p = \begin{array}{r} -Y(XY+1) \\ \hline -Y+1 \\ -(-(Y+1)) \end{array}$$

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | $-1$ | |

$$
\begin{array}{rr}
 & -Y(XY+1) \\ \hline
 & -Y + 1 \\
 & -(-(Y+1)) \\ \hline
p = & 2
\end{array}
$$

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | $-1$ | |
| $-Y(XY + 1)$ | | | |
| $-Y + 1$ | | | |
| $-(-(Y + 1))$ | | | |
| $p = \quad 2$ | | | |

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | $-1$ | |

$$
\begin{array}{r}
-Y(XY+1) \\ \hline
-Y+1 \\
-(-(Y+1)) \\ \hline
p = \quad 2
\end{array}
$$

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| | $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | | $Y + 1$ | $-1$ | |
| | $-Y(XY + 1)$ | | | |
| | $-Y + 1$ | | | |
| | $-(-(Y + 1))$ | | | |
| $p =$ | $2$ | | | |

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|
| $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | $Y + 1$ | $-1$ | |
| $-Y(XY + 1)$ | | | |
| $-Y + 1$ | | | |
| $-(-(Y + 1))$ | | | |
| $p = \quad 2$ | $\rightarrow$ | | $2$ |

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

$$
\begin{array}{rrrrr}
 & f & f_1, f_2 & a_1, a_2 & r \\
 & XY^2 + 1 & XY + 1 & Y & \\
 & & Y + 1 & -1 & \\
 & \underline{-Y(XY+1)} & & & \\
 & -Y + 1 & & & \\
 & \underline{-(-(Y+1))} & & & \\
p = & 2 & \rightarrow & & 2
\end{array}
$$

# Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. "$\leq_{lex}$"

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| | $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | | $Y + 1$ | $-1$ | |
| | $\underline{-Y(XY + 1)}$ | | | |
| | $-Y + 1$ | | | |
| | $\underline{-(-(Y + 1))}$ | | | |
| | $\underline{2}$ | $\longrightarrow$ | | $2$ |
| $p =$ | $0$ | | | $2$ |

| | $f$ | $f_1, f_2$ | $a_1, a_2$ | $r$ |
|---|---|---|---|---|
| | $XY^2 + 1$ | $XY + 1$ | $Y$ | |
| | | $Y + 1$ | $-1$ | |
| | $\underline{-Y(XY + 1)}$ | | | |
| | $-Y + 1$ | | | |
| | $\underline{-(-(Y + 1))}$ | | | |
| | $\underline{2}$ | $\rightarrow$ | | $2$ |
| $p =$ | $0$ | | | $2$ |

$$XY^2 + 1 = Y(XY + 1) - (Y + 1) + 2$$