

Bsp. 6.2: Divisionsalgorithmus in $\mathbb{Q}[X, Y]$ bzgl. " \leq_{lex} "

$$\begin{array}{cccc} f & f_1, f_2 & a_1, a_2 & r \\ XY^2 + 1 & XY + 1 & Y + 1 & \end{array}$$

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$$p = \frac{f}{XY^2 + 1} = \frac{f_1, f_2}{XY + 1} - \frac{a_1, a_2}{Y + 1} r$$

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$$p = \frac{f}{XY^2 + 1} = \frac{f_1, f_2}{XY + 1} \cdot \frac{a_1, a_2}{Y + 1} r$$
$$\quad \quad \quad XY^2 + 1 \quad \quad \quad XY + 1 \quad \quad \quad Y + 1$$

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$$\begin{array}{rcl} p = & \begin{array}{c} f \\ XY^2 + 1 \\ - Y(XY + 1) \end{array} & \begin{array}{c} f_1, f_2 \\ XY + 1 \\ Y + 1 \end{array} \\ & & \begin{array}{c} a_1, a_2 \\ Y \end{array} \end{array} \quad r$$

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$$\begin{array}{cccccc} & f & f_1, f_2 & a_1, a_2 & r \\ p = & XY^2 + 1 & XY + 1 & Y & \\ & & Y + 1 & & \\ & -Y(XY + 1) & & & \end{array}$$

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$$p = \frac{\begin{array}{c} f \\ XY^2 + 1 \\ \hline -Y(XY + 1) \end{array}}{-Y + 1}$$
$$\begin{array}{c} f_1, f_2 \\ XY + 1 \\ Y + 1 \end{array}$$
$$\begin{array}{c} a_1, a_2 \\ Y \end{array}$$
$$r$$

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$$p = \frac{\begin{matrix} f \\ XY^2 + 1 \end{matrix}}{\begin{matrix} f_1, f_2 \\ XY + 1 \\ Y + 1 \end{matrix}} = \frac{-Y(XY + 1)}{-Y + 1} \quad r = Y$$

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$$p = \frac{\begin{array}{c} f \\ XY^2 + 1 \end{array}}{\begin{array}{c} f_1, f_2 \\ XY + 1 \\ Y + 1 \end{array}} \quad r$$

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$$p = \frac{\begin{array}{c} f \\ XY^2 + 1 \end{array}}{\begin{array}{c} f_1, f_2 \\ XY + 1 \\ Y + 1 \end{array}}$$
$$\frac{-Y(XY + 1)}{-Y + 1}$$

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$$p = \frac{\begin{array}{c} f \\ XY^2 + 1 \\ \hline -Y(XY + 1) \end{array}}{\begin{array}{c} f_1, f_2 \\ XY + 1 \\ Y + 1 \\ \hline -Y + 1 \end{array}}$$

a_1, a_2 r

Y -1

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$$\begin{array}{cccc} f & f_1, f_2 & a_1, a_2 & r \\ XY^2 + 1 & XY + 1 & Y & \\ & Y + 1 & -1 & \\ p = & \frac{-Y(XY + 1)}{-Y + 1} & & \\ & -(- (Y + 1)) & & \end{array}$$

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$$\begin{array}{cccc} f & f_1, f_2 & a_1, a_2 & r \\ XY^2 + 1 & XY + 1 & Y & \\ & Y + 1 & -1 & \\ p = & \frac{-Y(XY + 1)}{-Y + 1} & & \\ & -(-(Y + 1)) & & \end{array}$$

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$$\begin{array}{cccc} f & f_1, f_2 & a_1, a_2 & r \\ XY^2 + 1 & XY + 1 & Y & \\ & Y + 1 & -1 & \\ \hline -Y(XY + 1) & & & \\ \hline -Y + 1 & & & \\ \hline -(-(Y + 1)) & & & \\ \hline 2 & & & \end{array}$$

$p =$

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$$\begin{array}{cccc} f & f_1, f_2 & a_1, a_2 & r \\ XY^2 + 1 & \textcolor{red}{XY} + 1 & Y & \\ & Y + 1 & -1 & \\ \hline -Y(XY + 1) & & & \\ \hline -Y + 1 & & & \\ \hline -(-(Y + 1)) & & & \\ \hline 2 & & & \end{array}$$

$p =$

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$$\begin{array}{cccc} f & f_1, f_2 & a_1, a_2 & r \\ XY^2 + 1 & XY + 1 & Y & \\ & \textcolor{red}{Y} + 1 & & -1 \\ \hline -Y(XY + 1) & & & \\ \hline -Y + 1 & & & \\ \hline -(-(Y + 1)) & & & \\ \hline 2 & & & \end{array}$$

$p =$

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$$\begin{array}{cccc} f & f_1, f_2 & a_1, a_2 & r \\ XY^2 + 1 & XY + 1 & Y & \\ & Y + 1 & -1 & \\ \hline -Y(XY + 1) & & & \\ \hline -Y + 1 & & & \\ \hline -(-(Y + 1)) & & & \\ \hline 2 & & \rightarrow & 2 \end{array}$$

$p =$

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f	f_1, f_2	a_1, a_2	r
$XY^2 + 1$	$XY + 1$	Y	
	$Y + 1$	-1	
$\underline{-Y(XY + 1)}$			
$\underline{\quad -Y + 1}$			
$\underline{-(-(Y + 1))}$			
	$\underline{\quad \quad 2}$		2
$p =$	$\underline{\quad \quad 0}$		2

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	f	f_1, f_2	a_1, a_2	r
$XY^2 + 1$	$XY + 1$	Y		
	$Y + 1$		-1	
	$-Y(XY + 1)$			
	$-Y + 1$			
	$-(-(Y + 1))$			
	2			2
$p =$	0			2

$$XY^2 + 1 = Y(XY + 1) - (Y + 1) + 2$$