

(37) POSTPONED = VERTAGIT [4 points]

Let  $T$  be a 2-times covariant tensor field on a pseudo Riemannian manifold  $X$ , i.e.  $T \in T^{0,2}(X)$ .

a) What kind of mathematical object is the contraction of  $T$ ?

b) Please calculate the contraction of  $T$  with respect to the components  $T_{ij}$  for a given chart.  $\square$

(38) [2 points]

Please give one/the definition of a connexion on a smooth mfd.  $\square$

(39) [1 point]

Please give the 2 properties that allow you to check whether a given connexion is the Levi-Civita connexion with respect to a given metric.  $\square$

Remark: We have seen in class that there always exists such a connexion satisfying the above properties.

(40) [1 point]

Let  $X$  be either a smooth or a pseudo-Riemannian mfd (Pick what ever you like!). Please define the Riemannian curvature tensor for a given connexion on  $X$ .  $\square$

(41) [4 points]

Let  $D$  be the set  $\{(x_1, x_2) \in \mathbb{R}^2 : x_1 > 0 \ \& \ x_2 > 0\}$  and  $g$  the function  $D \rightarrow \mathbb{R}^{2 \times 2}$

$$(x_1, x_2) \mapsto \begin{pmatrix} 0 & x_1 x_2 \\ x_1 x_2 & 0 \end{pmatrix}$$

a) Please show that  $(D, g)$  is a pseudo-Riemannian domain

b) Please calculate the curvature of  $(D, g)$ .

Hint for b)

you may assume that  $D$  was a regular parametrization of a hypersurface in  $\mathbb{R}^3$  and calculate its gaussian curvature