

Problem sheet #10 DUE: 13th Jan.

Most exercise are just reminders.

(32) Given the map $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ [3 points]
 $(x_1, x_2) \mapsto (x_1 + x_2, x_1 \cdot x_2) = (f^1(x_1, x_2), f^2(x_1, x_2))$
 $= (y_1(x_1, x_2), y_2(x_1, x_2)).$

Please calculate the pullbacks $f^*(dy_1)$ and $f^*(dy_2)$. □

(33) Please give the definition of a pseudo-Riemannian and a Riemannian domain. [2 points] □

(34) Let $u: [0, 1] \rightarrow (D, g)$ be a smooth curve in a Riemannian domain. Please explain what
• is a variation of u [3 points]
• is the variational principle. □

Please characterize with the above items a geodesic curve. □

(35) [2 points]

Let X be a smooth mfd and a a point on X .

Please give the definition of
• a derivation in the point a
• the tangent space $T_a X$. □

(36) [1 point]

Once again X is a smooth mfd and $u: I \rightarrow X$ a smooth curve. What is denoted by $\dot{u}(t)$? □