Errata for:

Cohomology of Number Fields. Online Edition 2.3, May 2020
by J. Neukirch, A. Schmidt, K. Wingberg

This file lists known mistakes. If you find a mistake not listed below or have a comment, please send me an e-mail:
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-p. 23, l. -1 (noticed by D. Vogel) replace ‘§§’ by ‘§9’.

Errata for the printed version (Corrected Second Printing 2013)

-p. X, l. -18 and -16 (noticed by T. Wunder) replace ‘§4’ by ‘§5’ and ‘§5’ by ‘§6’.

-p. 8, l. 20–21 (noticed by Siyan “Daniel” Li) replace ‘If $A = A_U$ for some open subgroup $U \subseteq G$, then $\text{Hom}(A, B)$ is a discrete $G$-module. This is the case, for example,’ by ‘$\text{Hom}(A, B)$ is a discrete $G$-module’. (noticed by Siyan “Daniel” Li) replace ‘§5’ by ‘§8’.

-p. 23, l. -1 (noticed by D. Vogel) replace ‘§§’ by ‘§9’.

-p. 32, footnote (noticed by M. Lüdtke) replace ‘§7’ by ‘§8’.

-p. 39, l. 4 (noticed by D. Harari) replace ‘C’ by ‘C’.

-p. 62, l. 3 (noticed by S. Panda) Replace ‘, which’ by ‘. If $H$ is of finite index in $G$, it’ and replace ‘H is of finite index in $G$ and’ in line 5.

-p. 73, Exercises 2 and 3 (noticed by M. Lüdtke) In both exercises the subgroup $H$ should be normal.

-p. 89 (noticed by A. Holschbach) add before Lemma (1.9.8): ‘For an abelian profinite group $A$ and a prime number $p$, we denote by $A(p)$ the (unique) $p$-Sylow subgroup of $A$.’

-p. 181, Exercise 5 (noticed by an anonymous person) replace the assumption ‘$\text{cd}_p G / H \neq 0$’ by ‘$\text{cd}_p H \neq 0$’.

-p. 199, l. 9 (misleading argument) replace ‘$i$: $X \to F$. It satisfies condition (1) of (3.5.14), since $A$ is finite’ by ‘$i$: $X \to F$, which satisfies condition (1) of (3.5.14).’

-p. 210, l. 22 (noticed by O. Thomas) replace ‘for $A$’ by ‘for finite $A$’.

-p. 218, l. 12–15 (noticed by J. Minac) replace ‘$H$’ by ‘$U’ (three times).

-p. 228, l. 2 (noticed by A. Holschbach) replace ‘X §8’ by ‘X §10’.

-p. 228, l. -14 (noticed by A. Holschbach) replace ‘$\text{ker}(j|_G)$’ by ‘$\text{ker}(j|_A)$’.

-p. 239, l. 4 replace ‘$2 I$’ by ‘$2^{j-1}$’.

-p. 239, l. 5 replace ‘$2^I$’ by ‘$2$’.

-p. 239, l. 17 replace ‘alternating’ by ‘anti-symmetric’.

-p. 266, l. 12 (noticed by an anonymous person) replace ‘of $G_e \subseteq$’ by ‘of $G_e \times G_e \subseteq$’.

-p. 271, l. 12 (noticed by an anonymous person) replace ‘$1, \ldots, n$’ by ‘$1, \ldots, h$’.

-p. 290, l. 13 (noticed by L. Sauer) replace ‘$\text{deg}(v) = 0$’ by ‘the constant term of $v$ is a unit in $O$’.

-p. 310, l. -11 ff (noticed by O. Thomas) replace ‘for $D_{r-1}(M^\vee)$. Therefore $D_{r-1}(M^\vee) \otimes \mathbb{Q}/\mathbb{Z} = 0$ and’ by ‘for $D_{r-1}(M^\vee)$ and $D_r(M^\vee)$. Therefore $D_r(M^\vee) \otimes \mathbb{Q}/\mathbb{Z} = 0$ and’.

-p. 340, l. -7 (noticed by Siyan “Daniel” Li) the explicit formula for the map is wrong and has to be replaced by

$$(a_0, \ldots, a_{n-1}) \mapsto \bar{a}_0 p^{n-1} + \bar{a}_1 p^{n-2} + \cdots + \bar{a}_{n-1} p^{n-1} \mod p^n,$$

where $\bar{a}_i$ is some lift of $a_i$ to $\mathbb{Z}$.

-p. 393, (7.3.3) Lemma. The lemma is correct but not sufficient for the application in the proof of (7.3.2). It should be replaced by (7.3.3) Lemma.

(i) Let $A$ be a finite $G$-module. Then $[\iota A] = [A]_\iota$.

(ii) Let $V$ be a $G$-module such that $V_\ell$ and $\iota V$ are finite. If $W \subseteq V$ is a submodule of finite index, then $[V_\ell] = [\iota V] = [W_\ell] = [\iota W]$.

Proof: We prove (ii) first. Using a Jordan-Hölder series, we may assume that $V/W$ is a finite simple $G$-module. In particular, $\iota(V/W) \cong (V/W)_\ell$. Consider the diagram

$$
\begin{array}{cccccc}
0 & \to & W & \to & V & \to & V/W & \to & 0 \\
\downarrow{\iota} & & \downarrow{\iota} & & \downarrow{\iota} & & \\
0 & \to & W & \to & V & \to & V/W & \to & 0.
\end{array}
$$

The snake lemma gives the exact sequence

$$0 \to \iota W \to \iota V \to \iota(V/W) \to W_\ell \to V_\ell \to (V/W)_\ell \to 0,$$

and hence the result. Assertion (i) follows by applying (ii) to $V = A, W = 0$.

-p. 450, l. -11 (noticed by M. Leonhardt) Replace ‘$H^0(G, D_K) = D_\ell$’ by ‘$H^0(G, D_K) \cong D_\ell \oplus (\mathbb{Z}/2\mathbb{Z})^m$’.

-p. 450, l.-10 and l.-7 (noticed by M. Leonhardt) Replace ‘$S_\ell C' by 'S_\ell(K')'.

-p. 451, l.-9 (noticed by M. Leonhardt) Replace $\text{M}_{\ell}(D)$ by $\text{M}_{\ell}(D)$, $\text{N}_{\ell}(D_K) \cong \mathbb{Z}/2\mathbb{Z}$

-p. 451, l. -6 (noticed by A. Holschbach) replace ‘onto’ by ‘into’.

-p. 498, l.-1 replace ‘$\iota \lambda_\ell = \pi_\iota(\iota \lambda_\ell)$’ by ‘$\iota \lambda_\ell = \pi_\iota(\iota \lambda_\ell \circ \inf)$’.

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therefore obtain the commutative exact diagram (writing $E$)

- The word ‘realize’ might be misleading. To be more precise, one should replace the first sentence of the proof of (9.4.3) by the following: ‘We have to show that for every finite Galois extension $K_p|k_p$ with $G(K_p|k_p) \in \mathfrak{c}$ there exists a global Galois extension $L|k$ unramified outside $S$ with $G(L|k) \in \mathfrak{c}$ such that $K_p \subset L_p$."

- For $i \geq 2$, $A_i$ is not a $\Gamma$-module. However, $A_i$ is a $G/H_{i-1}$-module and, after refining the filtration, we may assume without loss of generality that it is simple.

- (noticed by T. Wunder) replace ‘For finite subextension’ by ‘finite, totally imaginary subextension’

- (noticed by O. Thomas) replace ‘realize’ by ‘is surjective’. In the proof replace ‘is an isomorphism’ by ‘is surjective’. In the proof replace ‘$E_{2,0} = \hat{E}_2^{2,0}$ by $E_{2,0}^{2,0} \to \hat{E}_2^{2,0}$."

- (noticed by O. Thomas) replace ‘finite number field (otherwise we cannot speak about density)"

- (noticed by O. Thomas) replace ‘$H^i(G(k_p|k_p'))' by ‘$H^i(G(k_p(p)|k_p'))'."

- (noticed by O. Thomas) replace ‘(10.5.1)(i)’ by ‘(10.5.1)’.

- (noticed by O. Thomas) replace ‘$\Gamma$’ by ‘$\mathfrak{c}$’.

- (noticed by T. Wunder) replace ‘chapter XII’ by ‘§11’.

- (noticed by A. Holschbach) The word ‘realize’ might be misleading. To be more precise, one should replace the first sentence of the proof of (9.4.3) by the following: ‘We have to show that for every finite Galois extension $K_p|k_p$ with $G(K_p|k_p) \in \mathfrak{c}$ there exists a global Galois extension $L|k$ unramified outside $S$ with $G(L|k) \in \mathfrak{c}$ such that $K_p \subset L_p$."

- (noticed by T. Wunder) replace ‘chapter XII’ by ‘§11’.

- (noticed by D. Neugber) replace ‘For finite $S$."

- (noticed by D. Neugber) replace ‘Comparing two copies of the upper sequence of (10.3.13) (for $T$ and $S$) we therefore obtain the commutative exact diagram (writing $E_{k'}$ for $O_{k'}^*$ and $G_{T'}$ for $G_{T'}(k')$)’ by ‘Passing to the inverse limit over the upper exact sequences of (10.3.13) for $T = \emptyset$ and all finite subsets of $S(k')$ containing $S_p \cup S_\infty$, and doing the same for $T^\prime$ instead of $S$, we obtain the commutative exact diagram (writing $E_{k'}$ for $O_{k'}^*$, $G_{S}$ for $G_S(k')$ and $G_{T'}$ for $G_{T'}(k')$)’."

- (noticed by M. Witte) replace ‘is a $\mathfrak{c}$-module and, after refining the filtration, we may assume without loss of generality that it is simple."

- (noticed by M. Witte) replace ‘is an isomorphism’ by ‘is surjective’. In the proof replace ‘$E_{2,0} = \hat{E}_2^{2,0}$ by $E_{2,0}^{2,0} \to \hat{E}_2^{2,0}$’."

- (noticed by O. Thomas) According to our convention in section 10.5, the number field $k$ in (10.5.8)–(10.5.11) should be a finite number field (otherwise we cannot speak about density)"