Errata for:
Cohomology of Number Fields. Online Edition 2.3, May 2020
by J. Neukirch, A. Schmidt, K. Wingberg

This file lists known mistakes. If you find a mistake not listed below or have a comment, please send me an e-mail: schmidt@mathi.uni-heidelberg.de.
No mistakes known at the moment.

Errata for the printed version (Corrected Second Printing 2013)
- p. X, l. 18 and -16 (noticed by T. Wunder) replace ‘§4’ by ‘§5’ and ‘§5’ by ‘§6’.
- p. 8, l. 18–21 (noticed by Siyan ‘Daniel’ Li) replace ‘If $A = A^U$ for some open subgroup $U \subseteq G$, then $\text{Hom}(A, B)$ is a discrete $G$-module. This is the case, for example, by $\text{Hom}(A, B)$ is a discrete $G$-module’.
- p. 32, footnote (noticed by M. Lüdtke) replace ‘§7’ by ‘§8’.
- p. 39, l. 4 (noticed by D. Harari) replace ‘$C$’ by ‘$C$’.
- p. 62, l. 3 (noticed by S. Panda) Replace ‘, which’ by ‘. If $H$ is of finite index in $G$, it’ and remove ‘$H$ is of finite index in $G$ and’ in line 5.
- p. 73, Exercises 2 and 3 (noticed by M. Lüdtke) In both exercises the subgroup $H$ should be normal.
- p. 89 (noticed by A. Holschbach) add before Lemma (1.9.8): ‘For an abelian profinite group $A$ and a prime number $p$, we denote by $A(p)$ the (unique) $p$-Sylow subgroup of $A$.’
- p. 181, Exercise 5 (noticed by an anonymous person) replace the assumption ‘$cd_p G/H \neq 0$’ by ‘$cd_p H \neq 0$’.
- p. 199, l. 9 (noticed by O. Thomas) replace ‘onto’ by ‘into’.
- p. 218, l. 12–15 (noticed by J. Minac) replace ‘$H$’ by ‘$U$’ (three times).
- p. 228, l. 2 (noticed by A. Holschbach) replace ‘$X$ §8’ by ‘$X$ §10’.
- p. 239, l. 4 (noticed by M. Leonhardt) Replace ‘$X$ §8’ by ‘$X$ §10’.
- p. 239, l. 5 replace ‘$2^{-1}$’ by ‘$2$’. 
- p. 266, l. 12 (noticed by an anonymous person) replace ‘of $G_t \subseteq \hat{X}$ by ‘of $G_t \times G_t \subseteq \hat{X}$’.
- p. 271, l. 12 (noticed by an anonymous person) replace ‘$1, \ldots, n$’ by ‘$1, \ldots, h$’.
- p. 290, l. -9 and l.-7 (noticed by M. Leonhardt) replace ‘$\hat{X}$’ by ‘$\hat{X}$’.
- p. 290, l. -11 (noticed by L. Sauer) replace ‘$\ker(j|G)$’ by ‘$\ker(j|A \cdot G)$’.
- p. 310, l. -11 ff (noticed by Siyan ‘Daniel’ Li) the explicit formula for the map is wrong and has to be replaced by

$$\begin{align*}
(a_0, \ldots, a_{n-1}) & \mapsto \bar{a}_0^{p^{n-1}} + \bar{a}_1^{p^{n-2}} p + \cdots + \bar{a}_{n-1} p^{n-1} & \mod p^n,
\end{align*}$$

where $\bar{a}_i$ is some lift of $a_i$ to $\mathbb{Z}$.
- p. 393, (7.3.3) Lemma. The lemma is correct but not sufficient for the application in the proof of (7.3.2). It should be replaced by (7.3.3) Lemma.

(i) Let $A$ be a finite $G$-module. Then

$$[\ell A] = [A]_\ell.$$

(ii) Let $V$ be a $G$-module such that $V_\ell$ and $\ell V$ are finite. If $W \subseteq V$ is a submodule of finite index, then

$$[V_\ell] = [\ell V] = [W_\ell] - [W].$$

Proof: We prove (ii) first. Using a Jordan-Hölder series, we may assume that $V/W$ is a finite simple $G$-module. In particular, $\ell(V/W) \cong (V/W)_\ell$. Consider the diagram

$$
\begin{array}{ccc}
0 & \longrightarrow & W & \longrightarrow & V & \longrightarrow & V/W & \longrightarrow & 0 \\
\downarrow \ell & & \downarrow \ell & & \downarrow \ell & & \\
0 & \longrightarrow & W & \longrightarrow & V & \longrightarrow & V/W & \longrightarrow & 0.
\end{array}
$$

The snake lemma gives the exact sequence

$$0 \rightarrow \ell W \rightarrow \ell V \rightarrow \ell(V/W) \rightarrow W_\ell \rightarrow V_\ell \rightarrow (V/W)_\ell \rightarrow 0,$$

and hence the result. Assertion (i) follows by applying (ii) to $V = A, W = 0$. 

- p. 450, l. -11 (noticed by M. Leonhardt) Replace ‘$H^0(G, D_K) = D_\ell$’ by ‘$H^0(G, D_K) \cong D_K \oplus (\mathbb{Z}/2\mathbb{Z})^m$’.
- p. 450, l. 10 and l.7 (noticed by M. Leonhardt) Replace ‘$SC$’ by ‘$SC(K)$’.
- p. 451, l. -9 (noticed by M. Leonhardt) Replace the last paragraph of the proof of (8.2.6) by: Finally note that $N_K\mid K D_K = D_K$ by (8.2.1)(iv) and that $D_K$ is divisible by (8.2.1)(vi). Hence the calculation of $H^0(G, D_K)$ follows from that of $H^0(G, D_K)$.
- p. 460, l. 6 (noticed by A. Holschbach) replace ‘onto’ by ‘into’.
- p. 498, l. -1 replace ‘$\im\lambda' \tau = \tau' \im(\lambda' \tau)$’ by ‘$\im\lambda' \tau = \tau' \im(\lambda' \tau \circ \im \tau)$’.
- p. 507, l. -2 (noticed by A. Holschbach) replace ‘finite subextension’ by ‘finite, totally imaginary subextension’.

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-p. 532, l. 5 (noticed by A. Holschbach) replace \((k, S, m)\) by \((k, m, S)\).

-p. 554, l. 17 (noticed by A. Holschbach) The word 'realize' might be misleading. To be more precise, one should replace the first sentence of the proof of (9.4.3) by the following: ‘We have to show that for every finite Galois extension \(K_p/k_p\) with \(G(K_p/k_p) \in \epsilon\) there exists a global Galois extension \(L|k\) unramified outside \(S\) with \(G(L|k) \in \epsilon\) such that \(K_p \subset L_p\).’

-p. 569, l. -7 (noticed by M. Jarden) For \(i \geq 2\), \(A_i\) is not a \(\Gamma\)-module. However, \(A_i\) is a \(G/H_{i-1}\)-module and, after refining the filtration, we may assume without loss of generality that it is simple.

-p. 600, l. -15 (noticed by T. Wunder) replace ‘1’ by ‘2’.

-p. 621, l. -12 (noticed by T. Wunder) replace ‘\(k\)’ by ‘\(K\)’ (twice).

-p. 622, l. 15 (noticed by T. Wunder) replace ‘\(G_T\)’ by ‘\(G_T(c)\)’.

-p. 630, l. 7 (noticed by D. Neugber) replace ‘For S’ by ‘For finite S’.

-p. 637, l. 7 (noticed by T. Wunder) replace ‘\(G\)’ by ‘\(G_K\)’.

-p. 649, l. 10ff (noticed by D. Neugber) replace ‘Comparing two copies of the upper sequence of (10.3.13) (for \(T\) and \(S\)) we therefore obtain the commutative exact diagram (writing \(E_{k'}^r\) for \(\mathcal{O}_{k'}^r\) and \(G_T\) for \(G_{T(k')}\)) by ‘Passing to the inverse limit over the upper exact sequences of (10.3.13) for \(T = \emptyset\) and all finite subsets of \(S(k')\) containing \(S_p \cup S_\infty\), and doing the same for \(T\) instead of \(S\), we obtain the commutative exact diagram (writing \(E_{k'}^r\) for \(\mathcal{O}_{k'}^r\), \(G_S\) for \(G_S(k')\) and \(G_T\) for \(G_T(k')\))’.

-p. 650 (10.5.5) Corollary. (noticed by M. Witte) replace ‘is an isomorphism’ by ‘is surjective’. In the proof replace ‘\(E_2^{2,0} = E_\infty^{2,0}\)’ by ‘\(E_2^{2,0} \twoheadrightarrow E_\infty^{2,0}\)’.

-p. 652 ff. (noticed by O. Thomas) According to our convention in section 10.5, the number field \(k\) in (10.5.8)–(10.5.11) should be a finite number field (otherwise we cannot speak about density)

-p. 653, l. -4 (noticed by O. Thomas) replace ‘\(H^i(G(k_p(p)|k_p'))\)’ by ‘\(H^i(G(k_p(p)|k_p'))\)’.

-p. 679, l. -12 (noticed by O. Thomas) replace ‘(10.5.1)(i)’ by ‘(10.5.1)’.

-p. 683, l. -4 (noticed by O. Thomas) replace ‘\(I_S(k)\)’ by ‘\(I_S(k)/p\)’.

-p. 695, l. 7 (noticed by T. Wunder) replace ‘\(k_S/K\)’ by ‘\(k_S|K\)’.

-p. 689, l. -5, 2, p. 690 l. 1 (noticed by M. Witte) replace ‘\(\mathcal{P}^2(k_S, S_0, -)\)’ by ‘\(\mathcal{P}^2(k_S, S \setminus S_0, -)\)’.

-p. 701, l. -10, (noticed by O. Thomas) replace ‘\(Cl(k)\)’ by ‘\(Cl_T(k)\)’.

-p. 781, l. 11, p. 784, l. 16 (noticed by H. Johnston) replace ‘[61]’ by ‘[64]’.

-p. 789, l. 16 (noticed by M. van Frankenhuijsen) replace ‘\(q=\)’ by ‘\(q-1=\)’.

-p. 796, l. 16 remove ‘\(i.e. p\) splits completely in \(k_2(Q)\)’.


last update: May 18, 2020 by Alexander Schmidt