Errata and complementary remarks for:

Cohomology of Number Fields, first ed. (2000) (by J. Neukirch, A. Schmidt, K. Wingberg)

This file lists a number of mistakes in the first edition and gives some remarks on the text which either did not find their way into the book, or refer to results which were proven after the book was written.

This file is not maintained anymore. If you find a mistake not listed below or have a comment, please have a look at the free online edition and its errata-file, available on my homepage.

-p.3 l.-14 (noticed S. Knorr) replace 'we use' by 'we make use'.
-p.5 l.-3 (noticed S. Knorr) replace 'exist' by 'exists'.
-p.7 l.-15 (noticed P. Barth) replace 'basis' by 'subbasis'.
-p.8 l.-5 The claimed universal property seems to be incorrect.
-p.30 l.-13 (noticed by P. Barth) replace '$x \mapsto \sum_{\sigma\in G} x(\sigma^{-1}) \otimes \sigma$' by '$x \mapsto \sum_{\sigma\in G} x(\sigma) \otimes \sigma$'
-p.30 l.-7 (noticed by M. Breuning) remove 'when A is induced' (in the general case, Hom(A,B) is not always discrete).
-p.39 l.2 Replace 'isomorphisms' by 'surjective'.
-p.39 l.3 ff (noticed by M. Breuning) The paragraph should be altered in the following way: For discrete G-modules A and B let G act on $\mathrm{Hom}(A,B)$ by $(g\phi)(a) = g\phi(g^{-1}a)$. If G is finite or if A is finitely generated as a $\mathbb{Z}$-module, then $\mathrm{Hom}(A,B)$ is a discrete G module and the canonical pairing .... Then, in prop.(1.4.5) one has to assume either that G is finite or that A is finitely generated as a $\mathbb{Z}$-module.
-p.41 l.18 (noticed by T. Schmidt) replace '$(f \cdot g)$' by '$\partial(f \cdot g)$'
-p.50 l.15 (noticed by S. Knorr) replace 'Hom_{cts}' by 'Hom_{cont}'.
-p.52 l.-13 add a second ')'
-p.69 l.-12 (noticed by C. Greither) replace this line by

$$G \cong \prod_{p\in T} \mathbb{Z}/p^{e_p}\mathbb{Z} \times \prod_{p\in S} \mathbb{Z}_p,$$

where T and S are disjoint sets of prime numbers and e_p, $p \in T$, are natural numbers.
-p.78 l.4 add the condition $E^n = 0$ for $n \ll 0$ and $n \gg 0$. (of course this can be weakened, in particular if $\mathcal{A}$ has limits.
-p.80 l.14 (noticed by S. Firouzian) replace '$F^0E^n = E^n$' by '$F^{n+1}E^n = 0$' and remove 'and $F^{n+1}E^n = 0$' in the next line.
-p.81 l.2 ff (noticed by J. Stix) Some arrows go into the false direction. Replace these lines by: 'Namely, in this case we have isomorphisms $E^{0,n}_{n+1} \xrightarrow{\sim} \cdots \xrightarrow{\sim} E^{0,n}_2$ and $E^{n+1,0}_2 \xrightarrow{\sim} \cdots \xrightarrow{\sim} E^{n+1,0}_{n+1}$. Therefore the differential $E^{0,n}_{n+1} \to E^{n+1,0}_{n+1}$ induces a homomorphism $E^{0,n}_2 \xrightarrow{d} E^{n+1,0}_2$.'
-p.81 l.9 add 'and all p' after '$0 < q < n$'.
-p.81 l.-12 better formulation: replace 'if $p > m$ or $q > n$' by 'if ($p > m$ and $q < n$) or ($p < m$ and $q > n$)'. In particular, after this modification, the following 'In particular' makes sense.
-p.83 l.-12 (noticed by J. Stix) replace '$\tau_0, \tau_1 \in H$' by '$\tau_0, \tau_1 \in G/H$'
-p.86 l.13 replace '$=$' by '$\equiv$'
-p.87 l.-1, l.-2 (noticed by J. Stix) replace '$\bar{G}$' by '$\bar{G}/\bar{H}'$' and 'G' by 'G/H'', where $U' = [U,U]$ for a profinite group U
-p.88 l.2 (noticed by J. Stix) replace 'U^{ab}' by 'H^{ab}' and '$\bar{U}^{ab}$' by '$\bar{H}^{ab}$'
-p.91 l.4 (noticed by J. Stix) replace '$H^m(G,A)^*$' by '$H^{n-(p+q)}(G,A)^*$'
-p.91 l.-11, l.-12 (noticed by J. Stix) replace '$H^p(G',A)^*$' by '$H^{n-p}(G',A)^*$' and '$H^p(G,A)^*$' by '$H^{n-p}(G,A)^*$'
-p.94 l.10 ff (noticed by J. Stix) replace '$H^p(G/U, H^0(U, \tilde{X}^{n-q}(G,A)^*))$' by '$C^p(G/U, H^0(U, \tilde{X}^{n-q}(G,A))^*)$'. The next lines should be replaced by: 'We have a canonical pairing $\varphi : (\tilde{X}^{n-q}(G,A)^U)^* \times A \longrightarrow (\tilde{X}^{n-q}(G,\mathbb{Z})^U)^*, \quad (\chi, a) \longmapsto \varphi(\chi,a) = f$, where $f : \tilde{X}^{n-q}(G,\mathbb{Z})^U \longrightarrow \mathbb{Q}/\mathbb{Z}$ is defined by $f(x) = \chi(ax)$. If $z(\sigma_0,\dots,\sigma_{p-j})$ is a $(p-j)$-cochain with coefficients in $(\tilde{X}^{n-q}(G,A)^U)^*$ and $t(\sigma_0,\dots,\sigma_j)$ is an j-cocycle in $Z^j(G,A)$, then $(z \cup t)$ is a p-cochain with coefficients in $(\tilde{X}^{n-q}(G,\mathbb{Z})^U)^*$. Thus we get a map $C^{p-j,q}(A) \times Z^j(G,A) \longrightarrow C^{p,q}(\mathbb{Z})$, i.e. for fixed $t \in Z^j(G,A)$ we have a morphism of double complexes $\cup t : C^{\bullet - j,\bullet}(A) \longrightarrow C^{\bullet,\bullet}(\mathbb{Z})$ of degree $(j,0)$ and hence a transformation of the associated edge morphisms.'
-p.98 l.2 (noticed by J. Stix) replace '$H^q(u) = 0$ for $q > 0$' by '$H^{n+1}(u) = 0$'; delete the diagram in l.4 - l.5
-p.99 l.13 (noticed by M. Föhl) replace 'Mod^t' by 'Mod_t'
-p.99 l.14 (noticed by M. Föhl) replace '$Mod^{(p)}$' by '$Mod_{(p)}$'
-p.99 l.-13 (noticed by M. Föhl) replace 'left derived' by 'right derived'
-p.101 l.13 replace 'discrete' by 'abstract'.
-p.101 l.14 replace 'a given a' by 'a given'.
-p.101 l.-7 (noticed by G. Wiese) replace 'exist' by 'exists'.
-p.102 l.13 This shows only that the E_2 and E_∞-terms coincide. A proof that the spectral sequences are actually isomorphic can be found in [60] (the functor 'homogeneous cochain complex' is a 'resolving functor').
-p.107 l.14 (noticed by P. Barth) as we are working with the homogenous cochain complex, the cup-product is defined by the formula

$$(f \cup g)(\sigma_0,\dots,\sigma_{p+q}) = f(\sigma_0,\dots,\sigma_p) \cdot g(\sigma_p,\dots,\sigma_{p+q}).$$

-p.110 l.4 replace '$(H^i(X^\bullet(G,A_n)))$' by '$(X^i(G,A_n)^G)$'.
-p.110 l.5 (noticed by J. Stix) replace the statement of Corollary (2.3.5) by the following: 'Let A be a compact G-module having a presentation $A = \varprojlim_{n\in\mathbb{N}} A_n$ as a countable inverse limit of finite, discrete G-modules. If $H^i(G,A_n)$ is finite for all n, then $H^{i+1}_{cts}(G,A) = \varprojlim_n H^{i+1}(G,A_n)$.'

-p.117 l.12 (noticed by T. Schmidt) replace 'then $H^1(H, C) = 0$, so ...' by 'then $H^1(H, C) = 0$ and ...'
-p.127 l.3 replace 'was' by 'has'
-p.132, l.-14 (noticed by A. Leesch) replace '0' by '1'
-p.133 l.5 remove the word 'been'
-p.142 l.13 ff (noticed by T. Wedhorn): In both diagrams of the proof of (3.3.8) one has to replace the module X^n by the (cohomologically trivial) module $A_n = ker(X^n \to X^{n+1})$.
-p.147 l.7 remove the $*$ in the second factor
-p.148 l.-8 (noticed by D. Harari) replace 'A^U' by '${}_m A^U$'.
-p.148 l.-4 (noticed by D. Harari) replace 'and we ...' by 'and, if A is finite, we ...'.
-p.158 l.7 (noticed by A. Matar) replace 'abelian' by 'cyclic'. By the Hochschild-Serre spectral sequence, the upper sequence in (3.6.2) can be completed on the left by $H_2(U/V, \hat{\mathbb{Z}})$, which is zero if U/V is finite cyclic.
-p.159 l.14 (noticed by L. Wan) replace '$\#U/V = p^{n-1}$' by '$\#U/W = p^{n-1}$'
-p.159, l. 1.-9 – -7 (noticed by L. Wan) replace the text following 'otherwise' by '$1 = p^{n-1}u_{U/V} = p^{n-2}i(u_{U/W})$ by (3.6.1). As i is injective, this implies $p^{n-2}u_{U/W} = 1$, contradicting the induction hypothesis.
-p. 165, l. -6 (noticed by L. Wan) replace 'scd' by 'scd_p' (twice)
-p. 168 l. 1 see Theorem 1 of our paper Extensions of profinite duality groups for the most general version of Theorem 3.7.4 known to us.
-p. 168 l. 11 replace '$E_2^{ij}(h, A)$' by '$E_2^{ij}(g, h, A)$'
-p. 168 l. -6 replace '$\varinjlim_{g} \varinjlim_{h'}$' by '$\varinjlim_{h'} \varinjlim_{g/h}$'
-p.175 l.-2 (noticed by D. Vogel) replace '$rs \neq 0$' by '$r + s \neq 0$'
-p.181, l.-12 (noticed by Z. Chen) replace 'In this case' by 'If $\mathscr{R}$ is finite and minimal'
-p.190 l.5 (noticed by J. Gärtner) replace '$\chi_1 \cup \chi_2$' by '$\chi_k \cup \chi_l$'
-p.196 l.5 (noticed by M. Föhl) replace '$F^{i+1} = (F^i)^p[F^i, F]$' by '$F^{i+1} = (F^i)^q[F^i, F]$'.
-p.196 l.9 (noticed by M. Föhl) replace '$r(y) = y^q(y_1, y_2)\cdots$' by '$r(y) = y_1^q(y_1, y_2)\cdots$'.
-p.196 l.-9 (noticed by M. Föhl) replace '$\mathrm{gr}_i(F)$' by '$\mathrm{gr}_1(F)$'.
-p.204 l.11 replace the first '=' by '$\longrightarrow$'.
-p.205 l.20 replace '$\pi \circ s' = s$' by '$\pi \circ s' = id$'
-p.208 l.-13 replace 'is the theorem' by 'the theorem is'
-p.224 l.14 (noticed by S. Schmidt) replace 'M' by '$T_A(M)$'.
-p.227 l.-4 ff We did not assume that $\mathcal{O}$ is noetherian, therefore we have to assume that no only $\mathcal{O}/\mathfrak{m}$ is finite, but that $\mathcal{O}/\mathfrak{m}^n$ is finite for all n, in order to obtain compactness (counterexample: $\mathcal{O}$ = power series ring in countable many variables over a finite field).
-p.237 l.3 remove 'that'
-p.237 l.13 (noticed by P. Schneider) Assertion a) in (5.2.16)(ii) follows from b)-d) only under the additional assumption that G is topologically finitely generated. The point is that the powers of Rad_G need not to be open, even if Rad_G is open. This happens, for example, if G is the abelianization of the free pro-p group on countable many generators.

To show b)⇒ a) if G is topologically finitely generated, we have to show that the powers of Rad_G are open if Rad_G is open. Choose $n \in \mathbb{N}$ and an open normal subgroup $U \subset G$ with $I := \mathfrak{m}^n\mathcal{O}[\![G]\!] + I(U) \subset \mathrm{Rad}_G$. It suffices to show that the powers of I are open. First of all, we note that a finitely generated ideal in a compact ring is closed, thus the notions 'topologically finitely generated' and 'finitely generated' coincide for ideals in $\mathcal{O}[\![G]\!]$. The ring $\mathcal{O}$ is compact in its $\mathfrak{m}$-adic topology. Hence $\mathfrak{m}/\mathfrak{m}^2 \subset \mathcal{O}/\mathfrak{m}^2$ is finite and the topological Nakayama lemma (5.2.18) (for $G = 1$) implies that $\mathfrak{m}$ and hence also $\mathfrak{m}^n$ is finitely generated. If G is topologically finitely generated, then so is U and therefore $I(U)$ is a finitely generated ideal in $\mathcal{O}[\![G]\!]$. We conclude that I and all its powers are finitely generated, in particular, they are closed ideals in $\mathcal{O}[\![G]\!]$. It therefore remains to show that I^m has finite index in $\mathcal{O}[\![G]\!]$ for all m. To begin with, the ring $\mathcal{O}[\![G]\!]/I$ is finite. Furthermore, for all m, the ideal I^m is finitely generated. Hence I^m/I^{m+1} is finitely generated over $\mathcal{O}[\![G]\!]/I$, thus finite. The assertion follows by induction.
-p.238 l.12 (noticed by M. Witte) It is not true in a general (noncommutative) ring that any nilpotent element is contained in the radical. Hence here is a gap in the argument which can be filled as follows. We start with the following observation: let K be a field of characteristic $p > 0$ and let P be a finite p-group. Then (same argument as in the proof 1.7.4), K with trivial P-action is the only simple left $K[P]$-module. Hence the augmentation ideal in $K[P]$ is the only left maximal ideal and thus equal to the radical. Since the radical of an artinian ring is nilpotent, we find a natural number n such that $\prod_{i=1}^n (x_i - 1) = 0$ in $K[P]$ for any elements $x_1, \ldots, x_n \in P$.

Now we return to the proof of 5.2.16 d)⇒b). Using the identity $g(u-1)g'(u'-1) = gg'((g')^{-1}ug' - 1)(u'-1)$ and the observation above, we conclude that the image of $a(u-1)$ in $\mathcal{O}/\mathfrak{m}[G/V]$ is nilpotent for any $a \in \mathcal{O}[\![G]\!]$, $u \in U$. Hence the image of $1 - a(u-1)$ is unit for any a, and so $u - 1$ is contained in the radical of $\mathcal{O}/\mathfrak{m}[G/V]$.
-p.238 l.-10 replace 'a pro-p-group' by 'a finitely generated pro-p-group'
-p.241 l.16 (noticed by M. Witte) replace 'generated' by 'presented'
-p.242 l.-9 replace '[15], chap.VIII, §3, no.8' by '[15], chap.VII, §3, no.8'
-p.245 l.-10 (noticed by O. Venjakob) replace 'polynomial' by 'Weierstraß polynomial'
-p.248 l.7 (noticed by S. Schmidt) 5.3.14 (v) is only true if M is a Λ-torsion module.
-p.251 l.3 (noticed by S. Schmidt) replace '$n \geq \frac{\lambda(\lambda-1)}{2}$' by '$n > \frac{\lambda(\lambda-1)}{2}$' (this correction is necessary if $p = 2$).
-p.251 l.10 (noticed by D. Vogel) replace 'n_0' by 'n_1' (2x) and insert 'where $n_1 > \max(n_0, \frac{1}{2}\lambda(\lambda-1))$'.

-p. 251, l.-12 (noticed by A. Leesch) one should not use T as the variable of the characteristic polynomial of the endomorphism T: replace 'T^λ' by 'X^λ'
-p.251 l.-8ff (noticed by S. Schmidt) To include the case $p=2$, the proof of (the corrected version of) (5.3.18) should read:
$\ldots$. Therefore $\gamma^{p^m} = 1$ on $M \otimes_{\mathbb{Z}_p} \mathbb{F}_p$ for $m \geq \frac{\lambda(\lambda-1)}{2}$. Now let $n > \frac{\lambda(\lambda-1)}{2}$ and let $A \in M(\lambda \times \lambda, \mathbb{Z}_p)$ be the matrix corresponding to the action of γ on M with respect to some basis. Then

$$A^{p^{n-1}} \equiv I \mod p$$

and so

$$A^{p^n} \equiv I \mod p^2,$$

where I denotes the unit matrix. It follows that

$$A^{p^n(p-1)} + \cdots + A^{p^n} + I \equiv pI \mod p^2,$$

-p.267 l.7. 'From (5.4.9)(iii) $\ldots$' This is somewhat unlucky since we have already used this reference in order to justify Definition (5.5.1). According to (5.5.1) the exactness of the sequence in line 9 is trivial.
-p.267 l.16 (shorter argument, noticed by D. Vogel) The exact sequence in line 15 shows pd $DM \leq 1$ and therefore DM has no nontrivial finite Λ-submodules by (5.3.19).
-p.268 l.14ff (noticed by S. Schmidt) These lines can be deleted, since the equivalence of (b) and (c) in (5.5.3)(iv) follows from (5.3.19)(i).
-p.291 l.-3ff (noticed by N. Naumann) The map $\wp := F - \mathrm{id}$ is not given explicitly as written here (addition in W_n is not component-wise). Furthermore, one needs the assumption that R is an integral domain in order to conclude that $W_n(\mathbb{F}_p) \cong \mathbb{Z}/p^n\mathbb{Z}$ is the kernel of $\wp : W_n(R) \to W_n(R)$.
-p.292 l.7 (noticed by N. Naumann) The surjectivity of $\wp : W_n(\bar{K}) \to W_n(\bar{K})$ in the sequence $(*)$ is obvious only in the case $n = 1$. For $n > 1$ one can argue by induction using the exact diagram

$$\begin{tikzcd}
0 \arrow[r] & W_n(\bar{K}) \arrow[r, "V"] \arrow[d, "\wp"] & W_{n+1}(\bar{K}) \arrow[r] \arrow[d, "\wp"] & W_1(\bar{K}) \arrow[r] \arrow[d, "\wp"] & 0 \\
0 \arrow[r] & W_n(\bar{K}) \arrow[r, "V"] & W_{n+1}(\bar{K}) \arrow[r] & W_1(\bar{K}) \arrow[r] & 0.
\end{tikzcd}$$

-p.295 l.2 replace 'of φ the' by 'of φ to the' (noticed by D. Vogel)
-p.303 (noticed by Y. T. Lam): theorem (6.3.8) and its proof are incorrect. A correct formulation is the following:
(6.3.8) Theorem. *For every Galois extension $L|K$ and every $n \geq 1$ there exists a natural injective map*

$$BS_n(L|K) \hookrightarrow Br(L|K)$$

whose image is the set of classes in $Br(L|K)$ which are represented by a central simple algebra of dimension n^2 over K. If $[L:K]$ is finite and divides n, then the above map is bijective.
Proof: The exact sequence of $G(L|K)$-groups

$$1 \longrightarrow L^\times \longrightarrow GL_n(L) \longrightarrow PGL_n(L) \longrightarrow 1$$

induces a map (see I §3 ex.8)

$$\delta : BS_n(L|K) \cong H^1(G(L|K), PGL_n(L)) \longrightarrow H^2(G(L|K), L^\times).$$

A K-algebra A is central simple of dimension n^2 and splits over L if and only if $A \otimes_K L \cong M_n(L)$. Since $\mathrm{Aut}_L(M_n(L)) = PGL_n(L)$, the same arguments as in the case of Brauer-Severi varieties show that the set $CSA_n(L|K)$ of K-isomorphism classes of such algebras is naturally isomorphic to $H^1(G(L|K), PGL_n(L))$. Thus we obtain a diagram

$$\begin{tikzcd}
H^1(G(L|K), PGL_n(L)) \arrow[r, "\delta"] \arrow[d, "\wr"] & H^2(G(L|K), L^\times) \arrow[d, "\wr"] \\
CSA_n(L|K) \arrow[r, "can"] & Br(L|K),
\end{tikzcd}$$

where the map can sends a central simple algebra to its similarity class in $Br(L|K)$. Two similar central simple algebras of the same dimension are isomorphic and therefore can is injective. The statement of the theorem now follows from the fact (see [94], V §30) that the above diagram commutes. Finally, if $n = m \cdot [L:K]$, then the class in $H^2(G(L|K), L^\times)$ of a 2-cocycle α is represented by the algebra $C(L, G(L|K), \alpha) \otimes_K M_m(K)$.
-p.306 l.-6 (noticed by D. Vogel) replace '$H^n(K, \mu_N^{\otimes n})$' by '$H^n(F, \mu_N^{\otimes n})$'
-p.307 l.1 replace 'X^n' by 'X^N'.
-p.307 l.9,10 (noticed by Z. Chen) add the subscript 'F_i' to '$(1-a_i, a_i)$' and '$(1-a_i, a_i^N)$'
-p.359 l.-2 (noticed by Z. Chen) replace 'K' by 'F'.
-p.307 l.-7 In the meantime a proof of the Milnor conjecture has been published by V. Voevodsky, see [V1], [V2].
-p.310 l.5 (noticed by J. Stix) replace 'number' by 'integer'.
-p. 316, l. 2 (noticed by L. Wan) replace 'algebraic' by 'separable'.
-p.317 l.1 replace 'K' by '$k(t_1, \ldots, t_n)$'.
-p.317 l.-12 (noticed by J.-L. Colliot-Thélène) In fact, this is known: $\mathbb{Q}_p$ is not C_i for any i, see [AK] (see also the review of D. Coray in mathscinet). In contrast, notice that the fields $\mathbb{F}_p((X))$ are C_2, see [54], Cor. 4.9.
-p.317 l.-6 In the meantime a proof of the Milnor conjecture has been published by V. Voevodsky, see [V1], [V2].
-p.319, l.-6 (noticed by A. Leesch) replace '0' by '1'

-p.323 l.12 (noticed by J. Stix) insert: 'assume that the order of tors(A) is prime to char(k). If ...' .
-p. 323, l.-15–-13 (noticed by T. Keller) replace these lines by
'together with $cd_p\Gamma = 1$ yields by (2.1.4)

$$H^{i+1}(k, A) \cong H^i(\Gamma, H^1(\tilde{k}, A)) = 0 \text{ for } i \geq 2,$$

hence $cd_p(k) \leq 2$.'
Furthermore, it would be quicker to refer to (3.3.7) in order to conclude that $cd_p(k) \leq 2$.
-p.329 l.10 (noticed by Rongzheng Jiao) replace '$K^\times$' by '$k^\times$'.
-p.332 l.-8 replace '(7.2.12)' by '(7.2.13)' (noticed by D. Vogel)
-p.332 l.-4 (noticed by J. Stix) replace '$\delta : k^\times/k^{\times m} \xrightarrow{\sim} H^1(k, \mu_m)$' by '$-\delta : k^\times/k^{\times m} \xrightarrow{\sim} H^1(k, \mu_m)$'.
-p.335 l.13 (noticed by M. Föhl) replace '$H^0(T_0, \mathrm{Hom}(\mathrm{tor}(A), \mu)^*$' by '$H^0(T_0, \mathrm{Hom}(\mathrm{tor}(A), \mu))^*$'
-p.339 l.4 (noticed by J. Stix) replace ' Let V be a finitely generated $\mathbb{Z}_\ell[G]$-module' by 'Let V be a $\mathbb{Z}_\ell[G]$-module such that V_ℓ and ${}_\ell V$ are finite'.
-p. 339, l. 7 (noticed by J. Stix) The proof of (i) is incorrect (A is not an $\mathbb{F}_p[G]$-module). Replace the proof by the following:
Proof: We prove (ii) first. Using a filtration of V/W by $\mathbb{Z}_\ell[G]$-modules such that the subquotients are $\mathbb{F}_\ell[G]$-modules, we may assume that V/W is an $\mathbb{F}_\ell$-module. Consider the diagram

$$\begin{tikzcd} 0 \arrow[r] & W \arrow[r] \arrow[d, "\ell"'] & V \arrow[r] \arrow[d, "\ell"'] & V/W \arrow[r] \arrow[d, "\ell"'] & 0 \\ 0 \arrow[r] & W \arrow[r] & V \arrow[r] & V/W \arrow[r] & 0. \end{tikzcd}$$

The snake lemma gives the exact sequence

$$0 \longrightarrow {}_\ell W \longrightarrow {}_\ell V \longrightarrow V/W \longrightarrow W_\ell \longrightarrow V_\ell \longrightarrow V/W \longrightarrow 0,$$

and hence the result. Assertion (i) follows by applying (ii) to $V = A$, $W = 0$.
-p.341, proof of (3.7.4) (noticed by L. Yongqi). Delete the sentence starting in line -15 and then argue as follows: Let $H = H^{(\ell')} \times H_\ell$ be a cyclic subgroup of G where H_ℓ is the ℓ-Sylow subgroup of H. Let M be a simple $\mathbb{F}_\ell[H]$-module. By (1.7.4), $M^{H_\ell} \neq 0$. Since M^{H_ℓ} is also an H-module, we obtain $M = M^{H_\ell}$, hence an isomorphism of $\mathbb{F}_\ell[H]$-modules $\mathrm{Ind}_H^{H^{(\ell')}} \mathrm{Res}^H_{H^{(\ell')}} M \cong M \otimes_{\mathbb{Z}} \mathbb{F}_\ell[H_\ell]$. Therefore the class of $\mathrm{Ind}_H^{H^{(\ell')}} \mathrm{Res}^H_{H^{(\ell')}} M$ in $K_0'(\mathbb{F}_\ell[H])$ is $n[M]$, where $\#H_\ell = \ell^n$, and so the images of $K_0'(\mathbb{F}_\ell[H]) \otimes \mathbb{Q}$ and $K_0'(\mathbb{F}_\ell[H^{(\ell')}]) \otimes \mathbb{Q}$ under $\mathrm{Ind} \otimes \mathbb{Q}$ in $K_0'(\mathbb{F}_\ell[G]) \otimes \mathbb{Q}$ are the same.
-p.342 l.10 (noticed by T. Schmidt) replace '$\|b\|_{k'} = \|b\|_k^{[k':k]} = \|a\|_k$' by '$\|a\|_{k'} = \|a\|_k^{[k':k]} = \|b\|_k$'
-p.345 l.-2 (noticed by J. Stix) Replace '$w_\ell^i := \max\left\{\ell^n \;\middle|\; [k(\mu_{\ell^n}) : k] \,|\, i\right\}$' by '$w_\ell^i := \max\left\{\ell^n \;\middle|\; \exp\Big(G(k(\mu_{\ell^n})|k)\Big) \,|\, i\right\}$, where $\exp(G)$ denotes the exponent of a finite group G'.
If $k(\mu_{\ell^\infty})/k$ is pro-cyclic (the contrary can only happen if k is a dyadic number field, i.e. a finite extension of $\mathbb{Q}_2$ and $\ell = 2$), then obviously $w_\ell^i = \max\left\{\ell^n \;\middle|\; [k(\mu_{\ell^n}) : k] \,|\, i\right\}$.
-p.346 l.15 (in the remark) replace ' $\cdots \cong H^{2-i}(G_k, \mathbb{Q}_\ell/\mathbb{Z}_\ell(1-i))^\vee \cong \cdots$ ' by ' $\cdots \cong H^{2-j}(G_k, \mathbb{Q}_\ell/\mathbb{Z}_\ell(1-i))^\vee \cong \cdots$ '
-p.348 theorem (7.4.2) (noticed by O. Venjakob): Part (ii) of this theorem is incorrect as it stands. The problem occurs if there are finitely many p-power roots of unity contained in K. In this case already the elements a_i are not uniquely determined. Let, for example $\#\mu_{p^\infty}(K) = p$ and $G \cong \mathbb{Z}_p$. Let $\gamma \in G$ be a topological generator and choose $\sigma_1 = \ldots = \sigma_{n+2} = \gamma$. Then the sequence on p.351. l.12 is not exact. The image of the first map is a principal ideal in $\mathbb{Z}_p[\![G]\!]$ while the kernel of the second map is the maximal ideal, which is *not* a principal ideal. A correct formulation of part (ii) of th.(7.4.2) is the following
(ii) Let $\sigma_1, \ldots, \sigma_{n+2}$ be topological generators of $\mathscr{G} = G(\bar{k}|k)$ and let $a_i \in \mathbb{Z}_p$ with $\sigma_i(\zeta) = \zeta^{a_i}$ for all $\zeta \in \mu_{p^\infty}(\bar{k})$, $i = 1, \ldots, n+2$. Let $\bar{\sigma}_i$ be the image of σ_i in G, $i = 1, \ldots, n+2$. Then there exists an exact sequence

$$\begin{array}{ccccccc} 0 \longrightarrow & \mathbb{Z}_p[\![G]\!] & \longrightarrow & \mathbb{Z}_p[\![G]\!]^{n+2} & \longrightarrow Y \longrightarrow 0, \\ & 1 & \longmapsto & (\bar{\sigma}_i - a_i)_i, & \end{array}$$

where $Y = I_{\mathscr{G}}/I_{\mathscr{H}} I_{\mathscr{G}}$, as in V §6. If $\mu_{p^\infty}(K) = 1$, then Y is a free $\mathbb{Z}_p[\![G]\!]$-module of rank $n+1$.
-p.348 lemma (7.4.3) replace '$A(K) \otimes \mathbb{Q}_p$' by '$A(K) \otimes \mathbb{Q}$' and '$U(K) \otimes \mathbb{Q}_p$' by '$U(K) \otimes \mathbb{Q}$'. Similar at several places in the proof.
-p.351 starting from l.11 the proof of (7.4.2)(ii) must be changed to the following:
Since $a_i \in \mathbb{Z}_p^\times$, (5.6.6) implies the exact sequence

$$\begin{array}{ccccccc} \mathbb{Z}_p[\![\mathscr{G}]\!]^{n+2} & \longrightarrow & \mathbb{Z}_p[\![\mathscr{G}]\!] & \longrightarrow & \mathbb{Z}_p & \longrightarrow & 0 \\ e_i & \longmapsto & a_i(\bar{\sigma}_i^{-1} - 1), & & & & \end{array}$$

where $\{e_i \,|\, i = 1, \ldots, n+2\}$ is a basis of $\mathbb{Z}_p[\![\mathscr{G}]\!]^{n+2}$. Tensoring by $\mathbb{Z}_p(-1)$, using the isomorphism

$$\begin{array}{ccc} \mathbb{Z}_p[\![\mathscr{G}]\!] \otimes \mathbb{Z}_p(-1) & \xrightarrow{\sim} & \mathbb{Z}_p[\![\mathscr{G}]\!] \\ g \otimes 1 & \longmapsto & \chi_{cycl}(g) \cdot g \end{array}$$

and taking $G(\bar{k}|K)$-coinvariants, we obtain the exact sequence

$$\begin{array}{ccccccc} \mathbb{Z}_p[\![G]\!]^{n+2} & \longrightarrow & \mathbb{Z}_p[\![G]\!] & \longrightarrow & (\mu_{p^\infty}(K))^\vee & \longrightarrow & 0 \\ e_i & \longmapsto & (\bar{\sigma}_i^{-1} - a_i), & & & & \end{array}$$

Since $(\mu_{p^\infty}(K)^\vee)_U$ is finite for every open normal subgroup U of G, we have $(\mu_{p^\infty}(K)^\vee)^+ = 0$. Thus the last sequence implies the exact sequence

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \mathbb{Z}_p[\![G]\!] & \longrightarrow & \mathbb{Z}_p[\![G]\!]^{n+2} & \longrightarrow & D((\mu_{p^\infty}(K))^\vee) & \longrightarrow & 0 \\ & & 1 & \longmapsto & (\bar\sigma_i - a_i)_i, & & & & \end{array}$$

Finally note that $Y \simeq D(\mu_{p^\infty}(K)^\vee)$.

-p.358 l.4 (noticed by M. Föhl) replace '(5.7)(ii)' by '(5.7)(i)'

-p.358 l.-5 (noticed by M. Föhl) replace '$\mathbb{Z}_p \oplus \cdots$' by '$\mathbb{Z} \oplus \cdots$'

-p.366 l.15 (noticed by M. Föhl) replace '0 for $i = 1$' by '1 for $i = 1$'

-p. 373, l. 7 (noticed by L. Wan) In the middle term replace '$cor_k^K(c)$' by '$cor_k^K(c)_\mathfrak{p}$'.

-p.373 l.-12 (noticed by T. Schmidt) replace 'inv_K' by 'inv_k'.

-p.374 l.8 (noticed by T. Schmidt) replace '$\sum_\mathfrak{p} \chi\big((\alpha_\mathfrak{p}, K_\mathfrak{p}|k_\mathfrak{p})\big)$' by '$\sum_\mathfrak{p} \chi_\mathfrak{p}\big((\alpha_\mathfrak{p}, K_\mathfrak{p}|k_\mathfrak{p})\big)$'.

-p. 375, proof of (8.1.15) (noticed by D. Harari and G. Kings) The first part of the proof uses Čebotarev density, a result whose proof uses global class field theory. The last argument of the proof uses the fact that $(a, K|k) = 1$ for $a \in k^\times$, also a result which we do not have at hand at this moment. To avoid a 'flavour of circularity', the proof should be replaced by the following:

Proof: We start by showing that $inv_{K|k} : H^2(G, I_K) \to \frac{1}{[K:k]}\mathbb{Z}/\mathbb{Z}$ is surjective. We first assume that $[K : k]$ is a prime power, say p^n. Let K' be the unique extension of degree p inside $K|k$. If all primes of k would split in K', then the norm map $N_{K'|k} : I_{K'} \to I_k$ would be surjective, and so would be the norm map $N_{K'|k} : C_{K'} \to C_k$. Since this contradicts $\#\hat{H}^0(G(K'|k), C_{K'}) = p$, we find a prime $\mathfrak{p}$ of k which is inert in K', and hence also in K. Therefore $G = G_\mathfrak{p} = G(K_\mathfrak{p}|k_\mathfrak{p})$, and we know that

$$inv_{K_\mathfrak{p}|k_\mathfrak{p}} : H^2(G_\mathfrak{p}, K_\mathfrak{p}^\times) \longrightarrow \tfrac{1}{[K_\mathfrak{p}:k_\mathfrak{p}]}\mathbb{Z}/\mathbb{Z}$$

is bijective. We conclude that $inv_{K|k} : H^2(G, I_K) \to \frac{1}{[K:k]}\mathbb{Z}/\mathbb{Z}$ is surjective if $[K : k]$ is a prime power. The general case now easily follows from (8.1.10).

Since G is cyclic, $H^3(G, K^\times) = H^1(G, K^\times) = 0$. Moreover, $H^1(G, C_K) = 0$, and so the exact sequence $0 \to K^\times \to I_K \to C_K \to 0$ yields the exact sequence

$$0 \longrightarrow H^2(G, K^\times) \longrightarrow H^2(G, I_K) \longrightarrow H^2(G, C_K) \longrightarrow 0;$$

in particular, the map $H^2(G, K^\times) \to H^2(G, I_K)$ is injective and, by (8.1.1), its cokernel has order $\#H^2(G, C_K) = \#\hat{H}^0(G, C_K) = [K : k]$.

It remains to show that inv is trivial on the image of $H^2(G, K^\times)$. We prove this without the assumption of $K|k$ being cyclic. So let $\alpha \in Br(k)$ be arbitrary. If k is a function field, then, by (8.1.14) and the following remark, we may assume that $\alpha \in Br(K|k)$, where $K = k(\zeta_n)$ for some n prime to $\mathrm{char}(k)$. If k is number field, let K be a finite extension of k which is Galois over $\mathbb{Q}$ and such that $\alpha \in Br(K|k)$. Then $inv_k(\alpha) = inv_{K|k}(\alpha) = inv_{K|\mathbb{Q}}(cor_\mathbb{Q}^k \alpha)$. Hence we may assume that $k = \mathbb{Q}$ and, by the same argument as in the function field case, $\alpha \in Br(K|\mathbb{Q})$ for a cyclic subextension K of $\mathbb{Q}(\zeta_n)|\mathbb{Q}$ for some n. In both cases, let $G = G(K|k)$ and let χ be a generator of $H^1(G, \mathbb{Q}/\mathbb{Z})$. Then $\delta\chi$ is a generator of $H^2(G, \mathbb{Z})$, and the cup-product

$$\delta\chi \cup\ : \hat{H}^0(G, K^\times) \longrightarrow H^2(G, K^\times)$$

is the periodicity isomorphism (see (1.6.12)). Hence every element of $H^2(G, K^\times)$ is of the form $\bar{a} \cup \delta\chi$ with $a \in k^\times$. By (8.1.11), we have

$$inv_{K|k}(\bar{a} \cup \delta\chi) = \chi((a, K|k)).$$

It therefore remains to show that $(a, K|k) = 1$ for $a \in k^\times$. Hence it suffices to show that $(a, k(\zeta_n)|k)\zeta_n = \zeta_n$, where k is a function field and $(n, \mathrm{char}(k)) = 1$ or $k = \mathbb{Q}$ and n arbitrary. Let k be a function field. Then, for any place $\mathfrak{p}$ of k, we have $(a, k_\mathfrak{p})\zeta_n = \zeta_n^{\#k(\mathfrak{p})^{v_\mathfrak{p}(a)}}$. The 'product formula' yields $\prod_\mathfrak{p} \#k(\mathfrak{p})^{v_\mathfrak{p}(a)} = 1$, hence $(a, k)\zeta_n = \zeta_n^{\prod_\mathfrak{p} \#k(\mathfrak{p})^{v_\mathfrak{p}(a)}} = \zeta_n$. The argument in the case $k = \mathbb{Q}$ is similar, but the computation of the local norm residue symbols at the primes $p \mid n$ is much more involved, see, e.g., [160], VI, (5.3). This proves the theorem. $\square$

-p.376 l.-10 (noticed by M. Föhl) replace 'theorem' by 'corollary'.

-p.383 l.2 (noticed by D. Vogel): replace '$I_\infty = \prod_{\mathfrak{p}\in S_\infty} k_\mathfrak{p}^\times$' by '$\tilde{U} = \prod_{\mathfrak{p}\in S_\infty} U_\mathfrak{p}$, where $U_\mathfrak{p} = \mathbb{R}_+^\times$ if $\mathfrak{p}|\infty$ is real and $U_\mathfrak{p} = k_\mathfrak{p}^\times = \mathbb{C}^\times$ if $\mathfrak{p}|\infty$ is complex'.

-p.385 l.9 The exponentiation does not make $U_k = \bar{U} \times \tilde{U}$ a $\bar{\mathbb{Z}} = \hat{\mathbb{Z}} \times \mathbb{R}$-**module**. $\bar{U}$ is a $\hat{\mathbb{Z}}$-module, but the map $\mathbb{R} \times \tilde{U} \to \tilde{U}$ has just the property that for any fixed $\tilde{u} \in \tilde{U}$, the map $\mathbb{R} \to \tilde{U}$, $\lambda \mapsto \tilde{u}^\lambda$ is a continuous homomorphism. But that is all we use in the following.

-p.385 l.-14 (noticed by J. Stix) replace '$\varepsilon_1^{z_1} \cdots \varepsilon_{r-1}^{z_{r-1}} = 1$' by '$\bar\varepsilon_1^{z_1} \cdots \bar\varepsilon_{r-1}^{z_{r-1}} = 1$'.

-p.391 l.-13 replace '$\mathbb{N}(S) = \{n \in \mathbb{N} \mid v_\mathfrak{p}(n) = 0 \quad \text{for all } \mathfrak{p} \notin S\}$' by '$\mathbb{N}(S) = \{n \in \mathbb{N} \mid n \in \mathcal{O}_{k,S}^\times\}$'

-p. 392, l. -7 (noticed by L. Wan) $d(\alpha)$ is not always -1 as asserted, but ± 1. Put $f = X^p - X - a$. We have $d(\alpha) = \mathrm{disc}(f) = (-1)^{p(p-1)/2}\mathrm{Res}(f, f') = \begin{cases} -1 & \text{if } p \equiv 1 \bmod 4 \\ +1 & \text{else.} \end{cases}$

-p.393 l.7 (noticed by G. Wiese) replace '$\mathfrak{P} \in S$' by '$\mathfrak{P} \notin S$'.

-p. 396, l. 15 (noticed by Martin Sigl) replace 'is 1 or 2' by 'is of order 1 or 2'.

-p.397, l.8 replace '$\mu_{p\infty}$' by 'μ_{p^∞}'

-p.399 (noticed by D. Vogel) The formulation of (8.3.13) is somewhat misleading as in the number field case every open subgroup of $C_S(k)$ has finite index and contains $D_S(k)$.

-p.401 l.-9 (noticed by T. Schmidt) replace '$\bar{C}_S(L)/C_S(K)$' by '$\bar{C}_S(L)/C_S(L)$'
-p.401 l.-1 (noticed by T. Schmidt) replace '$\bigcap\limits_{K\subseteq L\subseteq k_S} N_{L|K}\bar{C}_L = U_{K,S}K^\times/K^\times$' by '$\bigcap\limits_{K\subseteq L\subseteq k_S} N_{L|K}C_L = U_{K,S}K^\times/K^\times$'.
-p.402 l.2 (noticed by M. Föhl) replace 'strict' by '*strict*'.
-p.402 l.7 By the results of [S], the assumptions of (8.3.17) may be weakened to $S_p \subset S$ and, if $p = 2$, S contains no real primes.
-p.402 l.-10 By the results of [S], the statement of (8.3.18) may be sharpened in the following way. If to $S_2 \subset S$ then $cd_2(G_S(K)) = \infty$ if and only if S contains a real prime and $cd_2(G_S(K)) \leq 2$ otherwise.
-p.404 end of (8.3.20) (noticed by D. Harari) $\overline{\mathcal{O}_{k,S}^\times}$ is not the closure of $\mathcal{O}_{k,S}^\times$ in $I_{k,S}$ but the closure in $\prod_{\mathfrak{p}\in S} G_{k_\mathfrak{p}^{ab}}$
-p.410 l.-1 replace '$\mathbb{N}(S) = \{n \in \mathbb{N} \mid v_\mathfrak{p}(n) = 0 \quad \text{for all } \mathfrak{p} \notin S\}$' by '$\mathbb{N}(S) = \{n \in \mathbb{N} \mid n \in \mathcal{O}_{k,S}^\times\}$'
-p.411 l.2 (noticed by M. Föhl) replace 'modules which' by 'modules A which'.
-p.415 l.-1 (noticed by T. Schmidt) replace '$\prod\limits_{\mathfrak{p}\in S} I_\mathfrak{p}(A)^{\bar{G}_\mathfrak{P}}$' by '$\prod\limits_{\mathfrak{p}\in S} I_\mathfrak{p}(A)^{G_\mathfrak{P}}$'.
-p.416 l.-4 replace 'category discrete' by 'category of discrete'
-p.418 l.-14 The argument only works if S is finite or A is finite or A is torsion-free. If S is infinite, one has to add an argument how deduce the statement for general A from that for finite A and for torsion-free A.
-p.419 l.12 (noticed by L. Wan) The diagram is erroneous and the argument should be replaced by the following: Consider the canonical injections

$$\bigoplus_{\mathfrak{p}\in S\setminus T(k)} H^1(k_\mathfrak{p}, A)/H^1_{nr}(k_\mathfrak{p}, A) \hookrightarrow \bigoplus_{\mathfrak{p}\in S\setminus T(k)} H^1(T_\mathfrak{p}, A) \overset{diag}{\hookrightarrow} \prod_{\mathfrak{P}\in S\setminus T(k_T)} H^1(T_\mathfrak{P}, A),$$

where $T_\mathfrak{p}$ denotes the inertia group of the local group $G(\bar{k}_\mathfrak{p}|k_\mathfrak{p})$ for a prime $\mathfrak{p}$. We obtain the commutative and exact diagram

$$\begin{array}{ccc}
H^1(G(k_S|k_T), A) & \longrightarrow & \prod\limits_{\mathfrak{P}\in S\setminus T(k_T)} H^1(T_\mathfrak{P}, A) \\
\uparrow & & \uparrow \\
H^1(G_S, A) & \longrightarrow & P^1(G_S, A) \\
\uparrow & & \uparrow \\
H^1(G_T, A) & \longrightarrow & P^1(G_T, A) \times \prod\limits_{\mathfrak{p}\in S\setminus T(k)} H^1_{nr}(k_\mathfrak{p}, A) = P_T.
\end{array}$$

Since A is a trivial $G(k_S|k_T)$-module and since the images of the inertia groups $T_\mathfrak{P}$, $\mathfrak{P} \in S \smallsetminus T(k_T)$, generate $G(k_S|k_T)$ as a normal subgroup, the upper horizontal map is injective. Thus ...
-p.420 l.7 (noticed by M. Föhl) replace 'VIII §6' by '(8.5.2)'
-p.423 l.-12 $\mathrm{im}(\xi^\vee)$ is not closed if k is a function field and A is infinite. But this does not matter. All one needs is that $\mathrm{im}(\xi^\vee)$ is dense in $\ker(\varepsilon)$
-p.423 l.-3 (noticed by G. Wiese) Replace this paragraph by: 'For finite modules we can describe this pairing explicitly. For every finite module $A \in \mathit{Mod}_S(G_S)$ we consider the "new" pairing ...'
-p.424 l.3 Replace 'As ...' by 'As $H^3(G_S, \mathcal{O}_S)(p) = 0$ for every prime number $p \in \mathbb{N}(S)$ (see (8.3.10)), there is a cochain ...'
-p.429 l.6 replace $N_{G_S} I(\mu_m)$ by $\varprojlim\limits_{K,\text{Norm}} {}_m I_{K,S}$.
-p.429 l.-1 (noticed by T. Schmidt) replace '$\prod\limits_{\mathfrak{p}\in S_\infty} N_{G_\mathfrak{p}} \mathrm{Hom}(\mu_m, I_S)$' by '$\prod\limits_{\mathfrak{p}\in S_\infty} N_{G_\mathfrak{p}} \mathrm{Hom}(\mu_m, \mathbb{C}^\times)$'.
-p.430 l.-1 ff. (noticed by Ch. Kaiser): The argument must be modified in the following way: Obviously, $\mathrm{im}\,\lambda'_S \subseteq (\mathrm{im}\,\lambda_S)^\perp$. Let $x \in (\mathrm{im}\,\lambda_S)^\perp$. Then, for all sufficiently large finite subsets $T \subset S$, we have $\pi_T(x) \in (\mathrm{im}\,\lambda_T)^\perp = \mathrm{im}\,\lambda'_T$. Hence

$$x \in \bigcap_T \pi_T^{-1}(\mathrm{im}\,\lambda'_T) = \overline{\mathrm{im}\,\lambda'_S} = \mathrm{im}\,\lambda'_S.$$

-p.431 l.-10 (noticed by T. Schmidt) replace 'using (8.6.13)(i)' by 'using (8.6.8) and (8.6.3)'.
-p.432 l.5 (noticed by M. Föhl) in the index of the sum replace '$i - 1$ by 'i=1'.
-p.432 l.14 (noticed by M. Föhl) replace '$\chi(G, \mu_p)$' by '$\chi(G_S, \mu_p)$'.
-p.432 l.10 (noticed by T. Schmidt) replace '$p^{\Theta\left(\sum_{i=0}^{2}(-1)^i[H^i(G_S,M)]\right)}$' by '$p^{\Theta\left(\sum_{i=0}^{2}(-1)^i[H^i(G(k_S|K),M)]\right)}$'.
-p.433 l.12ff (noticed by M. Föhl) in the diagram on the middle of the page replace 'λ_3' by 'λ^3' (twice).
-p.433 l.-12 replace (8.6.13)(ii) by (8.6.13)(i)
-p.437 l.7 (noticed by T. Schmidt) replace
'$H^0(\bar{G}, \mathrm{Ind}_{\bar{G}}^{\bar{G}_\mathfrak{P}} A) = \bigoplus\limits_{\sigma\in\bar{G}|\bar{G}_\mathfrak{P}} \sigma H^0(\bar{G}_\mathfrak{P}, A)$' by '$H^0(\bar{G}, \mathrm{Ind}_{\bar{G}}^{\bar{G}_\mathfrak{P}} A) = H^0(\bar{G}_\mathfrak{P}, A)$'.
-p.437 l.11 (noticed by T. Schmidt) replace '$-\sum\limits_{\mathfrak{P}|p} \dim_{\mathbb{F}_p} A^{\bar{G}_\mathfrak{P}}$' by '$-\dim_{\mathbb{F}_p} A^{\bar{G}_\mathfrak{P}}$'
-p.437 l.-9 replace 'k' by 'k' (noticed by D. Vogel)
-p.446 l.8 replace '$\sum\limits_{i=1}^{2}$' by '$\sum\limits_{i=0}^{2}$' (noticed by A. Matar)

-p.450 l.-1 replace '$\mathbb{N}(S) = \{n \in \mathbb{N} \mid v_\mathfrak{p}(n) = 0 \quad \text{for all } \mathfrak{p} \notin S\}$' by '$\mathbb{N}(S) = \{n \in \mathbb{N} \mid n \in \mathcal{O}_{k,S}^\times\}$'
-p.453 Prop. (9.1.4) contains a mistake (noticed by M.Fenn). As a result, several modifications are necessary. In particular, there is also a 'special case' for function fields:
— p.452 l.16 add: 'or k is a function field of characteristic ℓ, $\ell \equiv -1 \bmod 2^r$ and $\mathbb{F}_\ell(\mu_{2^r}) \cap k = \mathbb{F}_\ell$'
— p.452 l.-7 add: 'If k is a number field, $\delta(S) = 1$ and T is ...'
— p.452 l.-2 add: 'If k is a function field, $\delta(S) = 1$ and $\delta(T) < 1/[k(\mu_{2^r}) : k]$, then the special case does not occur.'
— p.453 l.4 add: 'or $p = 2, m \geq 2$, char$(k) = \ell \equiv -1 \bmod 2^m$ and $\mathbb{F}_\ell(\mu_{2^m}) \cap k = \mathbb{F}_\ell$.'
— p.453 l.7 remove: 'or char$(k) \neq 0$'
— p.453 l.9 remove: 'If p is odd,'
— p.453 l.10 remove: 'and if $p = 2$, then $\alpha = \pm(1 + 2^s u)$, $2 \nmid u$, $s \geq 2$, since $\langle\sigma\rangle$ is cyclic.'
— p.454 l.6 replace '$2v'$' by '$2^{m-1}v'$'
— p.454 l.-13 add: 'Now let $p = 2$, $m \geq 2$ and char$(k) = \ell \neq 0$. Then

$$\hat{H}^i(G(k(\mu_{2^m})|k), \mu_{2^m}) = \hat{H}^i(G(\mathbb{F}_\ell(\mu_{2^m})|k \cap \mathbb{F}_\ell(\mu_{2^m})), \mu_{2^m})$$

and this group is non-zero (and then equal to μ_2) if and only if $\ell^{[\mathbb{F}_\ell(\mu_{2^m}) \cap k : \mathbb{F}_\ell]} \equiv -1 \bmod 2^m$. Indeed, $G := G(\mathbb{F}_\ell(\mu_{2^m})|k \cap \mathbb{F}_\ell(\mu_{2^m})) = \langle\sigma\rangle \cong \mathbb{Z}/2^{m-s}\mathbb{Z}$, $1 \leq s \leq m$, is cyclic. Write $\zeta^\sigma = \zeta^\alpha$, $\alpha \in \mathbb{Z}_2^\times$, then

$$\alpha = -1 \quad \text{or} \quad \alpha = \pm(1 + 2^s u),\ 2 \nmid u,\ s \geq 2\,.$$

It follows that

$$N_G(\zeta) = \begin{cases} \zeta^{2^{m-s}v}, & \text{if } \alpha = 1 + 2^s u\,, \\ \zeta^{2^{m-1}v'}, & \text{if } \alpha = -(1 + 2^s u)\,, \\ 1\,, & \text{if } \alpha = -1\,, \end{cases} \quad \text{with } 2 \nmid vv', \text{ and } \ (\mu_{2^m})^G = \begin{cases} \mu_{2^s}\,, & \text{if } \alpha = 1 + 2^s u\,, \\ \mu_2\,, & \text{if } \alpha = -(1 + 2^s u)\,, \\ \mu_2\,, & \text{if } \alpha = -1\,. \end{cases}$$

Hence $\hat{H}^0(G, \mu_{2^m})$ is trivial in the first two cases (and then all cohomology groups are trivial) and equal to μ_2 if $\alpha = -1$. This is equivalent to the statement $\ell^{[\mathbb{F}_\ell(\mu_{2^m}) \cap k : \mathbb{F}_\ell]} \equiv -1 \bmod 2^m$ resp. $\ell \equiv -1 \bmod 2^m$ and $\mathbb{F}_\ell(\mu_{2^m}) \cap k = \mathbb{F}_\ell$.
— p.455 l.-11 and p.456 l.5 add: 'or k is a function field of characteristic ℓ, $p = 2$, $r \geq 2$ and $\ell \equiv -1 \bmod 2^r$, $\mathbb{F}_\ell(\mu_{2^r}) \cap k = \mathbb{F}_\ell$'
— p.456 l.8 Replace 'If T is finite' by 'If k is a number field, $\delta(S) = 1$ and $\delta(T) < \frac{1}{2}$'
— p.456 l.11 (typographical) replace 'does' by 'do'
— p.456 l.13 add: 'If k is a function field and S is the set of all primes, then $k(\mu_{2^r})|k$ is cyclic and there exists a prime $\mathfrak{p}$ not contained in T such that $G(k(\mu_{2^r})|k) \cong G(k_\mathfrak{p}(\mu_{2^r})|k_\mathfrak{p})$.'
— p.457 l.-2 and p.461 l.-12 add: 'except k is a number field and we are in the special case (k, m, T)'
-p.458 l.6 Obviously, one can weaken the assumption of (9.1.8)(i) by replacing '$p^r|\#A$' by '$p^r|\exp(A)$.
-p.459 l.8 replace 'Assume $\zeta_s \in k$ but $\zeta_{s+1} \notin k$.' by 'Assume $\eta_s \in k$ but $\eta_{s+1} \notin k$.'
-p.460 l.-1 (noticed by D. Vogel): in (9.2.2)(vi) replace '$cs(k(A')|k) \subsetneqq S$' by '$\delta(S) = 1$'
-p.461 l.-15 ff There is some logical confusion in the formulation of corollary (9.2.3). Add as a second sentence: 'Assume we are not in the special case $(k, \exp(A), T)$.' and remove the 'except ...' from the last sentence.
-p.461 l.-11 Replace 'The above statement can be formulated as follows:' by 'We show the following stronger statement:'
-p.463, l.5 replace '(9.2.3)' by '(9.2.2)'
-p.463 l.12 ff This should be formulated in a more defensive way (see below)
-p.463 l.-8 and -p.464 l.1 (noticed by G. Wiese) In the theorems (9.3.2) and (9.3.3) the assumption that $\mathfrak{P}_1, \ldots, \mathfrak{P}_r$ have pairwise different restrictions to the fixed field Z of H is missing. (This assumption is used in the first paragraph of the proof. It is obviously fulfilled, if they have pairwise different restrictions to k.) If this assumption is not true, then H is the free product of the decomposition groups of any maximal subset of $\{\mathfrak{P}_1, \ldots, \mathfrak{P}_r\}$ which has pairwise different restrictions to Z. This does not occur if $r = 2$, but for $r \geq 3$ this actually occurs: Let us consider the case $r = 2$ first. Let Z_1 and Z_2 be the decomposition fields of $\mathfrak{P}_1$ and $\mathfrak{P}_2$ in $k_\mathfrak{M}(p)$ and $Z = Z_1 \cap Z_2$. If $\mathfrak{P}_1 \cap Z \neq \mathfrak{P}_2 \cap Z$, then the proof of (9.3.3) shows that H is the free product of G_1 and G_2. If the restrictions to Z are equal, then G_1 and G_2 are conjugated in H and therefore (say) G_1 generates H as a normal subgroup. Since we are dealing with pro-p-groups, we conclude that $H = G_1 = G_2$ and $Z = Z_1 = Z_2$. But $\mathfrak{P}_1 \cap Z_1 = \mathfrak{P}_2 \cap Z_2$ has exactly one extension to $k_\mathfrak{M}(p)$, which shows $\mathfrak{P}_1 = \mathfrak{P}_2$ in contrast to our assumption.

Now take any element in $\sigma \in H = G_1 * G_2$ not contained in G_1 and put $\mathfrak{P}_3 = \sigma\mathfrak{P}_1$. We immediately see that G_3 is contained in H and is conjugated to G_1. This gives the generic counterexample.

Now let $\mathfrak{P}_1, \ldots, \mathfrak{P}_r$ be arbitrary and, after renumbering, assume that for an $s \leq r$ the set $\{\mathfrak{P}_1, \ldots, \mathfrak{P}_s\}$ is a maximal subset with pairwise different restrictions to the fixed field Z of $H =< G_1, \ldots, G_r >$. Then $G_1, \ldots, G_s$ generated H as a normal subgroup, hence they generate H. Therefore H is the free product of $G_1, \ldots, G_s$ and each G_i, $s < i \leq r$ is conjugated to one of the $G_1, \ldots, G_s$.
-p.466 l.13 Add: '$\mathcal{E}(G)$ is called finite if E is.'
-p.467 l.4 and l.9 (noticed by F. Löwner) replace 'tr' by 'tg'
-p.468, l.-8 (noticed by L. Wan) replace '$\psi'(\sigma)\psi^{-1}(\sigma)$' by '$\psi'(\sigma)\psi(\sigma)^{-1}$'
-p.471 l.4 (noticed by G. Wiese) Replace 'Every embedding' by 'Every finite embedding'.
-p.471 l.6 replace 'Let an embedding' by 'Let a finite embedding'.
-p.475 l.10 replace 'Then by (9.2.2)(ii)' by 'If $p \neq$ char(k), then by (9.2.2)(ii)'; add in line 13: 'If $p =$ char(k), then the same is true by the strong approximation theorem using (8.3.2) and (6.1.2).'
-p.475 l.-12 replace '$N \cong \mathbb{Z}/p^m\mathbb{Z}$' by '$N \cong (\mathbb{Z}/p\mathbb{Z})^m$'.
-p.476 l.17 For those who need a slightly more refined statement: The proof of theorem (9.5.1) shows that we can choose K in such a way that finitely many given primes split completely in K.

-p.489 l.9 replace '$y \in H^1(k_S|K, \mu_p)$' by '$\frac{1}{2}y \in H^1(k_S|K, \mu_p)$'.
-p.496 l.-12 (noticed by T. Keller) replace 'v' by 'ν' (twice)
-p.496 l.-8 this is not true, condition (i) might be affected. There are many ways to overcome this calamity, for one possibility, see the proof in the second edition of our book.
-p.514 l.4 (noticed by T. Keller) replace 'chap.IV, §4' by 'chap.IV, §2'.
-p.517 l.4 (noticed by T. Keller) replace '$i \in \mathbb{Z}$' by '$i > 0$'.
-p.517 l.18 (noticed by M. Föhl) replace 'VII §6' by 'VII §5'.
-p.518 l.10ff We find a complete local ring A with residue field k and algebraically closed quotient field K of characteristic 0 but, of course, not a dvr. Therefore the following should be modified as follows. Let A be a complete dvr with residue field k and quotient field K of characteristic 0 (e.g., the ring of Witt vectors over k). Then replace X_K by $X_{\bar{K}}$ in in the definition of sp and in (10.1.4), where $\bar{K}$ is an algebraic closure of K.
-p.525 l.1 (noticed by T. Keller) replace '$G_\varnothing(K\bar{k})$' by '$G_\varnothing(K\bar{k})(p)$' (twice).
-p.528 l.14. replace '§6' by '§7' , replace 'Later in chapter XII' by 'In §9' (noticed by D. Vogel)
-p.530 l.-5. Remove the sentence starting with "Since .." and replace (3.3.4) by (3.3.8) in the next sentence.
-p.534 l.-1. Replace 'U_K^1' by 'U_k^1'.
-p.536 l.15. There is of course another possibility: K might be a cubic field which is not totally real.
-p.548 l.-8 (noticed by S. Schmidt) replace 'surjectivity' by 'injectivity' and in the next line '$\twoheadrightarrow$' by '$\hookleftarrow$'.
-p.549 l.8 (noticed by S. Schmidt) replace 'be' by 'be a' (twice).
-p.550 l.15 Replace 'finite' by 'finitely generated'
-p.550 l.-10 See [S] for an extension of the results of §4 to the case that $p = 2$ and S does not necessarily contain all infinite primes.
-p.551 l. -11 (noticed by J. Bartels) replace '$i \geq 1$' by '$i \geq 2$'
-p.552 l.-16 Replace 'finite field' by 'finite number field'
-p.553 l.10 replace 'exact sequence' by 'inclusion' and the sequence in l.11 by

$$_pH^2(G(L|K'), \mathcal{O}_{L,S}^\times) \hookrightarrow {}_p\Big(\prod_{\mathfrak{p}\in S} H^2(K'_\mathfrak{p}, \mu_{p^\infty})\Big)$$

-p.554 l.3 replace 'is a finite torsion group' by 'and $Cl_S(K')$ is a torsion group'
-p.555 l.8 remove the second and the third 'be'.
-p.556 l.1 Riemann's existence theorem is also true, if we only assume that $S \supset T \supset S_p$, see [S]. (This is a nontrivial statement only for $p = 2$.)
-p.556 l.2 remove the second and third 'be'
-p.557 l.10 It might be helpful to notice at this point that $k_T(p)$ contains the cyclotomic $\mathbb{Z}_p$-extension of k.
-p.570 l.-4 remove '(observe that $S_p \not\subseteq S_0$)'
-p.573 l.9 In definition (10.7.3) replace 'every subset' by 'a subset'
-p.579 l.-11 remove the second 'be'
-p.579 l.-10 remove the word 'be'
-p.581 l.6 (noticed by G. Wiese) replace 'H^{-1}' by '$\hat{H}^{-1}$'
-p.581 l.9 (noticed by O. Thomas) replace '$U_\mathfrak{P}^\times$' by '$U_\mathfrak{P}$'
-p.586 l.13 replace '$G_S(K)$' by '$G_S(k)$'
-p.586 l.14 replace '$G_S(K)$' by '$G_S(k)$'
-p.588 1st footnote l.1 (noticed by D. Vogel) replace 'originally' by 'original'
-p.592 l.12 (noticed by M. Föhl) replace '$\max\{g-1, 0\})$' by '$\max(g-1, 0)$'.
-p.597 l.-10 (noticed by S. Schmidt) replace 'this' by 'these'.
-p.601 l.9 (noticed by D. Vogel) replace the exact sequence by the following:
$T_n/[T_n, G(H|k_n)] \to G(H|k_n)^{ab} \to G(H_n|k_n) \to 0.$
-p.601 l.-13ff (noticed by S. Schmidt) These lines are confusing and should be replaced by: 'where $\tau_i \in T_{\mathfrak{P}_i}(H|k_n)$, $i = 1, \ldots, s_\infty$, lifts the generator $\gamma^{p^n} \in \Gamma_n$, where γ is a chosen generator of Γ'.
-p.607 l.-6 One should note that (5.6.11) does not apply directly but in an obvious manner.
-p.613 l.15–16 (noticed by S. Schmidt) remove '$P(X)$' and replace '$\mathrm{supp}(X)$' by '$\mathrm{supp}(\mathrm{tor}_\Lambda X)$'.
-p.626 l. -13ff: -l.-13,-12 replace these lines by: Let $K|k$ be a finite Galois extension of degree prime to p with Galois group Δ and assume that k is totally imaginary if $p = 2$. Let K_∞ be the cyclotomic $\mathbb{Z}_p$-extension of K. Then
-l.-8,-7 replace in both lines '$\Lambda[\Delta]^{r_2+r_1}$' by '$\Lambda[\Delta]^{r_1+2r_2}$'. (noticed by S. Schmidt)
-l.-6 Add (before X_Σ): Assume in addition that $K = k(\mu_p)$. Then
-p.627 l.17 (noticed by S. Schmidt) add 'if multiplication by 2 is an isomorphism on A'.
-p.629 l.2ff replace this paragraph by
'It is easy to see that the map φ_2 is given explicitly by

$$\varphi_2 : \mathfrak{M} \longrightarrow Cl_\Sigma(k_\infty)(p),\ \alpha \otimes p^{-n} \longmapsto [\mathfrak{a}_1] \in Cl_\Sigma(k_m)(p),$$

where $\alpha \in k_m^\times$ and $\alpha\mathcal{O}_{k_m} = \mathfrak{a}_1^{p^n}\mathfrak{a}_2$ with an ideal $\mathfrak{a}_2$ having only prime divisors in $S_p(k_m)$. Since all primes dividing p have an infinite ramification index in $k_\infty|k$, we see that, for sufficiently large s, $\alpha\mathcal{O}_{k_{m+s}} = \mathfrak{a}^{p^n}$ for some ideal $\mathfrak{a}$ in k_{m+s}. We define φ_1 by sending $\alpha \otimes p^{-n}$ to the class $[\,\mathfrak{a}\,] \in Cl(k_\infty)(p)$. The map φ_2 is the composition of φ_1 with the natural projection $Cl(k_\infty)(p)$ to $Cl_\Sigma(k_\infty)(p)$.'

-p.634 l.-8ff (noticed by R. Sharifi) replace $\bigoplus_{\substack{\mathfrak{p} \text{ prime of } k_\infty^+ \\ \mathfrak{p}\nmid p}} \mathrm{Ind}_G^{G_\mathfrak{P}} I_{G_\mathfrak{P}}$ by $\bigoplus_{\substack{\mathfrak{p} \text{ prime of } k_\infty^+ \\ \mathfrak{p}\nmid p,\, \mathfrak{p} \text{ splits in } k_\infty}} \mathrm{Ind}_G^{G_\mathfrak{P}} I_{G_\mathfrak{P}}$

–p.634 l.-7 replace $\bigoplus_{\substack{\mathfrak{p}\in S(k_\infty^+) \\ \mu_p\subseteq k_{\infty,\mathfrak{p}}^+}} \mathrm{Ind}_G^{G_\mathfrak{P}} I_{G_\mathfrak{P}}$ by $\bigoplus_{\substack{\mathfrak{p}\in S(k_\infty^+) \\ \mathfrak{p} \text{ splits in } k_\infty}} \mathrm{Ind}_G^{G_\mathfrak{P}} I_{G_\mathfrak{P}}$

–p.635 l.-6 replace $\bigoplus_{\substack{\mathfrak{p} \text{ prime of } k_\infty^+ \\ \mathfrak{p}\nmid p}} I_{G_\mathfrak{P}}$ by $\bigoplus_{\substack{\mathfrak{p} \text{ prime of } k_\infty^+ \\ \mathfrak{p}\nmid p,\, \mathfrak{p} \text{ splits in } k_\infty}} I_{G_\mathfrak{P}}$

–p.635 l.-3 replace $\bigoplus_{\substack{\mathfrak{P}\in S_p(K_\infty^+) \\ \mu_p\subseteq K_{\infty,\mathfrak{p}}^+}} \mathbb{Q}_p$ by $\bigoplus_{\substack{\mathfrak{p}\in S_p(k_\infty^+) \\ \mathfrak{p} \text{ splits in } k_\infty}} \bigoplus_{\mathfrak{P}|\mathfrak{p}} \mathbb{Q}_p$

–p.635 l.-2 replace $\lambda_{nr}^-(k) = \lambda_{cs}^-(k) + \#\{\mathfrak{p}\in S_p(k_\infty^+) \mid \mu_p\subseteq k_{\infty,\mathfrak{p}}^+\}$ by $\lambda_{nr}^-(k) = \lambda_{cs}^-(k) + \#\{\mathfrak{p}\in S_p(k_\infty^+) \mid \mathfrak{p} \text{ splits in } k_\infty\}$

–p.636 l.5 replace $\sum_{\substack{\mathfrak{p} \text{ prime of } k_\infty^+ \\ \mathfrak{P}\nmid p}} \sum_{\mathfrak{P}|\mathfrak{p}} (e_\mathfrak{P}-1)$ by $\sum_{\substack{\mathfrak{p} \text{ prime of } k_\infty^+ \\ \mathfrak{p}\nmid p,\, \mathfrak{p} \text{ splits in } k_\infty}} \sum_{\mathfrak{P}|\mathfrak{p}} (e_\mathfrak{P}-1)$

–p.636 l.7 replace $\sum_{\substack{\mathfrak{p}\in S(k_\infty^+) \\ \mu_p\subseteq k_{\infty,\mathfrak{p}}^+}} \sum_{\mathfrak{P}|\mathfrak{p}} (e_\mathfrak{P}-1)$ by $\sum_{\substack{\mathfrak{p}\in S(k_\infty^+) \\ \mathfrak{p} \text{ splits in } k_\infty}} \sum_{\mathfrak{P}|\mathfrak{p}} (n_\mathfrak{P}-1)$,

where $n_\mathfrak{P}$ is the local degree of $K_\infty^+|k_\infty^+$ with respect to a prime $\mathfrak{P}$ of K_∞^+.

-p.641 in the diagram above proposition (11.5.2) replace $\tilde{k}(p)$ by $\tilde{k}$

-p.649 l.-5 (noticed by D. Vogel) replace 'Cassou-Nougués' by 'Cassou-Noguès'

-p.657 l.3 (noticed by O. Venjakob) replace '$\mu(X)$' by 'μ_χ' .

-p.658 l.4-5 (noticed by O. Venjakob) replace these lines by:

$L(0,\omega^i)$ for $i = 1, 3, \ldots, p-4$. Finally the congruence

$$L(0,\omega^i) \equiv -\frac{B_{i+1}}{i+1} \bmod p$$

-p. 668, l. -2,-1 (noticed by Martin Sigl) replace 'group' by 'set' (twice)

Remarks on the bibliography

[6] replace 'Artin, M.' by 'Artin, E.' (noticed by S. Böge)

[35] the correct title is 'The group of the maximal p-extension of a local field' (in Russian)

[45] replace 'FESENKOV' by 'FESENKO' (noticed by I. Fesenko)

[51] remove the point after 'Nauk'

[52] the correct title is: *Infinitude of the number of relations in the Galois group of the maximal p-extension of a local field with restricted ramification* (noticed by M. Fenn)

[54] replace 'of' by 'on'

[61] replace 'Revètements Étales et Groupe de Fondamental' by 'Revêtements Étales et Groupe Fondamental' (noticed by M. Föhl)

[90] replace '71–79' by '71–98' (noticed by T. Keller)

[94] replace 'Viehweg' by 'Vieweg' (noticed by D. Vogel)

[122] replace '(1986)' by '(1984)' (noticed by D. Vogel)

[128] replace 'carieties' by 'varieties'

[133] has appeared in Invent. math. **138** (1999) 319–423

[140] the correct reference is: *Freie Produkte pro-endlicher Gruppen und ihre Kohomologie*. Archiv der Math. **22** (1971) 337–357

[173] the page numbers 295–319 have to be added (noticed by D. Vogel)

[178] has appeared in J. reine angew. Math. **517** (1999) 145–160

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[S] Schmidt, A. *On the relation between 2 and ∞ in Galois cohomology of number fields.* Comp. Math. **133** (2002) 267–288

[V1] Voevodsky, V. *Motivic cohomology with $\mathbb{Z}/2$-coefficients.* Publ. Math. Inst. Hautes Études Sci. **98** (2003) 59–104

[V2] Voevodsky, V. *Reduced power operations in motivic cohomology.* Publ. Math. Inst. Hautes Études Sci. **98** (2003) 1-57

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