Emmy Noether and Hermann Weyl.*

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Contents

1 Preface

We are here for a conference in honor of Hermann Weyl and so I may be allowed, before touching the main topic of my talk, to speak about my personal reminiscences of him.

It was in the year 1952. I was 24 and had my first academic job at München when I received an invitation from van der Waerden to give a colloquium talk at Zürich University. In the audience of my talk I noted an elder gentleman, apparently quite interested in the topic. Afterwards – it turned out to be Hermann Weyl – he approached me and proposed to meet him next day at a specific point in town. There he told me that he wished to know more about my doctoral thesis, which I had completed two years ago already but which had not yet appeared in print. Weyl invited me to join him on a tour on the hills around Zürich. On this tour, which turned out to last for several hours, I had to explain to him the content of my thesis which contained a proof of the Riemann hypothesis for function fields over finite base fields. He was never satisfied with sketchy explanations, his questions were always to the point and he demanded every detail. He seemed to be well informed about recent developments.

This task was not easy for me, without paper and pencil, nor blackboard and chalk. So I had a hard time. Moreover the pace set by Weyl was not slow and it was not quite easy to keep up with him, in walking as well as in talking.

Much later only I became aware of the fact that this tour was a kind of examination, Weyl wishing to find out more about that young man who was myself. It seems that I did not too bad in this examination, for some time later he sent me an application form for a grant-in-aid from the Institute for Advanced Study in Princeton for the academic year 1954/55. In those years Weyl was commuting between Zürich and Princeton on a half-year basis. In Princeton he had found, he wrote to me, that there was a group of people who were working in a similar direction.

Hence I owe to Hermann Weyl the opportunity to study in Princeton. The two academic years which I could work and learn there turned out to be important for my later mathematical life. Let me express, posthumously, my deep gratitude and appreciation for his help and concern in this matter.

The above story shows that Weyl, up to his last years, continued to be active helping young people find their way into mathematics. He really cared. I did not meet him again in Princeton; he died in 1955.

Let us now turn to the main topic of this talk as announced in the title.

2 Introduction

Both Hermann Weyl and Emmy Noether belonged to the leading group of mathematicians in the first half of 20th century, who shaped the image of mathematics as we see it today.

Emmy Noether was born in 1882 in the university town of Erlangen, as the daughter of the renowned mathematician Max Noether. We refer to the literature for information on her life and work, foremost to the empathetic

biography by Auguste Dick [Dic70] which has appeared in 1970, the 35th year after Noether's tragic death. It was translated into English in 1981. For more detailed information see, e.g., the very carefully documented report by Cordula Tollmien [Tol90]. See also Kimberling's publications on Emmy Noether, e.g., his article in [BS81].

When the Nazis had come to power in Germany in 1933, Emmy Noether was dismissed from the University of Göttingen and she emigrated to the United States. She was invited by Bryn Mawr College as a visiting professor where, however, she stayed and worked for 18 months only, when she died on April 14, 1935 from complications following a tumor operation.[1]

Quite recently we have found the text, hitherto unknown, of the speech which Hermann Weyl delivered at the funeral ceremony for Emmy Noether on April 17, 1935.[2] That moving text puts into evidence that there had evolved a close emotional friendship between the two. There was more than a feeling of togetherness between immigrants in a new and somewhat unfamiliar environment. And there was more than high esteem for this women colleague who, as Weyl has expressed it[3], was "superior to him in many respects". This motivated us to try to find out more about their mutual relation, as it had developed through the years.

We would like to state here already that we have not found many documents for this. We have not found letters which they may have exchanged.[4] Neither did Emmy Noether cite Hermann Weyl in her papers nor vice versa[5]. After all, their mathematical activities were going into somewhat different directions. Emmy Noether's creative power was directed quite generally towards the clarification of mathematical structures and concepts through abstraction, which means leaving all unnecessary entities and properties aside and concentrating on the essentials. Her basic work in this direction can be subsumed under algebra, but her methods eventually penetrated all mathematical fields, including number theory and topology.

On the other side, Hermann Weyl's mathematical horizon was wide-spread, from complex and real analysis to algebra and number theory, mathematical physics and logic, also continuous groups, integral equations and much more. He was a mathematical generalist in a broad sense, touching also philosophy of science. His mathematical writings have a definite flair of art and poetry, with his book on symmetry as a culmination point [Wey52].

We see that the mathematical style as well as the extent of Weyl's research work was quite different from that of Noether. And from all we know the same can be said about their way of living. So, how did it come about that there developed a closer friendly relationship between them? Although we cannot offer a clear cut answer to this question, I hope that the reader may find something of interest in the following lines.

[1] See footnote 48.

[2] See [Roq07b]. We have included in the appendix an English translation of Weyl's text; see section 9.2.

[3] See [Wey35].

[4] With one exception; see section 5.3.

[5] There are exceptions; see section 4.

3 The first period: until 1915

In the mathematical life of Emmy Noether we can distinguish four periods.[6] In her first period she was residing in Erlangen, getting her mathematical education and working her way into abstract algebra guided by Ernst Fischer, and only occasionally visiting Göttingen. The second period starts in the summer of 1915 when she came to Göttingen for good, in order to work with Klein and Hilbert. This period is counted until about 1920. Thereafter there begins her third period, when her famous paper "*Idealtheorie in Ringbereichen*" (Ideal theory in rings) appeared, with which she "*embarked on her own completely original mathematical path*" – to cite a passage from Alexandroff's memorial address [Ale83]. The fourth period starts from 1933 when she was forced to emigrate and went to Bryn Mawr.

3.1 Their mathematical background

Hermann Weyl, born in 1885, was about three years younger than Emmy Noether. In 1905, when he was 19, he entered Göttingen University (after one semester in München). On May 8, 1908 he obtained his doctorate with a thesis on integral equations, supervised by Hilbert.

At about the same time (more precisely: on December 13, 1907) Emmy Noether obtained her doctorate from the University of Erlangen, with a thesis on invariants supervised by Gordan. Since she was older than Weyl we see that her way to Ph.D. was longer than his. This reflects the fact that higher education, at that time, was not as open to females as it is today; if a girl wished to study at university and get a Ph.D. then she had to overcome quite a number of difficulties arising from tradition, prejudice and bureaucracy. Noether's situation is well described in Tollmien's article [Tol90].[7]

But there was another difference between the status of Emmy Noether and Hermann Weyl at the time of their getting the doctorate.

On the one side, Weyl was living and working in the unique Göttingen mathematical environment of those years. Weyl's thesis belongs to the theory of integral equations, the topic which stood in the center of Hilbert's work at the time, and which would become one of the sources of the notion of "Hilbert space". And Weyl's mathematical curiosity was not restricted to integral equations. In his own words, he was captivated by all of Hilbert's mathematics. Later he wrote:[8]

> *I resolved to study whatever this man* [Hilbert] *had written. At the end of my first year I went home with the "Zahlbericht" under my arm, and during the summer vacation I worked my way through it - without any previous knowledge of elementary number theory or Galois theory. These were the happiest months of my life, whose shine, across years burdened with our common share of doubt and*

[6] Weyl [Wey35] distinguishes three epochs but they represent different time intervals than our periods.

[7] For additional material see also Tollmien's web page: www.tollmien.com.

[8] Cited from the Weyl article in "MacTutor History of Mathematics Archive".

failure, still comforts my soul.

We see that Weyl in Göttingen was exposed to and responded to the new and exciting ideas which were sprouting in the mathematical world at the time. His mathematical education was strongly influenced by his advisor Hilbert.

On the other side, Noether lived in the small and quiet mathematical world of Erlangen. Her thesis, supervised by Paul Gordan, belongs to classical invariant theory, in the framework of so-called symbolic computations. Certainly this did no longer belong to the main problems which dominated mathematical research in the beginning of the 20th century. It is a well-known story that after Hilbert in 1888 had proved the finiteness theorem of invariant theory which Gordan had unsuccessfully tried for a long time, then Gordan did not accept Hilbert's existence proof since that was not constructive in his (Gordan's) sense. He declared that Hilbert's proof was "theology, not mathematics". Emmy Noether's work was fully integrated into Gordan's formalism and so, in this way, she was not coming near to the new mathematical ideas of the time.[9] In later years she described the work of her thesis as rubbish ("*Mist*" in German[10]). In a letter of April 14, 1932 to Hasse she wrote:

> Ich habe das symbolische Rechnen mit Stumpf und Stil verlernt.
>
> *I have completely forgotten the symbolic calculus.*

We do not know when Noether had first felt the desire to update her mathematical background. Maybe the discussions with her father helped to find her way; he corresponded with Felix Klein in Göttingen and so was well informed about the mathematical news from there. She herself reports that it was mainly Ernst Fischer who introduced her to what was then considered "modern" mathematics. Fischer came to Erlangen in 1911, as the successor of the retired Gordan.[11] In her curriculum vitae which she submitted in 1919 to the Göttingen Faculty on the occasion of her *Habilitation*, Noether wrote:

> Wissenschaftliche Anregung verdanke ich wesentlich dem persönlichen mathematischen Verkehr in Erlangen und in Göttingen. Vor allem bin ich Herrn E. Fischer zu Dank verpflichtet, der mir den entscheidenden Anstoß zu der Beschäftigung mit abstrakter Algebra in arithmetischer Auffassung gab, was für all meine späteren Arbeiten bestimmend blieb.
>
> *I obtained scientific guidance and stimulation mainly through personal mathematical contacts in Erlangen and in Göttingen. Above all I am indebted to Mr. E. Fischer from whom I received the decisive impulse to study abstract algebra from an arithmetical viewpoint, and this remained the governing idea for all my later work.*

[9] Well, Noether had studied one semester in Göttingen, winter 1903/04. But she fell ill during that time and had to return to her home in Erlangen, as Tollmien [Tol90] reports. We did not find any indication that this particular semester has had a decisive influence on her mathematical education.

[10] Cited from Auguste Dick's Noether biography [Dic70].

[11] More precisely: Gordan retired in 1910 and was followed by Erhard Schmidt who, however, left Erlangen one year later already and was followed in turn by Ernst Fischer. – The name of Fischer is known from the *Fischer-Riesz theorem* in functional analysis.

Thus it was Fischer under whose direction Emmy Noether's mathematical outlook underwent the "*transition from Gordan's formal standpoint to the Hilbert method of approach*", as Weyl stated in [Wey35].

We may assume that Emmy Noether studied, like Weyl, all of Hilbert's papers, at least those which were concerned with algebra or arithmetic. In particular she would have read the paper [Hil90] where Hilbert proved that every ideal in a polynomial ring is finitely generated; in her famous later paper [Noe21] she considered arbitrary rings with this property, which today are called "Noetherian rings". We may also assume that Hilbert's *Zahlbericht* too was among the papers which Emmy Noether studied; it was the standard text which every young mathematician of that time read if he/she wished to learn algebraic number theory. We know from a later statement that she was well acquainted with it – although at that later time she rated it rather critically[12], in contrast to Weyl who, as we have seen above, was enthusiastic about it. But not only Hilbert's papers were on her agenda; certainly she read Steinitz' great paper "*Algebraische Theorie der Körper*" [Ste10] which marks the start of abstract field theory. This paper is often mentioned in her later publications, as the basis for her abstract viewpoint of algebra.

3.2 Meeting in Göttingen 1913

Hermann Weyl says in [Wey35], referring to the year 1913:

> *. . . She must have been to Göttingen about that time, too, but I suppose only on a visit with her brother Fritz. At least I remember him much better than her from my time as a Göttinger Privatdozent, 1910-1913.*

We may conclude that he had met Emmy Noether in Göttingen about 1913, but also that she did not leave a lasting impression on him on that occasion.

As Tollmien [Tol90] reports, it was indeed 1913 when Emmy Noether visited Göttingen for a longer time (together with her father Max Noether). Although we have no direct confirmation we may well assume that she met Weyl during this time. In the summer semester 1913 Weyl gave two talks in the *Göttinger Mathematische Gesellschaft*. In one session he reported on his new book "*Die Idee der Riemannschen Fläche*" (The idea of the Riemann surface) [Wey13], and in another he presented his proof on the equidistribution of point sequences modulo 1 in arbitrary dimensions [Wey16] – both pieces of work have received the status of a "classic" by now. Certainly, Max Noether as a friend of Klein will have been invited to the sessions of the *Mathematische Gesellschaft*, and his daughter Emmy with him. Before and after the session people would gather for discussion, and from all we know about Emmy Noether she would not have hesitated to participate in the discussions. From what we have said in the foregoing section we can conclude that her mathematical status was up-to-date and well comparable to his, at least with respect to algebra and number theory.

[12] In a letter of November 17, 1926 to Hasse; see [LR06]. Olga Taussky-Todd [TT81] reports from later time in Bryn Mawr, that once "*Emmy burst out against the* Zahlbericht*, quoting also Artin as having said that it delayed the development of algebraic number theory by decades*".

Unfortunately we do not know anything about the possible subjects of the discussions of Emmy Noether with Weyl. It is intriguing to think that they could have talked about Weyl's new book "*The Idea of the Riemann surface*". Weyl in his book defines a Riemann surface axiomatically by structural properties, namely as a connected manifold X with a complex 1-dimensional structure. This was a completely new approach, a structural viewpoint. Noether in her later period used to emphasize on every occasion the structural viewpoint. The structure in Weyl's book is an *analytic* one, and he constructs an algebraic structure from this, namely the field of meromorphic funtions, using the so-called Dirichlet principle – whereas Emmy Noether in her later papers always starts from the function field as an *algebraic* structure. See, e.g., her report [Noe19]. There she did not cite Weyl's book but, of course, this does not mean that she did not know it.

We observe that the starting idea in Weyl's book was the definition and use of an axiomatically defined topological space[13]. We wonder whether this book was the first instance where Emmy Noether was confronted with the axioms of what later was called a topological space. It is not without reason to speculate that her interest in topology was inspired by Weyl's book. In any case, from her later cooperation with Paul Alexandroff we know that she was acquainted with problems of topology; her contribution to algebraic topology was the notion of "Betti group" instead of the "Betti number" which was used before. Let us cite Alexandroff in his autobiography [Ale80]:

> *In the middle of December Emmy Noether came to spend a month in Blaricum. This was a brilliant addition to the group of mathematicians around Brouwer. I remember a dinner at Brouwer's in her honour during which she explained the definition of the Betti groups of complexes, which spread around quickly and completely transformed the whole of topology.*

This refers to December 1925. Blaricum was the place where L. E. J. Brouwer lived.

We have mentioned this contact of Emmy Noether to the group around Brouwer since Weyl too did have mathematical contact with Brouwer. In fact, in his book "*The idea of the Riemann surface*" Weyl mentioned Brouwer as a source of inspiration. He writes:

> In viel höherem Maße, als aus den Zitaten hervorgeht, bin ich dabei durch die in den letzten Jahren erschienenen grundlegenden topologischen Untersuchungen Brouwers, dessen gedankliche Schärfe und Konzentration man bewundern muss, gefördert worden ...
>
> *I have been stimulated – much more than the citations indicate – by the recent basic topological investigations of Brouwer, whose ideas have to be admired in their sharpness and concentration.*

Brouwer's biographer van Dalen reports that Weyl and Brouwer met several times in the early 1920s [vD99]. By the way, Emmy Noether, Hermann Weyl

[13]The Hausdorff axiom was not present in the first edition. This gap was filled in later editions.

and L. E. J. Brouwer met in September 1920 in Bad Nauheim, at the meeting of the DMV.[14]

Returning to the year 1913 in Göttingen: In the session of July 30, 1913 of the *Göttinger Mathematische Gesellschaft*, Th. v. Kármán reported on problems connected with a recent paper on turbulence by Emmy Noether's brother Fritz. Perhaps Fritz too was present in Göttingen on this occasion, and maybe this was the incident why Weyl had remembered not only Emmy but also Fritz? That he remembered Fritz "much better" may be explained by the topic of Fritz' paper; questions of turbulence lead to problems about partial differential equations, which was at that time more close to Weyl's interests than were algebraic problems which Emmy pursued.

4 The second period: 1915-1920

In these years Emmy Noether completed several papers which are of algebraic nature, mostly about invariants, inspired by the Göttingen mathematical atmosphere dominated by Hilbert. She also wrote a report in the *Jahresbericht der DMV* on algebraic function fields, in which Noether compares the various viewpoints of the theory: analytic, geometric and algebraic (which she called "arithmetic") and she points out the analogies to the theory of number fields. That was quite well known to the people working with algebraic functions, but perhaps not written up systematically as Emmy Noether did. Generally, these papers of hers can be rated as good work, considering the state of mathematics of the time, but not as outstanding. It is unlikely that Hermann Weyl was particularly interested in these papers; perhaps he didn't even know about them.[15]

But this would change completely with the appearance of Noether's paper on invariant variation problems [Noe18] ("*Invariante Variationsprobleme*"). The main result of this paper is of fundamental importance in many branches of theoretical physics even today. It shows a connection between conservation laws in physics and the symmetries of the theory. It is probably the most cited paper of Emmy Noether up to the present day. In 1971 an English translation appeared [Tav71], and in 2004 a French translation with many comments [KS04].

In 1918, when the paper appeared, its main importance was seen in its applicability in the framework of Einstein's relativity theory. Einstein wrote to Hilbert in a letter of May 24, 1918:

> Gestern erhielt ich von Frl. Noether eine sehr interessante Arbeit über Invariantenbildung. Es imponiert mir, dass man diese Dinge von so allgemeinem Standpunkt übersehen kann ... Sie scheint ihr Handwerk zu verstehen.
>
> *Yesterday I received from Miss Noether a very interesting paper on the formation of invariants. I am impressed that one can handle those things from such a general viewpoint ... She seems to understand her job.*

[14] DMV = *Deutsche Mathematiker Vereinigung* = German Mathematical Society.
[15] In those years Weyl was no more in Göttingen but held a professorship at ETH in Zürich.

Einstein had probably met Emmy Noether already in 1915 during his visit to Göttingen.

Emmy Noether's result was the fruit of a close cooperation with Hilbert and with Klein in Göttingen during the past years. As Weyl [Wey35] reports, "*Hilbert at that time was over head and ears in the general theory of relativity, and for Klein, too, the theory of relativity brought the last flareup of his mathematical interest and production*". Emmy Noether, although she was doubtless influenced, not only assisted them but her work was a genuine production of her own. In particular, the connection of invariants with the symmetry groups, with its obvious reference to Klein's Erlanger program, caught the attention of the world of mathematicians and theoretical physicists.[16] Noether's work in this direction has been described in detail in, e.g., [Row99], [KS04], [Wue05].

It is inconceivable that Hermann Weyl did not take notice of this important work of Emmy Noether. At that time Weyl, who was in correspondence with Hilbert and Einstein, was also actively interested in the theory of relativity; his famous book "*Raum, Zeit, Materie*" (Space, Time, Matter) had just appeared. Emmy Noether had cited Weyl's book[17], and almost certainly she had sent him a reprint of her paper. Thus, through the medium of relativity theory there arose mathematical contact between them.[18] Although we do not know, it is well conceivable that there was an exchange of letters concerning the mathematical theory of relativity. From now on Weyl would never remember her brother Fritz better than Emmy.

In 1919 Emmy Noether finally got her *Habilitation*. Already in 1915 Hilbert and Klein, convinced of her outstanding qualification, had recommended her to apply for *Habilitation*. She did so, but it is a sad story that it was unsuccessful because of her gender although her scientific standing was considered sufficient. The incident is told in detail in Tollmien's paper [Tol90]. Thus her *Habilitation* was delayed until 1919 after the political and social conditions had changed.

We see again the difference between the scientific careers of Weyl and of Emmy Noether. Weyl had his *Habilitation* already in 1910, and since 1913 he held a professorship in Zürich. Emmy Noether's *Habilitation* was possible only nine years later than Weyl's. As is well known, she never in her life got a permanent position; although in the course of time she rose to become one of the leading mathematicians in the world.

5 The third period: 1920-1932

The third period of Noether's mathematical life starts with the great paper "*Idealtheorie in Ringbereichen*" (Ideal theory in rings) [Noe21].[19] After Hilbert

[16] In [Row99] it is said that nevertheless "few mathematicians and even fewer physisists ever read Noether's original article ...".

[17] The citation is somewhat indirect. Noether referred to the literature cited in a paper by Felix Klein [Kle18], and there we find Weyl's book meontioned. In a second paper of Noether [Noe23] Weyl's book is cited directly.

[18] Added in proof: We read in [Row99] that there is a reference to Noether in Weyl's book, tucked away in a footnote.

[19] Sometimes the earlier investigation jointly with Schmeidler [NS20] is also counted as belonging to this period.

had shown in 1890 that in a polynomial ring (over a field as base) every ideal is finitely generated, Noether now takes this property as an axiom and investigates the primary decomposition of ideals in arbitrary rings satisfying this axiom. And she reformulates this axiom as an "ascending chain condition" for ideals. Nowadays such rings are called "Noetherian". The paper appeared in 1921.

We note that she was nearly 40 years old at that time. The mathematical life of Emmy Noether is one of the counterexamples to the dictum that mathematics is a science for the young and the most creative work is done before 40. Emmy Noether would not have been a candidate for the Fields medal if it had already existed at that time.

5.1 Innsbruck 1924 and the method of abstraction

We do not know whether and how Weyl took notice of the above-mentioned paper of Noether [Noe21]. But her next great result, namely the follow-up paper [Noe26] on the ideal theory of what are now called Dedekind rings, was duly appreciated by Weyl. At the annual DMV meeting in 1924 in Innsbruck Noether reported about it [Noe25]. And Weyl was chairing that session; so we know that he was informed first hand about her fundamental results.

In her talk, Emmy Noether defined Dedekind rings by axioms and showed that every ring satisfying those axioms admits a unique factorization of ideals into prime ideals. Well, Noether did not use the terminology "Dedekind ring"; this name was coined later. Instead, she used the name "5-axioms-ring" since in her enumeration there were 5 axioms. Then she proved that the ring of integers in a number field satisfies those axioms, and similarly in the funtion field case. This is a good example of Noether's "method of abstraction". By working solely with those axioms she first *generalized* the problem, and it turned out that by working in this generalization the proof of prime decomposition is *simplified* if compared with the former proofs (two of which had been given by Hilbert [Hil94]).

How did Weyl react to Noether's method of abstraction? At that time, this method met sometimes with skepticism and even rejection by mathematicians. But Hilbert in various situations had already taken first steps in this direction and so Weyl, having been Hilbert's doctorand, was not against Noether's method. After all, in his book "Space, Time, Matter" Weyl had introduced vector spaces by axioms, not as n-tuples[20].

Weyl's reaction can be extracted implicitly from an exchange of letters with Hasse which happened seven years later. The letter of Weyl is dated December 8, 1931. At that time Weyl held a professorship in Göttingen (since 1930) as the successor of Hilbert. Thus Emmy Noether was now his colleague in Göttingen. Hasse at that time held a professorship in Marburg (also since 1930) as the successor of Hensel. The occasion of Weyl's letter was the theorem that every simple algebra over a number field is cyclic; this had been established some weeks ago by Brauer, Hasse and Noether, and the latter had informed Weyl about it. So Weyl congratulated Hasse for this splendid achievement. And he recalled the meeting in Innsbruck 1924 when he first had met Hasse.

[20]This has been expressly remarked by MacLane [ML81a].

For us, Hasse's reply to Weyl's letter is of interest.[21] Hasse answered on December 15, 1931. First he thanked Weyl for his congratulations, but at the same time pointed out that the success was very essentially due also to the elegant theory of Emmy Noether, as well as the p-adic theory of Hensel. He also mentioned Minkowski in whose work the idea of the Local-Global principle was brought to light very clearly. And then Hasse continued, recalling Innsbruck:

> Auch ich erinnere mich sehr gut an Ihre ersten Worte zu mir anläßlich meines Vortrages über die erste explizite Reziprozitätsformel für höheren Exponenten in Innsbruck. Sie zweifelten damals ein wenig an der inneren Berechtigung solcher Untersuchungen, indem Sie ins Feld führten, es sei doch gerade Hilberts Verdienst, die Theorie des Reziprozitätsgesetzes von den expliziten Rechnungen früherer Forscher, insbesondere Kummers, befreit zu haben.
>
> *I too remember very well your first words to me on the occasion of my talk in Innsbruck, about the first explicit reciprocity formula for higher exponent. You somewhat doubted the inner justification of such investigations, by pointing out that Hilbert had freed the theory of the reciprocity law from the explicit computations of former mathematicians, in particular Kummer's.*

We conclude: Hasse in Innsbruck had talked on explicit formulas and Weyl had critized this, pointing out that Hilbert had embedded the reciprocity laws into more structural results. Probably Weyl had in mind the product formula for the so-called Hilbert symbol which, in a sense, comprises all explicit reciprocity formulas.[22] For many, like Weyl, this product formula was the final word on reciprocity while for Hasse this was the starting point for deriving explicit, constructive reciprocity formulas, using heavily the p-adic methods of Hensel.

We can fairly well reconstruct the situation in Innsbruck: Emmy Noether's talk had been very abstract, and Hasse's achievement was in some sense the opposite since he was bent on explicit formulas, and quite involved ones too.[23] Weyl had been impressed by Emmy Noether's achievements which he considered as continuing along the lines set by Hilbert's early papers on number theory. In contrast, he considered Hasse's work as pointing not to the future but to the mathematical past.

We have mentioned here these letters Weyl-Hasse in order to put into evidence that already in 1924, Weyl must have had a very positive opinion on Emmy Noether's methods, even to the point of preferring it to explicit formulas.

But as it turned out, Hasse too had been impressed by Noether's lecture. In the course of the years after 1924, as witnessed by the Hasse-Noether correspondence [LR06], Hasse became more and more convinced about Noether's abstract methods which, in his opinion, served to clarify the situation; he used the word

[21] We have found Hasse's letter in the Weyl legacy in the archive of the ETH in Zürich.

[22] But Hilbert was not yet able to establish his product formula in full generality. We refer to the beautiful and complete treatment in Hasse's class field report, Part 2 [Has30a] which also contains the most significant historic references.

[23] Hasse's Innsbruck talk is published in [Has25]. The details are found in volume 154 of Crelle's Journal where Hasse had published 5 papers on explicit reciprocity laws.

"*durchsichtig*" (lucid). Hasse's address at the DMV meeting in Prague 1929 [Has30b] expresses his views very clearly. Hensel's p-adic methods could also be put on an abstract base, due to the advances in the theory of valuations.[24] But on the other hand, Hasse was never satisfied with abstract theorems only. In his cited letter to Weyl 1931 he referred to his (Hasse's) class field report Part II [Has30a] which had appeared just one year earlier. There, he had put Artin's general reciprocity law[25] as the base, and from this structural theorem he was able to derive all the known reciprocity formulas. Hasse closed his letter with the following:

> . . . Ich kann aber natürlich gut verstehen, daß Dinge wie diese expliziten Reziprozitätsformeln einem Manne Ihrer hohen Geistes- und Geschmacksrichtung weniger zusagen, als mir, der ich durch die abstrakte Mathematik Dedekind-E. Noetherscher Art nie restlos befriedigt bin, ehe ich nicht zum mindesten *auch* eine explizite, formelmäßige konstruktive Behandlung daneben halten kann. Erst von der letzteren können sich die eleganten Methoden und schönen Ideen der ersteren wirklich vorteilhaft abheben.
>
> *. . . But of course I well realize that those explicit reciprocity formulas may be less attractive to a man like you with your high mental powers and taste, as to myself. I am never fully satisfied by the abstract mathematics of Dedekind-E. Noether type before I can* also *supplement it by at least one explicit, computational and constructive treatment. It is only in comparison with the latter that the elegant methods and beautiful ideas of the former can be appreciated advantageously.*

Here, Hasse touches a problem which always comes up when, as Emmy Noether propagated, the abstract methods are put into the foreground. Namely, abstraction and axiomatization is not to be considered as an end in itself; it is a method to deal with concrete problems of substance. But Hasse was wrong when he supposed that Weyl did not see that problem. Even in 1931, the same year as the above cited letters, Weyl gave a talk on abstract algebra and topology as two ways of mathematical comprehension [Wey32]. In this talk Weyl stressed the fact that axiomatization is not only a way of securing the logical truth of mathematical results, but that it had become a powerful tool of concrete mathematical research itself, in particular under the influence of Emmy Noether. But he also said that abstraction and generalization do not make sense without mathematical substance behind it. This is close to Hasse's opinion as expressed in his letter above.[26] The mathematical work of both Weyl and Hasse puts their opinions into evidence.

At the same conference [Wey32] Weyl also said that the "*fertility of these abstracting methods is approaching exhaustion*". This, however, met with sharp protests by Emmy Noether, as Weyl reports in [Wey35]. In fact, today most of us would agree with Noether. The method of abstracting and axiomatizing

[24] For this see [Roq02].

[25] By the way, this was the first treatment of Artin's reciprocity law in book form after Artin's original paper 1927. We refer to our forthcoming book on the Artin-Hasse correspondence.

[26] Even more clearly Hasse has expressed his view in the foreword to his beautiful and significant book on abelian fields [Has52].

has become a natural and powerful tool for the mathematician, with striking successes until today. In Weyl's letter to Hasse (which we have not cited fully) there are passages which seem to indicate that in principle he (Weyl) too would agree with Emmy Noether. For, he encourages Hasse to continue his work in the same fashion, and there is no mention of an impending "exhaustion". He closes his letter with the following sentence which, in our opinion, shows his (Weyl's) opinion of how to work in mathematics:

> Es freut mich besonders, daß bei Ihnen die in Einzelleistungen sich bewährende wissenschaftliche Durchschlagskraft sich mit geistigem Weitblick paart, der über das eigene Fach hinausgeht.
>
> *In particular I am glad that your scientific power, tested in various special accomplishments, goes along with a broad view stretching beyond your own special field.*

5.2 Representations: 1926/27

We have made a great leap from 1924 to 1931. Now let us return and proceed along the course of time. In the winter semester 1926/27 Hermann Weyl stayed in Göttingen as a visiting professor, and he lectured on representations of continuous groups. In [Wey35] he reports:

> *I have a vivid recollection of her* [Emmy Noether]*... She was in the audience; for just at that time the hypercomplex number systems and their representations had caught her interest and I remember many discussions when I walked home after the lectures, with her and von Neumann, who was in Göttingen as a Rockefeller Fellow, through the cold, dirty, rain-wet streets of Göttingen.*

This gives us information not only about the weather conditions in Göttingen in winter time but also that a lively discussion between Weyl and Emmy Noether had developed.

We do not know precisely when Emmy Noether first had become interested in the representation theory of groups and algebras, or "hypercomplex systems" in her terminology. In any case, during the winter semester 1924/25 in Göttingen she had given a course on the subject. And in September 1925 she had talked at the annual meeting of the DMV in Danzig on "*Group characters and ideal theory*". There she advocated that the whole representation theory of groups should be subsumed under the theory of algebras and their ideals. She showed how the Wedderburn theorems for algebras are to be interpreted in representation theory, and that the whole theory of Frobenius on group characters is subsumed in this way. Although she announced a more detailed presentation in the *Mathematische Annalen*, the mathematical public had to wait until 1929 for the actual publication [Noe29][27]. Noether was not a quick writer; she developed her ideas again and again in discussions, mostly on her walks with students and colleagues into the woods around Göttingen, and in her lectures.

[27] This appeared in the "Mathematische Zeitschrift" and not in the "Annalen" as announced by Noether in Danzig.

The text of her paper [Noe29] consists essentially of the notes taken by van der Waerden at her lecture in the winter semester 1927/28. Although the main motivation of Noether was the treatment of Frobenius' theory of representations of finite groups, it turned out that finite groups are treated on the last two pages only – out of a total of 52 pages. The main part of the paper is devoted to introducing and investigating general abstract notions, capable of dealing not only with the classical theory of finite group representations but with much more. Again we see the power of Noether's abstracting methods. The paper has been said to constitute "*one of the pillars of modern linear algebra*".[28]

We can imagine Emmy Noether in her discussions with Weyl on the cold, wet streets in Göttingen 1926/27, explaining to him the essential ideas which were to become the foundation of her results in her forthcoming paper [Noe29]. We do not know to which extent these ideas entered Weyl's book [Wey39] on classical groups. After all, the classical groups which are treated in Weyl's book are *infinite* while Noether's theory aimed at the representation of *finite* groups. Accordingly, in Noether's work there appeared a *finiteness condition* for the algebras considered, namely the descending chain condition for (right) ideals. If one wishes to use Noether's results for infinite groups one first has to generalize her theory such as to remain valid in more general cases too. Such a generalization did not appear until 1945; it was authored by Nathan Jacobson [Jac45]. He generalized Noether's theory to simple algebras containing at least one irreducible right ideal.

At this point let me tell a story which I witnessed in 1947. I was a young student in Hamburg then. In one of the colloquium talks the speaker was F. K. Schmidt who recently had returned from a visit to the US, and he reported on a new paper by Jacobson which he had discovered there.[29] This was the above mentioned paper [Jac45]. F. K. Schmidt was a brilliant lecturer and the audience was duly impressed. In the ensuing discussion Ernst Witt, who was in the audience, commented that all this had essentially been known to Emmy Noether already.

Witt did not elaborate on his comment. But he had been one of the "Noether boys" in 1932/33, and so he had frequently met her. There is no reason to doubt his statement. It may well have been that she had told him, and perhaps others too, that her theory could be generalized in the sense which later had been found by Jacobson. Maybe she had just given a hint in this direction, without details, as was her usual custom. In fact, reading Noether's paper [Noe29] the generalization is obvious to any reader who is looking for it.[30] It is fascinating to think that the idea for such a generalization arose from her discussions with Weyl in Göttingen in 1927, when infinite groups were discussed and the need to generalize her theory became apparent.

By the way, Jacobson and Emmy Noether met in 1934 in Princeton, when she was running a weekly seminar. We cannot exclude the possibility that she had given a hint to him too, either in her seminar or in personal discussions.

[28] Cited from [Cur99] who in turn refers to Bourbaki.

[29] In those post-war years it was not easy to get hold of books or journals from foreign countries, and so the 1945 volume of the "Transactions" was not available at the Hamburg library.

[30] A particularly short and beautiful presentation is to be found in Artin's article [Art50] where he refers to Tate.

After all, this was her usual style, as reported by van der Waerden [vdW35].

5.3 A letter from N to W: 1927

As stated in the introduction we have not found letters from Emmy Noether to Weyl, with one exception. That exception is kept in the archive of the ETH in Zürich. It is written by Emmy Noether and dated March 12, 1927. This is shortly after the end of the winter semester 1926/27 when Weyl had been in Göttingen as reported in the previous section. Now Weyl was back in Zürich and they had to write letters instead of just talking.

The letter concerns Paul Alexandroff and Heinz Hopf and their plan to visit Princeton in the academic year 1927/28.

We have already mentioned Alexandroff in section 3.2 in connection with Noether's contributions to topology. From 1924 to 1932 he spent every summer in Göttingen, and there developed a kind of friendly relationship between him and Emmy Noether. The relation of Noether to her "Noether boys" has been described by André Weil as like a mother hen to her fledglings [Wei93]. Thus Paul Alexandroff was accepted by Emmy Noether as one of her fledglings. In the summer semester 1926 Heinz Hopf arrived in Göttingen as a postdoc from Berlin and he too was accepted as a fledgling. Both Alexandroff and Hopf became close friends and they decided to try to go to Princeton University in the academic year 1927/28.

Perhaps Emmy Noether had suggested this; in any case she helped them to obtain a Rockefeller grant for this purpose. It seems that Weyl also had lent a helping hand, for in her letter to him she wrote:

> ... Jedenfalls danke ich Ihnen sehr für Ihre Bemühungen; auch Alexandroff und Hopf werden Ihnen sehr dankbar sein und es scheint mir sicher, dass wenn die formalen Schwierigkeiten erst einmal überwunden sind, Ihr Brief dann von wesentlichem Einfluss sein wird.
>
> *... In any case I would like to thank you for your help; Alexandroff and Hopf too will be very grateful to you. And I am sure that if the formal obstacles will be overcome then your letter will be of essential influence.*

The "formal obstacles" which Noether mentioned were, firstly, the fact that originally the applicants (Alexandroff and Hopf) wished to stay for a period less than an academic year in Princeton (which later they extended to a full academic year), and secondly, that Hopf's knowledge of the English language seemed not to be sufficient in the eyes of the Rockefeller Foundation (but Noether assured them that Hopf *wanted* to learn English).[31] But she mentioned there had been letters sent to Lefschetz and Birkhoff and that at least the latter had promised to approach the Rockefeller Foundation to make an exception.

[31] It seems that his knowledge of English had improved in the course of time since Heinz Hopf had been elected president of the IMU (International Mathematical Union) in 1955 till 1958.

Alexandroff was in Moscow and Hopf in Berlin at the time, and so the mother hen acted as representative of her two chickens.[32]

About Alexandroff's and Hopf's year in Princeton we read in the Alexandroff article of MacTutor's History of Mathematics archive:

> *Aleksandrov and Hopf spent the academic year 1927-28 at Princeton in the United States. This was an important year in the development of topology with Aleksandrov and Hopf in Princeton and able to collaborate with Lefschetz, Veblen and Alexander.*

The letter from Noether to Weyl shows that both N. and W. were instrumental in arranging this important Princeton year for Alexandroff and Hopf. Both were always ready to help young mathematicians to find their way.

Remark: Later in 1931, when Weyl had left Zürich for Göttingen, it was Heinz Hopf who succeeded Weyl in the ETH Zürich. At those times it was not uncommon that the leaving professor would be asked for nominations if the faculty wished to continue his line. We can well imagine that Weyl, who originally would have preferred Artin[33], finally nominated Heinz Hopf for this position. If so then he would have discussed it with Emmy Noether since she knew Hopf quite well. It may even have been that she had taken the initiative and proposed to Weyl the nomination of Hopf. In fact, in the case of Alexandroff she did so in a letter to Hasse dated October 7, 1929 when it was clear that Hasse would change from Halle to Marburg. There she asked Hasse whether he would propose the name of Alexandroff as a candidate in Halle.[34] It seems realistic to assume that in the case of Heinz Hopf she acted similarly.

Remark 2: The above mentioned letter of Noether to Weyl contains a postscript which gives us a glimpse of the mathematical discussion between the two (and it is the only written document for this). It reads:

> Die Mertens-Arbeit, von der ich Ihnen sprach, steht Monatshefte, Bd. 4. Ich dachte an den Schluss, Seite 329. Es handelt sich hier aber doch nur um Determinanten-Relationen, sodass es für Sie wohl kaum in Betracht kommt.
>
> *The Mertens paper which I mentioned to you is contained in volume 4 of the* Monatshefte*. I had in mind the end of the paper, page 329. But this is concerned with determinant relations only, hence it will perhaps not be relevant to your purpose.*

The Mertens paper is [Mer93]. We have checked the cited page but did not find any hint which would connect to Weyl's work. Perhaps someone else will be able to interpret Noether's remark.

[32] Probably the letters from Noether and Weyl to the Rockefeller Foundation, i.e., to Trowbridge, on this matter are preserved in the Rockefeller archives in New York but we have not checked this.

[33] In fact, in 1930 Artin received an offer from the ETH Zürich which, however, he finally rejected.

[34] This however, did not work out. The successor of Hasse in Halle was Heinrich Brandt, known for the introduction of "Brandt's gruppoid" for divisor classes in simple algebras over number fields.

5.4 Weyl in Göttingen: 1930-1933

Weyl in [Wey35] reports:

> *When I was called permanently to Göttingen in 1930, I earnestly tried to obtain from the Ministerium a better position for her* [Emmy Noether]*, because I was ashamed to occupy such a preferred position beside her whom I knew my superior as a mathematician in many respects.*

We see that by now, Weyl was completely convinced about the mathematical stature of Emmy Noether. After all, Emmy Noether in 1930 was the world-wide acknowledged leader of abstract algebra, and her presence in Göttingen was the main attraction for young mathematicians from all over the world to visit the Mathematical Institute and study with her.

It would be interesting to try to find out which "better position" Weyl had in mind in his negotiations with the *Ministerium* in Berlin. Maybe he wished tenure for her, and an increase of her salary. The archives in Berlin will perhaps have the papers and reports of Weyl's negotiations. From those papers one may be able to extract the reasons for the rejection. But the opposition against Noether's promotion did not only come from the Ministerium in Berlin. It seems that a strong opposition came also from among the mathematician colleagues in Göttingen, for Weyl continues with his report as follows:

> *. . . nor did an attempt* [succeed] *to push through her election as a member of the Göttinger Gesellschaft der Wissenschaften. Tradition, prejudice, external considerations, weighted the balance against her scientific merits and scientific greatness, by that time denied by no one.*

I do not know whether there exist minutes of the meetings of the *Göttinger Mathematische Gesellschaft* in 1930. If so then it would be interesting to know the traditional, biased and external arguments which Weyl said were put forward against Emmy Noether from the members of the *Mathematische Gesellschaft* . Was it still mainly her gender? Or was it the opposition to her "abstract" mathematical methods? In any case, the decision not to admit Emmy Noether as a member of the *Göttinger Mathematische Gesellschaft* is to be regarded as an injustice to her and a lack of understanding of the development of modern mathematics. After all, the Emmy Noether of 1930 was quite different from the Emmy Noether of 1915. Now in 1930, she had already gone a long way "*on her own completely original mathematical path*" , and her "*working and conceptual methods had spread everywhere*". She could muster high-ranking colleagues and students who were fascinated by her way of mathematical thinking.

Nevertheless it seems that there was some opposition against her abstract methods, also in Göttingen among the mathematicians. Olga Taussky-Todd recalls in [TT81] her impression of the Göttingen mathematical scene:

> *. . . not everybody liked her* [Emmy Noether]*, and not everybody trusted that her achievements were what they later were accepted to be.*

One day Olga Taussky had been present when one of the senior professors talked very roughly to Emmy Noether. (Later he apologized to her for this insult.) When Emmy Noether had her 50th birthday in 1932 then, as Olga Taussky recalls, nobody at Göttingen had taken notice of it, although at that time all birthdays were published in the *Jahresbericht of the DMV*.[35] Reading all this, I can understand Emmy Noether when later in 1935 she said to Veblen about her time in the USA:

> *The last year and a half had been the very happiest in her whole life, for she was appreciated in Bryn Mawr and Princeton as she had never been appreciated in her own country.*[36]

Thus it seems that Weyl's statement that "*her scientific merits and scientific greatness by that time was denied by no one*" did not describe the situation exactly. Perhaps, since Weyl was the "premier professor" of mathematics at Göttingen, and since he was known to respect and acknowledge Noether's merits and scientific greatness, nobody dared to tell him if he disagreed. Olga Taussky-Todd remembers that

> "*outside of Göttingen, Emmy was greatly appreciated in her country.*"

We may add that this was not only so in her country but also world-wide. And of course also in Göttingen there was an ever-growing fraction of mathematicians, including Weyl, who held Noether in high esteem.

As to Hermann Weyl, let us cite MacLane who was a student at Göttingen in the period 1931-33. We read in [ML81b]:

> *When I first came to Göttingen I spoke to Professor Weyl and expressed my interest in logic and algebra. He immediately remarked that in algebra Göttingen was excellently represented by Professor Noether; he recommended that I attend her courses and seminars . . . By the time of my arrival she was Ausserordentlicher Professor. However, it was clear that in the view of Weyl, Hilbert, and the others, she was right on the level of any of the full professors. Her work was much admired and her influence was widespread.*

MacLane sometimes joined the hiking parties (*Ausflug*) of Emmy Noether and her class to the hills around Göttingen. Noether used these hiking parties to discuss "*algebra, other mathematical topics and Russia*".[37] It seems that Weyl too joined those excursions occasionally. There is a nice photo of Noether with Weyl and family, together with a group of mathematicians posing in front of the "*Gasthof Vollbrecht*". The photo is published in [BS81] and dated 1932. Since Artin is seen as a member of the hiking party, it seems very probable that the photo was taken on the occasion of Artin's famous Göttingen lectures

[35] But Hasse in Marburg had sent her a birthday cake, together with a paper which he had dedicated to her. The paper was [Has33]. See [LR06].

[36] Cited from a letter of Abraham Flexner to President Park of Bryn Mawr; see [Roq07b].

[37] 1928/29 Emmy Noether had been in Moscow as a visiting professor, on the invitation of Alexandroff whom she knew from Göttingen.

on class field theory which took place from February 29 to March 2, 1932.[38] This was a big affair and a number of people came from various places in order to listen to Artin lecturing on the new face of class field theory. The lectures were organized by Emmy Noether. Since she was not a full professor and, accordingly, had no personal funds to organize such meetings we suppose that one of her colleagues, probably Weyl, had made available the necessary financial means for this occasion. In any case we see that by now she was able to get support for her activities in Göttingen, not only for the Artin lectures but also for other speakers.

The International Congress of Mathematicians took place in September 1932 in Zürich. Emmy Noether was invited to deliver one of the main lectures there. Usually, proposals for invited speakers at the IMU conferences were submitted by the presidents of the national mathematical organizations which were members of the IMU. In 1931/32 Hermann Weyl was president ("*Vorsitzender*") of the DMV. So it appears that Weyl had his hand in the affair when it came to proposing Emmy Noether as a speaker from Germany. The proposal had to be accepted by the executive committee. The nomination of Emmy Noether was accepted and this shows the great respect and admiration which Emmy Noether enjoyed on the international scale.

Emmy Noether's Zürich lecture can be considered as the high point in her mathematical career.

6 Göttingen exodus: 1933

The year 1933 brought about the almost complete destruction of the unique mathematical scene in Göttingen. In consequence of the antisemitic political line of the Nazi government many scientists of Jewish origin had to leave the university, as well as those who were known to be critical towards the new government. The Göttingen situation in 1933 has often been described, and so we can refer to the literature, e.g., [Sch87], [Seg03].

Emmy Noether was of Jewish origin and so she too was a victim of the new government policy. On May 5, 1933 Emmy Noether obtained the message that she was put "temporarily on leave" from lecturing at the university. When Hasse heard this, he wrote a letter to her; we do not know the text of his letter but from her reply we may conclude that he asked whether he could be of help. Emmy Noether replied on May 10, 1933:

> Lieber Herr Hasse! Vielen herzlichen Dank für Ihren guten freundschaftlichen Brief! Die Sache ist aber doch für mich sehr viel weniger schlimm als für sehr viele andere: rein äußerlich habe ich ein kleines Vermögen (ich hatte ja nie Pensionsberechtigung), sodaß ich erst einmal in Ruhe abwarten kann; im Augenblick, bis zur definitiven Entscheidung oder etwas länger, geht auch das Gehalt noch weiter. Dann wird wohl jetzt auch einiges von der Fakultät versucht, die Beurlaubung nicht definitiv zu machen; der Erfolg ist natürlich im Moment recht fraglich.

[38] The photo is also contained in the Oberwolfach photo collection online. Probably the photo was taken by Natascha Artin, the wife of Emil Artin.

> Schließlich sagte Weyl mir, daß er schon vor ein paar Wochen, wo alles noch schwebte, nach Princeton geschrieben habe wo er immer noch Beziehungen hat. Die haben zwar wegen der Dollarkrise jetzt auch keine Entschlußkraft; aber Weyl meinte doch daß mit der Zeit sich etwas ergeben könne, zumal Veblen im vorigen Jahr viel daran lag, mich mit Flexner, dem Organisator des neuen Instituts, bekannt zu machen. Vielleicht kommt einmal eine sich eventuell wiederholende Gastvorlesung heraus, und im übrigen wieder Deutschland, das wäre mir natürlich das liebste. Und vielleicht kann ich Ihnen sogar auch einmal so ein Jahr Flexner-Institut verschaffen - das ist zwar Zukunftsphantasie - wir sprachen doch im Winter davon ...
>
> *Dear Mr. Hasse! Thanks very much for your good, friendly letter! But for myself, the situation is much less dire than for many others: in fact I have a small fortune (after all I was never entitled to pension) and hence for the time being I can quietly wait and see. Also, the salary payments continue until the final decision or even somewhat longer. Moreover the Faculty tries to avert my suspension to become final; at the moment, however, there is little hope for success. Finally, Weyl told me that some weeks ago already when things were still open, he had written to Princeton where he still has contacts. At the moment, however, because of the dollar crisis they don't have much freedom there for their decisions; but Weyl believes that in the course of time there may arise something, in particular since Veblen last year was eager to introduce me to Flexner, the organizer of the new Institute. Perhaps there will emerge a visiting professorship which may be iterated, and in the meantime Germany again, this would be the best solution for me, naturally. And maybe I will be able to manage for you too a year in the Flexner Institute – but this is my fantasy for the future – we have talked last winter about this...*

The first impression while reading this letter is her complete selflessness, which is well-known from other reports on her life and which is manifest here again. She does not complain about her own situation but only points out that for other people things may be worse. Reading further, we see that the Faculty in Göttingen tries to keep her; this shows that she was respected there as a scientist and teacher although she still did not have a tenured position. Hermann Weyl was a full professor and hence a member of the Faculty committee; we can surely assume that he was one of the driving forces in trying to save Emmy Noether for a position in Göttingen. In fact, in his memorial speech [Wey35] Weyl said:

> *It was attempted, of course, to influence the Ministerium and other responsible and irresponsible bodies so that her position might be saved. I suppose there could hardly have been in any other case such a pile of enthusiastic testimonials filed with the Ministerium as was sent in on her behalf. At that time we really fought; there was still hope left that the worst could be warded off ...*

And finally, in the above Noether letter we read that, independent of these attempts, Weyl had written to Princeton on her behalf. We do not know whom

in Princeton Weyl had adressed. Since Noether mentions in her letter Veblen and Flexner, it seems probable that Weyl had written to one or both of them. Abraham Flexner was the spiritual founder and the first director of the newly-founded Institute for Advanced Study in Princeton. Oswald Veblen was the first permanent mathematics professor of the IAS. Certain indications suggest that Weyl had written to Lefschetz too; see next section. Solomon Lefschetz had the position of full professor at Princeton University.

One year earlier, in the late summer of 1932, Weyl had rejected an offer to join the IAS as a permanent member. But now, since the political situation had deteriorated, he inquired whether it was possible to reverse his decision. (It was.) From Noether's letter we infer that Weyl did not only write on his own behalf but also on Noether's. This fact alone demonstrates the very high esteem in which he held Noether as a mathematician and as a personality.[39]

But of course, the best solution would be that Noether could stay in Göttingen. This was what Weyl wished to achieve foremost, as we cited above. (It was in vain.) Weyl reports in [Wey35]:

> *I have a particularly vivid recollection of these months. Emmy Noether, her courage, her frankness, her unconcern about her own fate, her conciliatory spirit, were, in the middle of all the hatred and meanness, despair and sorrow surrounding us, a moral solace.*

That stormy time of struggle in the summer of 1933 in Göttingen drew them closer together. This is also evident from the words Weyl used two years later in his speech at her funeral:[40]

> *You did not believe in evil, indeed it never occurred to you that it could play a role in the affairs of man. This was never brought home to me more clearly than in the last summer we spent together in Göttingen, the stormy summer of 1933. In the midst of the terrible struggle, destruction and upheaval that was going on around us in all factions, in a sea of hate and violence, of fear and desperation and dejection - you went your own way, pondering the challenges of mathematics with the same industriousness as before. When you were not allowed to use the institute's lecture halls you gathered your students in your own home. Even those in their brown shirts were welcome; never for a second did you doubt their integrity. Without regard for your own fate, openhearted and without fear, always conciliatory, you went your own way. Many of us believed that an enmity had been unleashed in which there could be no pardon; but you remained untouched by it all.*

Parallel to the attempts of the Faculty to keep Noether in Göttingen, Hasse took the initiative and collected testimonials[41] which would put into evidence

[39] In the course of time, Weyl used his influence in American academic circles to help many other mathematicians as well.

[40] See section 9.2.

[41] The German word is "*Gutachten*". I am not sure whether the translation into "*testimonial*" is adequate. My dictionary offers also "*opinion*" or "*expertise*" or "*letter of recommendation*". I have chosen "*testimonial*" since Weyl uses this terminology.

that Emmy Noether was a scientist of first rank and hence it would be advantageous for the scientific environment of Göttingen if she did not leave. Hasse collected 14 such testimonials. Together they were sent to the *Kurator* of the university who was to forward them to the *Ministerium* in Berlin. Recently we have found the text of those testimonials which are kept in the Prussian State archives in Berlin; we plan to publish them separately. The names of the authors are:

H. Bohr, Kopenhagen
Ph. Furtwängler, Wien
G.H. Hardy, Cambridge
H. Hasse, Marburg
O. Perron, München
T. Rella, Wien
J.A. Schouten, Delft
B. Segre, Bologna
K. Shoda, Osaka
C. Siegel, Frankfurt
A. Speiser, Zürich
T. Takagi, Tokyo
B.L. van der Waerden, Leipzig
H. Weyl, Göttingen

We see that also Hermann Weyl wrote a testimonial. We have included it in the appendix, translated into English; see section 9.1. Note that Weyl compared Emmy Noether to Lise Meitner, the nuclear physicist. In the present situation this comparison may have been done since Meitner, also of Jewish origin, was allowed to stay in Berlin continuing her research with Otto Hahn in their common laboratory. After all, the initiatives of Hasse and of Weyl were to obtain a similar status for Emmy Noether in Göttingen.

As is well-known, this was in vain. Perhaps those testimonials were never read after the *Kurator* of Göttingen University wrote to the *Ministerium* that Emmy Noether's political opinions were based on "Marxism".[42]

Let us close this section with some lines from a letter of Weyl to Heinrich Brandt in Halle. The letter is dated December 15, 1933; at that time Weyl and Noether were already in the USA. Brandt was known to be quite sceptical towards abstract methods in mathematics; he did not even like Artin's beautiful presentation of his own (Brandt's) discovery, namely that the ideals and ideal classes of maximal orders in a simple algebra over a number field form a groupoid under multiplication.[43] (The notion of "groupoid" is Brandt's invention.) Weyl's letter is a reply to one from Brandt which, however, is not known to us. Apparently Brandt had uttered some words against Noether's abstracting method, and Weyl replied explaining his own viewpoint:[44]

[42] See [Tol90]. – By the way, there was another such initiative started, namely in favor of Courant who also had been "*beurlaubt*" from Göttingen University. That was signed by 28 scientists including Hermann Weyl and Helmut Hasse. Again this was not successful, although this time the *Kurator's* statement was not as negative as in Noether's case. (We have got this information from Constance Reid's book on Courant [Rei76].)

[43] Artin's paper is [Art28].

[44] I would like to thank M. Göbel for sending me copies of this letter from the Brandt

> . . . So wenig mir persönlich die "abstrakte" Algebra liegt, so schätze ich doch ihre Leistungen und ihre Bedeutung offenbar wesentlich höher ein, als Sie das tun. Es imponiert mir gerade an Emmy Noether, daß ihre Probleme immer konkreter und tiefer geworden sind.
>
> *Personally, the "abstract" algebra doesn't suit me well, but apparently I do estimate its achievements and importance much higher than you are doing. I am particularly impressed that Emmy Noether's problems have become more and more concrete and deep.*

Weyl continues as follows. It is not known whether Brandt had written some comments on Noether's Jewish origin and connected this with her abstract way of thinking, or perhaps Weyl's letter was triggered by the general situation in Germany and especially in Göttingen:

> Warum soll ihr, der Hebräerin, nicht zustehen, was in den Händen des "Ariers" Dedekind zu großen Ergebnissen geführt hat? Ich überlasse es gern Herrn Spengler und Bieberbach, die mathematische Denkweise nach Völkern und Rassen zu zerteilen. Daß Göttingen den Anspruch verloren hat, mathematischer Vorort zu sein, gebe ich Ihnen gerne zu – was ist denn überhaupt von Göttingen übrig geblieben? Ich hoffe und wünsche, daß es eine seiner alten Tradition würdige Fortsetzung durch neue Männer finden möge; aber ich bin froh, daß ich es nicht mehr gegen einen Strom von Unsinn und Fanatismus zu stützen brauche!
>
> *Why should she, as of Hebrew descent, not be entitled to do what had led to such great results in the hands of Dedekind, the "Arian"? I leave it to Mr. Spengler and Mr. Bieberbach to divide the mathematical way of thinking according to nations and races. I concede that Göttingen has lost its role as a high-ranking mathematical place – what is actually left of Göttingen? I hope and wish that Göttingen would find a continuation by new men, worthy of its long tradition; but I am glad that I do not have to support it against a torrent of nonsense and fanaticism.*

7 Bryn Mawr: 1933-1935

As we have seen in the foregoing section, Weyl had written to Princeton on behalf of Emmy Noether, and this was in March or April 1933 already. Since he was going to join the Institute for Advanced Study in Princeton, one would assume that he had recommended accepting Emmy Noether as a visiting scientist of the Institute. We know that some people at the Institute were interested in getting Noether to Princeton, for at the International Zürich Congress Oswald Veblen had been eager to introduce Emmy Noether to the Institute's director, Abraham Flexner. (See Noether's letter to Hasse, cited in the foregoing section.)

But as it turned out, Emmy Noether did not receive an invitation as a visitor to the Institute. We do not know the reason for this; perhaps the impending

archive in Halle. The letter is published in [Jen86], together with other letters Brandt-Weyl and Brandt-Noether.

dollar crisis, mentioned in Noether's letter to Hasse, forced the Institute to reduce its available funds. Or, may there have been other reasons as well? On the other hand, from the documents which we found in the archive of Bryn Mawr College it can be seen that the Institute for Advanced Study contributed a substantial amount towards the salary of Emmy Noether in Bryn Mawr.

We do not know who was the first to suggest that Bryn Mawr College could be a suitable place for Emmy Noether. Some evidence points to the conclusion that it was Solomon Lefschetz. In fact, we have found a letter, dated June 12, 1933 already, adressed to the "*Emergency Committee in Aid of Displaced German Scholars*", where he discusses future aspects for Emmy Noether and proposes Bryn Mawr.[45] Lefschetz had visited Göttingen two years ago and so he knew Emmy Noether personally. Lefschetz' letter is quite remarkable since, firstly, he clearly expresses that Emmy Noether, in his opinion, was a leading figure in contemporary mathematics; secondly we see that he had taken already practical steps to provide Bryn Mawr with at least part of the necessary financial means in order to offer Emmy Noether a stipend. Let us cite the relevant portions of that letter:

> *Dear Dr. Duggan: I am endeavoring to make connections with some wealthy people in Pittsburgh, one of them a former Bryn Mawr student, with a view of raising a fund to provide a research associateship at Bryn Mawr for Miss Emmy Noether. As you may know, she is one of the most distinguished victims of the Hitler cold pogrom and she is victimized doubly; first for racial reasons and second, owing to her sex. It occured to me that it would be a fine thing to have her attached to Bryn Mawr in a position which would compete with no one and would be created ad hoc; the most distinguished feminine mathematician connected with the most distinguished feminine university. I have communicated with Mrs. Wheeler, the Head of the Department at Bryn Mawr, and she is not only sympathetic but thoroughly enthusiastic for this plan,*
>
> *So far as I know, your organization is the only one which is endeavoring to do anything systematic to relieve the situation of the stranded German scientists. As I do not think random efforts are advisable, I wish first of all to inform you of my plan. Moreover, if I were to succeed only partially, would it be possible to get any aid from your organization? I would greatly appreciate your informing me on this point at your earliest convenience.*
>
> *In the preliminary communication with my intended victims I mentioned the following proposal: to contribute enough annually to provide Miss Noether with a very modest salary, say $ 2000, and a retiring allowance of $ 1200.*
>
> *Yours very sincerely, S. Lefschetz.*

Already one month later the committee granted the sum of $ 2000 to Bryn Mawr for Emmy Noether.

[45] We have found this letter in the archives of the New York Public Library.

There arises the question from whom Lefschetz had got the information, at that early moment already, that Emmy Noether had been suspended.[46] We are inclined to believe that it was Hermann Weyl. I do not know whether the correspondence of Lefschetz of those years has been preserved in some archive, and where. Perhaps it will be possible to find those letters and check.

Emmy Noether arrived in Bryn Mawr in early November 1933. Her first letter from Bryn Mawr to Hasse is dated March 6, 1934. She reported, among other things, that since February she gave a lecture once a week at the Institute for Advanced Study in Princeton. In this lecture she had started with representation modules and groups with operators. She mentions that Weyl too is lecturing on representation theory, and that he will switch to continuous groups later. It appears that the Göttingen situation of 1926/27 was repeating. And we imagine Hermann Weyl and Emmy Noether walking after her lectures around the Campus of Princeton University[47] instead of Göttingen's narrow streets, vividly discussing new aspects of representation theory.

In the book [Rei76] on Courant we read:

> *Weyl sent happy letters from Princeton. In Fine Hall, where Flexner's group was temporarily housed, German was spoken as much as English. He frequently saw Emmy Noether ...*

Perhaps in the Courant legacy we can find more about Weyl and Noether in Princeton, but we have not been able yet to check those sources.

Every week Emmy Noether visited the Brauers in Princeton; Richard Brauer was assistant to Weyl in that year and perhaps sometimes Hermann Weyl also joined their company. The name of Hermann Weyl appears several times in her letters to Hasse from Bryn Mawr. In November 1934 she reports that she had studied Weyl's recent publication on Riemann matrices in the Annals of Mathematics.

Emmy Noether died on April 14, 1935. One day later Hermann Weyl cabled to Hasse:

> *hasse mathematical institute gottingen – emmy noether died yesterday – by sudden collapse after successful – operation of tumor*[48] *few days ago – burial wednesday bryn mawr – weyl*

At the burial ceremony on Wednesday Weyl spoke on behalf of her German friends and colleagues. We have included an English translation of this moving text in the appendix; see section 9.2. One week later he delivered his memorial

[46] Emmy Noether had been "beurlaubt", i.e., temporarily suspended from her duties, in May 1933. Observing that mail from Europe to USA used about 2-3 weeks at that time, we conclude that Lefschetz must have started working on his Noether-Bryn Mawr idea immediately after receiving the news about her suspension. Noether was finally dismissed from university on September 9, 1933.

[47] The Institute's Fuld Hall had not yet been built and the School of Mathematics of the Institute was temporarily housed in Fine Hall on the University Campus.

[48] President Park of Bryn Mawr had sent a detailed report, dated May 16, 1935, to Otto Nöther in Mannheim, a cousin of Emmy Noether. A copy of that letter is preserved. There it is stated that according to the the medical diagnosis of the doctors who operated her, Emmy Noether suffered from a "pelvis tumor".

lecture in the large auditorium of Bryn Mawr College. That text is published and well known [Wey35].

8 The Weyl-Einstein letter to the NYT

On Sunday May 5, 1935 the New York Times published a "Letter to the Editor", signed by Albert Einstein and headed by the following title:

Professor Einstein Writes in Appreciation of a Fellow-Mathematician.

We have included the text of this letter in our appendix; see section 9.3.

Reading this letter one is struck by the almost poetic style which elevates the text to one of the pearls in the literature on mathematics. The text is often cited, the last citation which I found is in the "*Mitteilungen*" of the DMV, 2007 where Jochen Brüning tries to connect mathematics with poetry [Brü07]. But because of this character of style it has been doubted whether the text really was composed by Einstein himself. If not then this would not have been the first and not the last incident where Einstein had put his name under a text which was not conceived by himself – provided that in his opinion the subject was worth-while to support. Since Weyl's poetic style was known it was not considered impossible that the text was composed by Hermann Weyl.

Some time ago I have come across a letter signed by Dr. Ruth Stauffer-McKee. I include a copy of that letter in the appendix; see section 9.4. In particular I refer to the last paragraph of the letter. Based on the information provided by Stauffer I came to the conclusion that, indeed, the text was essentially written by Weyl. I have expressed this opinion in my talk in Bielefeld and also in a "Letter to the Editor" of the "*Mitteilungen der Deutschen Mathematiker-Vereinigung*" [Roq07a].

However, recently I have been informed that Einstein's draft of this letter in his own handwriting has been found by Siegmund-Schultze[49] in the Einstein archive in Jerusalem. The article is to appear in the next issue of the *Mitteilungen der DMV* [SS07]. This then settles the question of authorship in favor of Einstein. But what had induced Ruth Stauffer to claim that Weyl had "inspired" Einstein's letter?

In order to understand Stauffer's letter let us explain its background.

In 1972 there appeared a paper on Emmy Noether in the American Mathematical Monthly, authored by Clark Kimberling [Kim72]. Among other information the paper contains the text of Einstein's letter to the New York Times. Kimberling had obtained the text from an article in the Bryn Mawr Alumnae Bulletin where it had been reprinted in 1935. Together with that text, we find in [Kim72] the following:

> *A note in the files of the Bryn Mawr Alumnae Bulletin reads, "The above was inspired, if not written, by Dr. Hermann Weyl, emi-*

[49] I would like to thank R. Siegmund-Schultze for a number of interesting comments and corrections to this article.

nent German mathematician. Mr. Einstein had never met Miss Noether."

(Here, by "above" was meant the text of the Einstein letter to the New York Times.)

While the first sentence of that "note" can be considered as an affirmation of the guess that Weyl had conceived the text of Einstein's letter, the second sentence is hard to believe. Emmy Noether often visited the Institute for Advanced Study in Princeton, the same place where Einstein was, and it seems improbable that they did not meet there. After all, Einstein was already in May 1918 well informed about Noether's achievements, when he wrote to Hilbert praising her work [Noe18]. And in December that year, after receiving the printed version of this work, he wrote to Felix Klein and recommended her *Habilitation*. In the 1920s, Einstein had a correspondence with Emmy Noether who acted as referee for papers which were submitted to the *Mathematische Annalen*. It is hard to believe that in Princeton he would have avoided meeting Emmy Noether, whom he esteemed so highly. Moreover, we have already mentioned in section 4 that Einstein probably had met Noether in 1915 in Göttingen. Also, on the DMV-meetings 1909 in Salzburg and 1913 in Wien both Einstein and Emmy Noether presented talks and there was ample opportunity for them to meet.

Thus it seemed that the "note" which Kimberling mentioned had been written by someone who was not well informed about the situation in the early thirties. Actually, that "note" was not printed in the Bryn Mawr Alumnae Bulletin but it was added later by typewriter, maybe only on the copy which was sent to Kimberling. It is not known who had been the author of that "note".

In the same volume of the American Mathematical Monthly where his article [Kim72] had appeared, Kimberling published an *Addendum* saying that Einstein's former secretary, Miss Dukas, had objected to the statement that the letter written by Einstein was "*inspired, if not written by Dr. Hermann Weyl*". She insisted that the letter was written by Einstein himself at the request of Weyl.

This, however, induced Ruth Stauffer to write the above mentioned letter to the editor of the American Mathematical Monthly, which we are citing in section 9.4. Ruth Stauffer had been a Ph.D. student of Emmy Noether in Bryn Mawr and in her letter she recalls vividly the mathematical atmosphere in Princeton at that time.

On this evidence we were led to believe that the statement of Einstein's secretary Dukas may be due to a mix-up on her part. For, only shortly before Noether's death Einstein had written another letter in which he recommends that Emmy Noether's situation in Bryn Mawr College should be improved and put on a more solid base. At that time President Park of Bryn Mawr had tried to obtain testimonies on Emmy Noether, which could be used in order to get funds for a more permanent position.[50] Einstein's testimony is dated January 8, 1935 and is written in German; we have found it in the archives of the Institute for Advanced Study in Princeton. Its full text reads:

[50]This was successful, but Emmy Noether died before she got to know about it. – Other testimonials, by Solomon Lefschetz, Norbert Wiener and George D. Birkhoff are published in Kimberling's article [Kim81].

> Fräulein Dr. Emmy Noether besitzt unzweifelhaft erhebliches schöpferisches Talent, was jeweilen von nicht sehr vielen Mathematikern einer Generation gesagt werden kann. Ihr die Fortsetzung der wissenschaftlichen Arbeit zu ermöglichen, bedeutet nach meiner Ansicht die Erfüllung einer Ehrenpflicht und wirkliche Förderung wissenschaftlicher Forschung.
>
> *Without doubt Miss Dr. Emmy Noether commands significant and creative talent; this cannot be said of many mathematicians of one generation. In my opinion it is an obligation of honor to provide her with the means to continue her scientific work, and indeed this will be a proper support of scientific research.*

It is apparent that the style of this is quite different from the style of the letter to the New York Times.

Although we now know that Miss Dukas was right and Einstein had composed his NYT-letter with his own hand, there remains the question as to the basis of Stauffer's contentions.

Stauffer was a young student and what she reports is partly based on what she heard from Mrs. Wheeler. But the latter, who was head of the mathematics department of Bryn Mawr College at the time, had studied in Göttingen with Hilbert in the same years as Hermann Weyl had; so they were old acquaintances and it seems probable that Weyl himself had told her the story as it had happened. Thus it may well have been that first Weyl had sent his obituary on Emmy Noether to the New York Times, and that this was returned with the suggestion that Einstein should write an obituary – as Ruth Stauffer narrates. And then Einstein wrote his letter "at the request of Weyl", as Miss Dukas has claimed. Whether there was any cooperation between Einstein and Weyl while drafting the letter is not known. But we can safely assume that both had talked if not about the text of the letter but certainly about Emmy Noether's personality, her work and her influence on mathematics at large. In this way Stauffer's claim may be justified that Weyl had "inspired" Einstein in writing his letter.

Remark: It has been pointed out to me by several people that the very last sentence in the English version of Einstein's letter deviates in its meaning from the original German text wheras otherwise the translation seems to be excellent.[51] In the English version it is said that Noether's last years in Bryn Mawr were made the "*happiest and perhaps most fruitful years of her entire career*", but the German text does not refer to her entire career and only pointed out that death came to her "*mitten in froher und fruchtbarer Arbeit*". I do not know who had translated the German text into English. There is a letter of Abraham Flexner, the director of the Institute for Advanced Study in Princeton, addressed to Einstein and dated April 30, 1935, in which Flexner thanks Einstein for the "*beautiful tribute to Miss Noether*" and continues: "*I shall translate it into English and send it to the New York Times, through which it will reach, I think, many of those who should know of her career.*" But it does not seem justified, I believe, to conclude that Flexner personally did the translation job. He was quite busy with all kinds of responsibilities and certainly he had contacts

[51] The German text is published in my "Letter to the Editor" [Roq07a].

to experts who would have been willing and competent to do it.[52]

Final Remark: Weyl's solidarity with Emmy Noether extended to her brother and family. Emmy's brother Fritz had emigrated to Russia where he got a position at the university in Tomsk. In 1937 he was arrested and sentenced to 25 years in prison because of alleged espionage for Germany. In the Einstein archive in Jerusalem we have found a letter, dated April 1938 and signed by Einstein, addressed to the Russian minister of foreign affairs Litvinov. In this letter Einstein appeals to the minister in favor of Fritz Noether, whom he (Einstein) is sure to be innocent. In the Einstein archive, right after this letter, is preserved a curriculum vitae of Fritz Noether in Weyl's handwriting. Thus again it appears that Weyl has "inspired" Einstein to write such a letter.[53]

Among Weyl's papers I found a number of letters from 1938 and the following years, which show that he cared for the two sons of Fritz Noether, Hermann and Gottfried, who had to leave the Soviet Union after their father had been sentenced. Weyl saw to it that they obtained immigrant visa to the United States, and that they got sufficient means to finance their university education. Both became respected members of the scientific community.

9 Appendix: documents

9.1 Weyl's testimony

The following text[54] *is from the testimonial, signed by Hermann Weyl on July 12, 1933 and sent by Hasse to the Ministerium in Berlin together with 13 other testimonials. We have found these testimonials in the Prussian state archive Berlin.*

Emmy Noether has attained a prominent position in current mathematical research – by virtue of her unusual deep-rooted prolific power, and of the central importance of the problems she is working on together with their interrelationships. Her research and the promising nature of the material she teaches enabled her in Göttingen to attract the largest group of students. When I compare her with the two woman mathematicians whose names have gone down in history, *Sophie Germain* and *Sonja Kowalewska*, she towers over them due to the originality and intensity of her scientific achievements. The name Emmy Noether is as important and respected in the field of mathematics as *Lise Meitner* is in physics.

She represents above all "Abstract Algebra". The word "abstract" in this context in no way implies that this branch of mathematics is of no practical use. The prevailing tendency is to solve problems using suitable visualizations, i.e. appropriate formation of concepts, rather than blind calculations. Fräulein Noether is in this respect the legitimate successor of the great German number

[52] Siegmund-Schultze [SS07] advocates reasons to assume that indeed, Flexner himself did the translation job.

[53] The appeal of Einstein was in vain. In 1941, when German troops were approaching the town of Orjol where Fritz was kept in prison, he was sentenced to death and immediately executed. See, e.g., [Sch91].

[54] Translated from German by Ian Beaumont.

theorist R. Dedekind. In addition, Quantum Theory has made Abstract Algebra the area of mathematics most closely related to physics.

In this field, in which mathematics is currently experiencing its most active progress, *Emmy Noether* is the recognised leader, both nationally and internationally.

Hermann Weyl

9.2 Weyl's funeral speech

The following text[55] *was spoken by Hermann Weyl on Emmy Noether's funeral on April 18, 1935. We have found this text in the legacy of Grete Hermann, which is preserved in the "Archiv der sozialen Demokratie" in Bonn.*

The hour has come, Emmy Noether, in which we must forever take our leave of you. Many will be deeply moved by your passing, none more so than your beloved brother Fritz, who, separated from you by half the globe, was unable to be here, and who must speak his last farewell to you through my mouth. His are the flowers I lay on your coffin. We bow our heads in acknowledgement of his pain, which it is not ours to put into words.

But I consider it a duty at this hour to articulate the feelings of your German colleagues - those who are here, and those in your homeland who have held true to our goals and to you as a person. I find it apt, too, that our native tongue be heard at your graveside - the language of your innermost sentiments and in which you thought your thoughts - and which we hold dear whatever power may reign on German soil. Your final rest will be in foreign soil, in the soil of this great hospitable country that offered you a place to carry on your work after your own country closed its doors on you. We feel the urge at this time to thank America for what it has done in the last two years of hardship for German science, and to thank especially Bryn Mawr, where they were both happy and proud to include you amongst their teachers.

Justifiably proud, for you were a great woman mathematician - I have no reservations in calling you the greatest that history has known. Your work has changed the way we look at algebra, and with your many gothic letters you have left your name written indelibly across its pages. No-one, perhaps, contributed as much as you towards remoulding the axiomatic approach into a powerful research instrument, instead of a mere aid in the logical elucidation of the foundations of mathematics, as it had previously been. Amongst your predecessors in algebra and number theory it was probably Dedekind who came closest.

When, at this hour, I think of what made you what you were, two things immediately come to mind . The first is the original, productive force of your mathematical thinking. Like a too ripe fruit, it seemed to burst through the shell of your humanness. You were at once instrument of and receptacle for the intellectual force that surged forth from within you. You were not of clay, harmoniously shaped by God's artistic hand, but a piece of primordial human rock into which he breathed creative genius.

[55] translated from German by Ian Beaumont

The force of your genius seemed to transcend the bounds of your sex - and in Göttingen we jokingly, but reverentially, spoke of you in the masculine, as "den Noether". But you were a woman, maternal, and with a childlike warmheartedness. Not only did you give to your students intellectually - fully and without reserve - they gathered round you like chicks under the wings of a mother hen; you loved them, cared for them and lived with them in close community.

The second thing that springs to mind is that your heart knew no malice; you did not believe in evil, indeed it never occurred to you that it could play a role in the affairs of man. This was never brought home to me more clearly than in the last summer we spent together in Göttingen, the stormy summer of 1933. In the midst of the terrible struggle, destruction and upheaval that was going on around us in all factions, in a sea of hate and violence, of fear and desperation and dejection - you went your own way, pondering the challenges of mathematics with the same industriousness as before. When you were not allowed to use the institute's lecture halls you gathered your students in your own home. Even those in their brown shirts were welcome; never for a second did you doubt their integrity. Without regard for your own fate, openhearted and without fear, always conciliatory, you went your own way. Many of us believed that an enmity had been unleashed in which there could be no pardon; but you remained untouched by it all. You were happy to go back to Göttingen last summer, where, as if nothing had happened, you lived and worked with German mathematicians striving for the same goals. You planned on doing the same this summer.

You truly deserve the wreath that the mathematicians in Göttingen have asked me to lay on your grave.

We do not know what death is. But is it not comforting to think that souls will meet again after this life on Earth, and how your father's soul will greet you? Has any father found in his daughter a worthier successor, great in her own right?

You were torn from us in your creative prime; your sudden departure, like the echo of a thunderclap, is still written on our faces. But your work and your disposition will long keep your memory alive, in science and amongst your students, friends and colleagues.

Farewell then, Emmy Noether, great mathematician and great woman. Though decay take your mortal remains, we will always cherish the legacy you left us.

Hermann Weyl

9.3 Letter to the New York Times

The following text was published on Sunday, May 5, 1935 by the New York Times, with the heading: "Professor Einstein Writes in Appreciation of a Fellow-Mathematician".

To the Editor of The New York Times:

The efforts of most human-beings are consumed in the struggle for their

daily bread, but most of those who are, either through fortune or some special gift, relieved of this struggle are largely absorbed in further improving their worldly lot. Beneath the effort directed toward the accumulation of worldly goods lies all too frequently the illusion that this is the most substantial and desirable end to be achieved; but there is, fortunately, a minority composed of those who recognize early in their lives that the most beautiful and satisfying experiences open to humankind are not derived from the outside, but are bound up with the development of the individual's own feeling, thinking and acting. The genuine artists, investigators and thinkers have always been persons of this kind. However inconspicuously the life of these individuals runs its course, none the less the fruits of their endeavors are the most valuable contributions which one generation can make to its successors.

Within the past few days a distinguished mathematician, Professor Emmy Noether, formerly connected with the University of Göttingen and for the past two years at Bryn Mawr College, died in her fifty-third year. In the judgment of the most competent living mathematicians, Fräulein Noether was the most significant creative mathematical genius thus far produced since the higher education of women began. In the realm of algebra, in which the most gifted mathematicians have been busy for centuries, she discovered methods which have proved of enormous importance in the development of the present-day younger generation of mathematicians. Pure mathematics is, in its way, the poetry of logical ideas. One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulas are discovered necessary for the deeper penetration into the laws of nature.

Born in a Jewish family distinguished for the love of learning, Emmy Noether, who, in spite of the efforts of the great Göttingen mathematician, Hilbert, never reached the academic standing due her in her own country, none the less surrounded herself with a group of students and investigators at Göttingen, who have already become distinguished as teachers and investigators. Her unselfish, significant work over a period of many years was rewarded by the new rulers of Germany with a dismissal, which cost her the means of maintaining her simple life and the opportunity to carry on her mathematical studies. Farsighted friends of science in this country were fortunately able to make such arrangements at Bryn Mawr College and at Princeton that she found in America up to the day of her death not only colleagues who esteemed her friendship but grateful pupils whose enthusiasm made her last years the happiest and perhaps the most fruitful of her entire career.

Albert Einstein.

Princeton University, May 1, 1935.

9.4 Letter of Dr. Stauffer-McKee

The following letter was sent by Dr. Ruth Stauffer-McKee on October 17, 1972 to the editor of the American Mathematical Monthly, Professor H. Flanders. A carbon copy had been sent to Professor Kimberling. I am indebted to Clark Kimberling for giving me access to his private archive.

Dear Mr. Flanders,

After reading the Addendum to "Emmy Noether" in the August September issue of the American Mathematical Monthly, I was much disturbed by the apparent lack of information concerning the thirties at Princeton! Rechecking the reference to the original article which appeared in February 1972 I was even more disturbed to note that the quote was attributed to a note in the files of Bryn Mawr Alumnae Bulletin. A telephone conversation and a careful check by the Staff of the Bulletin assured me that there was nothing in the files of the Bulletin to even imply that "Mr. Einstein had never met Miss Noether."

In respect to the "thirties at Princeton", I should like to note that there was an air of continued excitement at the Institute for Advanced Study. Solomon Lefschetz, a guiding spirit who worked diligently to help the displayed mathematicians, Hermann Weyl, a leading mathematician of that time who had learned to know Miss Noether in Göttingen, and John von Neumann, then considered a brilliant young genius, were all at the Institute when Einstein arrived in December of 1933. Mrs. Wheeler, of Bryn Mawr, often told of the welcoming party which she and Miss Noether attended.

Mrs. Wheeler usually drove Miss Noether to Princeton for lectures and included Miss Noether's students in the parties. We listened to talks by these men who were the leaders in new exciting theories. It was a friendly group and after the talks everyone gathered for more talk and coffee in a long pleasant common room. There is no doubt that Einstein and Noether were acquainted. I saw them in the same group!

As regards the quote in the "addendum to 'Emmy Noether' " "inspired, if not written by Dr. Hermann Weyl" is certainly true. The writing of the obituary was a very natural occurence. Hermann Weyl was considered by the mathematicians as the mathematical leader of the time and at the peak of his productivity and he had probably the greatest knowledge and understanding of her work. Einstein had begun to slow down and Von Neumann was relatively young and still growing. It was, therefore, obvious to all the mathematicians that Weyl should write the obituary – which he did. He, furthermore, sent it to the New York Times, the New York Times asked who is Weyl? Have Einstein write something, he is the mathematician recognized by the world. This is how Einstein's article appeared. It was most certainly "inspired" by Weyl's draft. These facts were told to me at the time by Mrs. Wheeler who was indignant that the New York Times had not recognized the mathematical stature of Hermann Weyl.

Very truly yours,

Ruth Stauffer McKee

Senior Mathematician

10 Addenda and Corrigenda

Page 17, footnote 32: In Kimberling's article [Kim81] it is reported: "Handwritten letters dated 6/1/27 and 7/3/27 from Emmy Noether to W. W. Tisdale supporting Hopf's application are preserved in the International Educational Board Collection at the Rockefeller Archive Center."

Page 18: The words "*Göttinger Mathematische Gesellschaft*" are to be replaced by "*Göttinger Gesellschaft der Wissenschaften*".

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