Heinrich Weber, David Hilbert and Königsberg

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June 1, 1992

Preface

In the summer of 1992 I was invited to participate at a "Hilbert seminar" which was organized by the State University of Kaliningrad (formerly Königsberg). Königsberg was the town where David Hilbert was born, where he had his education and held his first university professorship (as the successor of Adolf Hurwitz). The Russian organizers of the "Hilbert seminar" had decided to hold the meeting in the nearby village of Svedlogorsk (formerly Rauschen), the Baltic sea resort where Hilbert used to spend his vacations every summer.

The following article consists of the notes for my talk at that seminar. I chose a subject which, on the one hand, seemed to be suitable to evoke the academic atmosphere of the Albertina, *the former university of Königsberg, in the last century. With respect to mathematics, it can be said that the* Albertina *was one of the leading places in the Germany of that period.[1] On the other hand, the subject was to be related to* Hilbert, *according to the title of the seminar.* H. Weber *was chosen because he had his roots in Heidelberg where I am living now.*

The lecture was directed to mathematicians as well as scientists from other fields, e.g., philosophy. The following text preserves the informal style of the spoken word, as befits true "Lecture Notes," which are written as notes for the lecturer and for the audience.

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*) I am indebted to Klaus Volkert (Heidelberg) for his help concerning the vita of Heinrich Weber.

[1] See e.g. Klein's lectures on Mathematics in the 19th century.

1 Introduction

Heinrich Weber was born in Heidelberg on 17 March 1842.[2]) The 150th anniversary of his birth was celebrated this year by a symposium, dedicated to Weber's memory, at the Mathematics Institute of Heidelberg University. In Weber's vitae[3]) we note the following items:

(1) Weber obtained his decisive mathematical education in Königsberg, during three years of postdoctoral study (1863-66) in the stimulating atmosphere of the school of RICHELOT and FRANZ NEUMANN.

(2) At a later period, Weber spent eight years as Professor of Mathematics at the University of Königsberg (1875-83), as the successor of Richelot. Among his students were DAVID HILBERT and HERMANN MINKOWSKI.

So we see that Königsberg as an academic place played an important role in Weber's life. This explains my motivation for speaking about Heinrich Weber particularly at this conference, which is organized by a university at the very place where Weber and Hilbert lived. I will address the following questions:

(1) Why did Weber choose Königsberg as the place for his postdoctoral studies? What do we know about the mathematical education and research at the Albertina of that time?

(2) What can be said about the relationship between Weber and the 20-years younger Hilbert, during Hilbert's student years at Königsberg and also later when Hilbert was a well established mathematician?

2 Heidelberg around 1863

Weber himself says in his memoirs that he went to Königsberg *"on the advice of his academic teachers in Heidelberg"*. Who were those teachers?

In the written record of Weber's doctoral examination at Heidelberg we find the names of HESSE, KIRCHHOFF and BUNSEN. In addition, his doctoral

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[2]) Not 17 May as has been erroneously stated in Voss' obituary, Jahresbericht der DMV vol. 23 (1914) p.431 and, from then on, in almost all biographical references.

[3]) For the convenience of the reader, at the end of these notes we present a brief *Zeittafel* on Weber's life. A similar table for Hilbert seems unnecessary as his dates are probably better known to mathematicians. The interested reader could consult the biograpy of Hilbert by C. Reid.

diploma contains the name of Helmholtz as Rector of the university. These four names are precisely those which Weber mentions in his memoirs, when he talks with great enthusiasm about the academic life in Heidelberg during his student years. Now if we look at the vitae of those four scientists, we observe that three of them came from Königsberg: Hesse, Kirchhoff and Helmholtz.

Hesse was born in Königsberg (1811). He was a student of the great Jacobi. He came to Heidelberg in 1857, and brought with him Jacobi's ideas of "modern" mathematics, as well as the new forms of academic teaching which encouraged students to critically participate in the scientific research of the teacher, within seminars and problem sessions, as well as in personal contacts outside of lecture rooms.

Kirchhoff too was born in Königsberg (1824). He was the "*Lieblings-schüler*" of Franz Neumann, and the son in law of Richelot. From today's viewpoint Kirchhoff would not be counted as a mathematician but as a physicist. However we should keep in mind that at that time the specialization was not as narrow as it is today; a mathematician would also be concerned with problems of physics and vice versa. In particular this was the case in the school of Franz Neumann in Königsberg. Kirchhoff came to Heidelberg in 1854, on the initiative of his close friend Bunsen (the chemist).

Helmholtz did not originally come from Königsberg, but he had held a professorship there for six years (1849–55). In Heidelberg he had held a chair since 1857, not in mathematics but in physiology. Nevertheless his most important mathematical papers were written during his Heidelberg years (from 1857 until 1871). We may safely assume that Helmholtz too, who was at home in many sciences from medicine to mathematics and physics, was well informed about the situation of mathematics at Königsberg University.

We see that academic life in Heidelberg around 1863, as far as mathematics and the neighbouring sciences were concerned, was flourishing, and that it was largely dominated by people who came from Königsberg.[4] They knew the stimulating and challenging atmosphere which Königsberg offered to young mathematicians at that time.

So it seems natural that young Weber, when he asked his academic teachers where he should go for his postdoctoral studies, was advised to move to Königsberg.

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[4] In Lorey's report we find the remark that there were two universities where, in the field of Mathematics, the Königsberg traditions were manifested with particular emphasis; one of those two was Heidelberg. (The other was Gießen where Clebsch was teaching.)

3   Königsberg around 1863

Now let us review the situation in Königsberg at that time. In the sixties of the last century, the mathematicians in Königsberg looked back upon a great tradition, characterized by the names of BESSEL (since 1810), F. NEUMANN and JACOBI (both since 1826). This triad of great scientists was considered to be, at the time, the *"nucleus of revival of mathematical research at German universities"*. In 1834 the first physico-mathematical *Seminar* in Germany was founded by Jacobi and F. Neumann. The new ideas for teaching scientific subjects included high quality requirements combined with the opportunity to participate in research activities. This proved to be attractive for many young students not only from Königsberg but also from various other places in Germany and from other countries, in particular Switzerland and Russia. This trend continued also after Jacobi moved away from Königsberg (in 1843) and was replaced by his faithful pupil Richelot who was a dedicated academic teacher, admired by his students.

Weber tells us in his memoirs that in Königsberg he met quite a number of young mathematicians and physicists who later became famous scientists in their fields. He explicitly mentions nine names of his own student generation, and he adds more names from earlier generations in order to explain the *genius loci*.[5]

So we see that Heinrich Weber, when he came to Königsberg in 1863, found a lively group of actively interested mathematicians. The atmosphere must have been stimulating and industrious, even more so than he had been used to in Heidelberg. He reports that he concentrated on the lectures by F. Neumann and Richelot. Those lectures and the problem sessions in the seminars required his full time and attention. And, we may add, in his Königsberg years Weber grew in scientific stature to become one of the few who had an intimate knowledge of Riemann's work and was able to apply it in various situations, including problems of Mathematical Physics.

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[5] We cannot discuss here all those names but perhaps it is worthwhile to note that CLEBSCH is among them. Clebsch later went to Gießen and then to Göttingen; he had his own "school" of algebraic geometry and founded the new journal "*Mathematische Annalen*" (jointly with CARL NEUMANN who also came from Königsberg) as a place where his and his pupils' modern ideas could be published. Clebsch was to edit Riemann's Collected Papers together with Dedekind; after his sudden death Heinrich Weber was asked to step in, and he succesfully completed this project. – At the 150th anniversary of Clebsch' birthday the *Mathematische Annalen* published an article to his memory written by the Russian mathematician I.R. Shafarevich.

4 Weber and Hilbert 1880-83; the Number Theory Course.

Now let us jump 14 years ahead, from 1866 to the year 1880. In the meantime Weber had returned to Heidelberg, where he did his *"Habilitation"* (i.e., his second doctorate), had accepted a professorship in Zürich and finally in 1875 had accepted an offer to return to Königsberg as the successor of Richelot. He had written a number of papers on Analysis and Theoretical Physics, was respected among mathematicians as the editor of Riemann's Collected Works which had appeared in 1876, and together with Dedekind had just completed the paper *Theorie der algebraischen Funktionen einer Veränderlichen* (Theory of algebraic functions of one variable), which was to become a milestone in the development of "modern" mathematical concepts. Weber was also well respected by the Königsberg academic community at large. This led to him being elected Rector of the university in 1880.[6]) In his presidential address *Über Causalität in den Naturwissenschaften* (On the notion of causality in the sciences) he sketched a multivalued logic based on Cantor's set theory.

For us, however, the interesting event in Königsberg in 1880 was the fact that DAVID HILBERT, who was 18 years old, enrolled at the university. We know that Hilbert attended lectures by Weber on number theory and on elliptic functions, and a seminar on the theory of invariants.[7]) What can be said about the effect of Weber's teaching on the mathematical development of young Hilbert?

In later years, when Hilbert himself spoke about his Königsberg years, he never mentioned any particular person as his "academic teacher". This may reflect the outstanding mathematical personality of Hilbert who from the beginning knew his way well into mathematics, without guidance from outside. Instead, he mentioned the many discussions with his friends HURWITZ and MINKOWSKI, who were about the same age as he was. Those discussions introduced him into almost every corner of mathematics, but this obviously refers to later years. Hurwitz came to Königsberg in 1884 only when Weber

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[6]) More precisely, Weber was *"Prorektor"*. The office of Rector was of formal nature, usually conferred upon the head of state.

[7]) In addition Hilbert spent one semester in Heidelberg where he attended lectures by L. FUCHS. By the way: In Constance Reid's biography of Hilbert we find the remark that Heidelberg was "the most delightful and most romantic of all the German universities". Would this characteristic have been sufficient for Hilbert to choose Heidelberg as his place of study? We may safely assume that he went to Heidelberg on the recommendation of Heinrich Weber who, being a son of Heidelberg, was well acquainted with the academic situation there. Surely Weber knew about the scientific calibre of L. Fuchs and hence could be sure that the Heidelberg semester would not be lost upon Hilbert.

had left already. It seems that we have to regard Hilbert's first semesters, when he attended Weber's lectures and seminars, as belonging to the quiet period, which he himself described as "leading to maturity".[8])

Fortunately at least one of Hilbert's student notebooks is preserved: the one on Weber's number theory course.[9])

According to these notes Weber's course started from the very beginning of number theory, working through congruence calculus, continued fractions, towards quadratic reciprocity. After that, binary quadratic forms are discussed, their classes, their composition, and the analytic class number formulas. Also, the existence of primes in arithmetic progressions, using L-series, is included. In this last part the reader gets the impression that things went quite fast but the proofs are recorded in detail. We conclude that much of the concept of the lecture was based on GAUSS' *Disquisitiones* and DIRICHLET's papers. In addition Weber was quite explicit in making use of modern notions, for instance the abstract notion of a group is to be found, as well as what we now call the "main theorem on finite abelian groups", i.e., decomposition of finite abelian groups into cyclic factors (in the context of description of characters).

It is clear from this list of contents that Weber's course was of a high level and required much from the young students in terms of comprehension and their own work. It is not known who else attended the course besides Hilbert, but perhaps Minkowski? In one of his *vitae* Minkowski mentions Weber among his university professors. Thus we may imagine both the young Hilbert and Minkowski together studying Dirichlet and Gauss under Weber's guidance. This may perhaps explain the wide horizon of Weber's course, who observing two extremely gifted students in his class, tried to satisfy their mathematical curiosity. For even today, such a course for students of the third semester (without training in abstract algebra and analysis) would be remarkable.

In any case, from those notes we see that young Hilbert had found a competent teacher who succeeded in stimulating his curiosity in number theory. Those notes, by the way, bear testimony that Hilbert indeed appreciated the mathematical challenge delivered with these lectures, for we find quite a number of comments and alterations by Hilbert's hand of a later date. Apparently he did look at his student notes again. Maybe this happened when he first had to deliver a number theory course himself, and wished to remember which subjects Weber had presented and how he had organized the

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[8]) *"Die langen Jahre in der Königsberger Geborgenheit waren eine Zeit der stetigen Reife."*

[9]) This book as well as the letters mentioned below, are in the possession of the *Handschriftenabteilung* of the university library in Göttingen.

lectures. Some details in Hilbert's comments and revisions seem to suggest this.

5   Weber and Hilbert: later years.

Judging by their correspondence Heinrich Weber remained in close contact with David Hilbert throughout his life. Most of the letters, which are preserved in Göttingen are Weber's, while the others, Hilbert's, seem to have been lost or at least have not yet been rediscovered.

Hilbert passed his doctoral examination in December 1884. Apparently he had sent Weber a telegram announcing this event, for there is a letter from Weber, dated February 1885, thanking Hilbert for a telegram. Weber adressed Hilbert as "*Lieber Herr Doctor*", thus alluding to Hilbert's newly acquired academic degree. He thanked Hilbert for sending him his dissertation, and then continued by giving him advice for further mathematical work: "*Algebraische Eigenschaften der Kugelfunktionen*" (Algebraic properties of spherical functions). We do not know what Hilbert replied to this good advice, but he never worked in the direction which Weber suggested. As we have said earlier, Hilbert found his way into mathematics himself and did not need any guidance.

Some years later it seems that Weber had realized the outstanding genius of Hilbert. For he never gave him any more advice on his work – instead, Weber approached Hilbert for *his opinion* and *his advice* in various mathematical situations. Hilbert's mathematical authority was undisputed.

Most of Weber's questions were connected, firstly, with the second edition of Riemann's Collected Papers and, secondly, with his "Algebra" textbook. In each instance Hilbert seems to have been interested in the question and to have freely given his opinion. Sometimes a whole new theory emerged from Hilbert's reply as we may indirectly conclude from Weber's reaction. For instance, in June 1894 Weber wrote (from Göttingen) that Hilbert's work would be published by the "Gesellschaft der Wissenschaften"[10]) and that, in addition, he (Weber) would put it into the second volume of his "Algebra".

It is not possible here to discuss in detail all this Weber–Hilbert correspondence. Perhaps this would only make sense in the framework of a complete work biography of Heinrich Weber which, unfortunately, is still missing. We shall restrict ourselves to the following four items:

(i) At the end of a letter to Hilbert in 1892, Weber sent "greetings to the friends in Königsberg". Surely Weber wouldn't have used this phrase if

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[10]) This became Hilbert's paper on Galois extensions in which he expounded what we today call "Hilbert theory" of Galois decomposition of primes.

Hilbert himself weren't to be counted among those friends. The friendship between those two men, 20 years apart by age, could most probably be traced back to the student years of Hilbert when he was attending lectures by Weber. After all, Weber stood in the tradition of the "Königsberg school" in mathematics and physics; from all we read about this "school" we see that the close contact between professor and student, not confined to the lecture room but extending into social and family life, was part of that tradition. We read this of Bessel, Jacobi and F. Neumann, of Richelot and also of Hesse (in Königsberg as well as in Heidelberg). And we may well assume that Weber continued in this tradition. This corresponds to what Voss said about Weber's character: *On closer contact he opened up without reservation to his friends whom he found through scientific cooperation and common interests.*

We can see this friendship growing when, in later years Weber, addressing Hilbert in his letters, had switched from the usual formal phrases to the more intimate "Dear friend".[11]) Still later, in 1903, Weber asked Hilbert for his consent to dedicate his latest work "to the three men to whom I feel closest in scientific respect". One of the three was Hilbert; the others were Dedekind and Minkowski. All three were friends from Weber's Königsberg years.[12]) The work in question was the famous third volume of Weber's "Algebra", the one which contained elliptic functions and complex multiplication. The dedication to the three men was given "in warm friendship".[13])

(ii) In a letter in 1894 Weber wrote that Hilbert's name had been put on the list of his successors in Göttingen. Weber himself had decided to move to Straßburg. At that time it was not unusual for the transferring professor to be asked for proposals for his successor (although the proposals would not be followed in every case). So it seems that Weber had some share (besides that of F. Klein) in Hilbert's nomination.

(iii) The friendly respect between Weber and Hilbert was mutual. As Hasse has pointed out, Hilbert had a high opinion of Weber. Hilbert repeatedly quoted and duly acknowledged Weber's ideas and results, even in those cases where he did simplify, generalize or extend Weber's work. Hilbert's high opinion of Weber was evident when in 1902 he successfully nominated him chairman of the next International Congress of Mathematicians, which was to be held at Heidelberg. After Weber's election he was informed by telegram from Hilbert about the outcome. In a letter dated 1902 Weber expressed his thanks for the nomination and election.

The historian Struik has voiced the opinion that Weber may have been

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[11]) *Lieber Freund...*

[12]) Dedekind had not been in Königsberg, but Weber's correspondence with Dedekind, which led to their common Crelle paper, dated from Weber's Königsberg time before 1880.

[13]) *... in herzlicher Freundschaft*

nominated as congress chairman for 1904 for certain political reasons. I did not find any evidence for this. It is, I believe, safe to assume that Weber was nominated because he was highly respected as a mathematician in Germany and in other countries, [14]) and in particular by Hilbert.

On the occasion of Weber's 70th birthday, in 1912, he was presented a "*Festschrift*" by friends and former students. Among the authors who contributed an article to this volume we find the name of David Hilbert (and that of Richard Dedekind).

(**iv**) In a letter dated 20 July 1894 from Göttingen to Königsberg, Weber said (among other things) that he thanked Hilbert for the invitation but regretted that he could not attend. Now what kind of invitation did he have in mind? If we compare the year date then we see that the Albertina in Königsberg celebrated her 350th anniversary in that year. Thus we learn that Weber, although invited, could not be present at the anniversary celebrations. Let us hope that in two years time, when the 450th anniversary is due, Weber will be alive and present at least in the minds of the participants.

Epilogue

The last words of my talk referred to certain plans to organize an International Scientific Conference in Kaliningrad in 1994, on the occasion of the 450th anniversary of the founding of the former Königsberg University. Indeed, that conference did take place and was organized by a joint German-Russian committee. There was a lively Mathematics/Physics section at that conference, including some talks on the history of mathematics at the Albertina. *There is a brief report on it in the* Mitteilungen der Deutschen Mathematiker-Vereinigung, *1995/2.*

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[14]) Weber was elected member of the scientific academies in Göttingen, Berlin, München, Stockholm, Uppsala, Rome. In 1902 he had received the academic title of Doctor honoris causa from the University of Kristiana, on the occasion of the 100th birthday of N.H. Abel.

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H. Weber, Festschrift zu seinem 70. Geburtstag am 5.März 1912. *Leipzig 1912*

Zeittafel: Heinrich Weber 1842–1913

1860   Academic studies in **Heidelberg**: Hesse, Kirchhoff, Bunsen, Helmholtz

1863   Doctorate (Ph.D.) in **Heidelberg**

1863   Postdoctoral studies in **Königsberg**: Richelot (Analysis), Franz Neumann (Mathematical Physics)

   1866   Singuläre Auflösungen partieller Differentialgleichungen erster Ordnung: Crelles Journal

1866   Habilitation in **Heidelberg**; Associate Professor 1869

   1869   Über die Integration der partiellen Differentialgleichung $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 u = 0$: Mathematische Annalen volume 1

1870   Full Professor ETH **Zürich**

   1874   Neuer Beweis des Abelschen Theorems: Mathematische Annalen

1875   **Königsberg** (Successor of Richelot); Rector 1880

   1876   Editing Riemann's Collected Papers (partly together with Dedekind)

   1880   (jointly with Dedekind) Theorie der algebraischen Funktionen einer Veränderlichen: Crelles Journal 1882

   1880   Minkowski (b.1864), Hilbert (b.1862) enrolled as students in Königsberg

1883   TH **Charlottenburg (Berlin)**

1884   **Marburg**; Rector 1890

   1886   Theorie der Abelschen Zahlkörper I,II,III: Acta Mathematica

   1891   Elliptische Funktionen und algebraische Zahlen (2nd ed. 1908)

1892   **Göttingen** (with Felix Klein)

   1893   Editor of Mathematische Annalen (founded 1869 by Clebsch and Carl Neumann)

   1893   Zahlentheoretische Untersuchungen aus dem Gebiet der elliptischen Funktionen I,II,III: Nachr. Ges. Wiss. Göttingen

1895   **Straßburg** (Weber's successor in Göttingen was Hilbert); Rector 1900

   1895   Lehrbuch der Algebra, 2 vols. (2nd ed. 1898/99)

   1896   Über Zahlgruppen in algebraischen Zahlkörpern I,II,III: Math. Annalen 1897

   1900   Riemann-Weber: Partielle Differentialgleichungen d. Mathematischen Physik, 2 vol. (5th ed. 1910/12)

   1903   (jointly with Wellstein et al.) Enzyklopädie d. Elementarmathematik

1904   President of International Congress of Mathematicians in Heidelberg