Peter Roquette *4. Nov. 2004*

Mathematisches Institut, Universität Heidelberg

Introduction of Langlands

at the Arf lecture, 11 November 2004.[1]

Ladies and Gentlemen!

It is my pleasure and a great honor to me to introduce Robert Langlands as the speaker of the Arf Lecture of this year 2004, here at the Middle East Technical University.

Professor Langlands is a permanent member of the Institute for Advanced Study in Princeton, which is one of the most prestigious places for mathematical research studies. He has received a number of highly valued awards, among them the Cole Prize in Number Theory from the American Mathematical Society in 1982, the National Academy of Sciences reward in Mathematics in 1988, and the Wolf prize from the Wolf foundation in Israel 1996. He is elected fellow of the Royal Society of Canada and the Royal Society of London, and he has received honorary doctorates from at least 7 universities (to my knowledge).

In the laudation for the Wolf prize it was said that he received the prize for his

> "*path-blazing work and extraordinary insights in the fields of number theory, automorphic forms and group representations*".

[1]Each year there is scheduled a special colloquium lecture at the Middle East Technical University in Ankara, commemorating Cahit Arf who had served as a professor for many years at METU. In the year 2004 the invited speaker was Robert Langlands, and I had been asked to chair his lecture. The following text was read to introduce the speaker. P.Rq.

Ladies and gentlemen, while I was preparing the text for this introduction it soon became clear to me that in this short time I would not be able to describe, not even approximately, Langlands' rich work, its underlying ideas, its enormous impact on the present mathematical research world wide, and its consequence for the future picture of Mathematics. After all, our speaker today with his own words will be the best man for explaining the main lines of his vision.

Instead, please allow me to add some comments of a more personal nature.

Perhaps I should explain that my own mathematical interests at present are predominantly of historical nature. I am interested in particular in the transmission of ideas from one mathematician to another, and from one generation to the next. Mathematics does not develop just by itself, it is shaped by people. In the process of history of science the communication between people plays a decisive role, and I find it fascinating to follow the lines of communication along which mathematical ideas have developed in the past.

Under this aspect I recently have found a very interesting statement by Professor Langlands himself. I have found it in an article in which he recollects his impressions during his first visit to Ankara in the academic year 1967-68. The article is written in Turkish language but a friend kindly translated it for me into English. From this I learned that Langlands' year at Ankara was quite decisive for his own research, in particular through his contact with Cahit Arf. Langlands writes that Arf had pointed out to him a paper by Helmut Hasse who had proved the first results in the direction which he, Langlands, was exploring just then. Citing Langlands' own words:

> " *Thanks to Cahit bey, I solved this problem during my stay in Ankara and proved the existence of the local ε-factor. Perhaps this theorem would never have been*

proved if I had been somewhere else that year..."

Indeed, I find this is a very interesting statement.

As a young student, Cahit Arf had worked with Hasse in Göttingen to obtain his doctorate. This was in the years 1937-38. In this formative period of his life, Arf became familiar with the mathematical ideas of that time in the circle around Hasse. Many of those ideas were directed towards the generalization of class field theory, from abelian to general Galois field extensions.

Please be not taken aback when I am using here some mathematical technical expressions whose precise meaning may be open to the specialist only. I hope that in any case I am able to convey to you that in the 1930s there were strong attempts concerned with a theory, called class field theory, which had been inherited from many generations of mathematicians, and now it was aimed to generalize it to a much wider and all-including scope than it was known at the time. In fact, the topic of Arf's doctoral thesis was a direct fall-out of those attempts. The main result of his thesis, nowadays known as the "Hasse-Arf Theorem" (more precisely: Arf's part of the Hasse-Arf theorem), had been envisaged already in 1930 by Artin in a letter to Hasse. Thus Arf's thesis was fulfilling eight years later what has been called "Artin's dream".

Helmut Hasse and Emil Artin belonged to the group of leading figures at the time of the said projects; other names were Emmy Noether, Claude Chevalley and Shokichi Iyanaga, André Weil. As I have mentioned, Cahit Arf was also working on those projects.

However, despite strong and penetrating attempts from the side of those people and other mathematicians, the desired general results for Galois extensions could not be reached in the 1930s, and class field theory remained to be essentially an affair of abelian extensions only. In a letter from Artin to Hasse dated March 9, 1932 Artin said:

> "*I have the impression that there has to be a completely new idea in order to obtain the desired general results.*"

A similar mood can be read from Emmy Noether's letters to Hasse, but she was somewhat more specific already. On April 26, 1934 she wrote:

> "*It seems to me that first we have to clear up the situation with respect to the irreducible representations, which means the Galois modules, and at the same time to Artin's conductors.*"

Even one week before Noether's death in 1935 she wrote that she was still thinking hard about this problem.

Certainly, Arf had become familiar with this situation when in 1937 he studied with Hasse; this can be concluded from his letters which are preserved in the library of Göttingen University. Also, Arf knew that a certain problem connected with the theory of Artin's L-functions, namely the existence of the local ε-factor, was considered to be a step towards further development. But in the 1930s, this and the big problem of Galois class field theory remained unsolved, with little essential progress in the following decades.

Finally, 30 years later in 1967 Langlands, after discussing with Arf and after studying Hasse's work, succeeded to have the "completely new idea" which Artin had wishfully foreseen in his letter to Hasse. And it was connected with Artin's conductors and with irreducible Galois representations, like Noether had envisaged. This was a step forward in the development of what is now called the *Langlands program* which implies a new form of class field theory valid for Galois extensions, and which Langlands had outlined in his famous letter to André Weil 1967.

Let me repeat: The historical moment when the first inspiration for the ε-factor came, was in 1967/68 and, according to

Langlands himself, it happened here at the Middle East Technical University, through the conveyance of Arf who had kept close and friendly relationship to his former teacher Hasse all through his life time, and who thus was informed about Hasse's ideas and work. In this way Langlands has become part of a long historical line of mathematical development, through Arf directly to Artin, Hasse and Noether, and from there on far back through Hecke, Takagi and Hilbert to Abel.

Certainly this will not be the only historical line which can be found leading to Langlands' widespread work. But today, at this occasion, this line through Arf seems to be of particular significance.

On this background I feel that this year's Arf lecture, here at the Middle East Technical University, will become something very special.

Professor Langlands, on behalf of the Mathematics Department at METU *I have the honor to invite you to start with your address.*