

Briefwechsel Hasse – Mordell

Version vom 30.9.2011

Hasse an Mordell 26.11.28 – 9.1.36

Mordell an Hasse 8.11.31 – 9.3.33

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Chapter 1

Letters Hasse–Mordell

1.1 26.11.1928, Hasse to Mordell

HALLE (SAALE), DEN
26. NOV. 1928

SEHR VEREHRTER HERR PROF. MORDELL !

Gerne komme ich Ihrer Bitte nach, und sende Ihnen mit gleicher Post die Separata der wichtigsten meiner bisherigen Veröffentlichungen. Es war gerade von allem noch *ein* Exemplar vorhanden.

Für die in Aussicht gestellte Zusendung Ihrer Arbeiten spreche ich Ihnen meinen besten Dank aus. Ich verfolge Ihre Arbeiten, soweit sie mir hier zugänglich werden, mit grossem Interesse.

Ich lege auch eine Arbeit von Artin (Hamburg) bei, die einen ganz grundlegenden Fortschritt in der Theorie der relativ-Abelschen Zahlkörper enthält. Vielleicht ist es nicht ohne Interesse für Sie, zu erfahren, dass ganz kürzlich Furtwängler, auf dem Boden dieser Artin'schen Arbeit, den Hauptidealsatz der Klassenkörpertheorie (vgl. meinen Bericht, S. 45) vollständig bewiesen hat, durch Reduction auf eine Frage der Theorie der endlichen Gruppen.

Mit ergebensten Grüßen bin ich Ihr

H. HASSE

1.2 25.11.1930, Hasse to Mordell

MARBURG-LAHN,
Nov. 25. 1930

DEAR PROF. MORDELL,

I have received your kind letter of Nov. 19th and the proofs of your paper for Crelles Journal. Many thanks for you have returned them so promptly. I shall try to make them appear so promptly too, in January next year perhaps.

You need not to be afraid there have been a large number of misprints, for the number of misprints in proofs of German papers is sometimes far larger. I found it better, to transfer your corrections in a neat copy, because our printer is not accustomed to your letters. I have ordered a second proof correction to your address. You will get it as soon as possible.

Naturally I am very interested in your new proof, that the class number is finite for indefinite quadratic forms in n variables.

I enclose, as you wanted, a few words about the work of Wedderburn. I am much enjoyed to have such an occasion for doing something for the glory of this big man.

I hope to agree with you that it was better to write that acknowledgement in German. It would be better, and easier for me too, to write this letter in German. But I am happy to have got an opportunity for practicing my knowledges in English. You may be interested to hear that I have continued my zealous studies in your language this summer. I have read many novels, the good one and the bad one, for example by Galsworthy and Wallace, by A. Huxley and Kay Sheila-Smith.

In order to have further occasion for applying and enriching my knowledges I would much like to get a young English fellow at home. It would be very kind of you, if you could send me one of your students during the next summer term (April — July). We would invite that student to dwell and eat with us. He would be obliged to speak English with us at any time we are together (at breakfast, dinner, tea, lunch etc.). Otherwise he would be allowed, naturally, to speak German with everyone else, in order to take some advantage from his German sojourn for himself. From my point of view it would be best, if he were student of pure mathematics out of an advanced course of

yours. This year I have read for the beginners „ Calculus”. I shall continue this course by a lecture on „ Theory of complex functions” next summer. My second lecture shall be on „ Theory of numbers” (without preliminary knowledges). Also at our University there will be the following lectures on higher mathematics: Potential theory, Differential geometry, Axiomata of geometry, Elements of algebra (Hensel), Calculus, Thermodynamics.

I would much like to hear from you, whether you know a clever and handsome fellow for this purpose.

Please notice the stamps on this letter. I have chosen them in order to enrich the collection of your children. They form a complete set of beneficence stamps.

With the kindest regards

very sincerely yours

H. HASSE

1.3 27.11.1930, Mordell to Hasse

DEPARTMENT OF
MATHEMATICS,
THE UNIVERSITY,
MANCHESTER,
27/11/30

DEAR PROF. HASSE,

I quite agree with the ; instead of the :
The latter is not usual in England. Thanks for your corrections etc. Very little escapes your notice, my eyes are not so sharp as yours. Thanks also for your stamps which my daughter is very pleased to have, and for your very helpful report on Wedderburn.

I can suggest the very person you want to go to Marburg.

Mr. Harold Davenport
Trinity College
Cambridge

he was formerly one of our students, the best we have had for many years, and has been at Cambridge for several years. He is now doing research, and lately he has proved some such result as $\sum_{n=0}^{p-1} \left(\frac{n^4+an^2+bn+c}{p} \right) = \mathcal{O} \left(p^{\frac{3}{4}} \right)$ (??) where the left hand () is the symbol of quadratic reciprocity. I think Hopf in the Zeitschrift a year or two ago showed right hand side $< \frac{p}{6}$!! He is interested in certain aspects of number theory and I believe he would be free to go. I have written to him and asked him to write direct to you.

Why not come to England for a few weeks ? We should be pleased to have you & Mrs Hasse as our guests in April or during the summer.

I was very glad to get your Report Teil II. I had been meaning for a long time to study the subject again and your report has given me the impetus to recommence it

Sincerely yours

L. J. MORDELL

P.S. I meant Selling (not Seeber) & Stouff in referring to indefinite quadratic forms.

P.S.S. In your proof sheet you write your rules
In England we don't use the \smile ¹

¹Added hand-written remark by H. Hasse: “Danken
und wegen
E. A. Western anfragen
,An extension of Eisenstein's law of reci-
procity' I, II
Proc. Lond. Math. Soc. (2) 6 (1908)”

1.4 10.12.1930, Hasse to Mordell

MARBURG-LAHN,
DEC. 10. 1930

DEAR PROF. MORDELL,

Many thanks for your kind letter, particularly for your writing to Mr. Davenport. Three days ago I received a very kind letter from him. I think he will come, though I am not at all interested in lattice points and only a little in Zetafunction. But I think that is no pity. We can learn from one another, each the interests of the other.

I received your corrected revision and returned it to our printer. I agree with you, that the type $\tilde{\omega}$ is still as bad as before and that it will be the best, to omit the \sim upon it.

The „ \bar{u} ” is also not usual in Germany writing Latin letters. We use it only writing German letters. But in order to avoid confoundings between „ u ” and „ n ” in mathematical MS, I have taken up the custom to write „ \bar{u} ” at any rate.

It is very kind of you and Mrs. Mordell to invite us for England for the following Easter holidays. You do not know what a misfortune you are conjuring to intrude your free time and your household. The next year you will still be sure, but in 1932 we will come certainly and see you and several other friends of ours in Manchester, provided indeed, that you and Mrs. Mordell will reply our visit and be our guests at Marburg as soon as possible.

Dare I ask you whether you know of an English mathematician E. A. Western, who had published two papers concerning the law of reciprocity in the Proc. Lond. Math. Soc. (2) 6 (1908), cited in my report you have received on page 201 ?

With kindest regards yours very sincerely

H. HASSE

1.5 19.10.1931, Postcard Hasse to Mordell

MARBURG,
POSTCARD 19. 10. 31

DEAR PROF. MORDELL,

Many thanks for your kind letter and the returning of the proofs. I look forward with great pleasure to your visit at Marburg. Our Christmas vacation, which only interrupts the running term, begins about Dec. 22. and ends about Jan. 7. Lectures go on, then, until Febr. 29. I hope you will give again a German lecture before our students and selves. It would be very nice if you'd bring your wife with you to Marburg and spend your time here together. Our guest room, though a little narrow for two, is by all means at your disposal. We have got a central heating, which renders all the flat completely and thoroughly warm, — and in the guest room, which is not in the flat but upstairs, a good cast-iron stove. Thus, I think, we'll be able to give your wife an appropriate equivalent of the southern climate, for which, at any rate in our opinion, there is time enough afterwards. Moreover, and last not least, my wife would be very pleased to make the acquaintance of yours and to talk with her their shop while we are talking ours. Perhaps you consider the matter once more and alter your decisions accordingly.

Which would very much please

“your sincerely devoted”
(lit. transl. fr. German)

H. HASSE

1.6 08.11.1931, Mordell to Hasse

DEPARTMENT OF
MATHEMATICS,
THE UNIVERSITY,
MANCHESTER,
NOV 8, 1931

DEAR PROF. HASSE,

It is very remiss of me not to have replied sooner to your very kind note. But during the last three weeks I became very interested in Davenport's note on the distribution of quadratic residues and I could not do anything else. I have only within the last few days proved that the number of solutions of $y^2 \equiv ax^3 + bx^2 + cx + d \pmod{p}$ is $p + \mathcal{O}\left(p^{\frac{3}{4}}\right)$ & more generally when y^2 is replaced by y^n except in one trivial case. Davenport has also found the theorem & proof of a different kind about the same time. If I remember any German, I might speak on this to your students etc. as the method is very elementary.

Thanks very much for your kind invitation, but I don't know yet whether my wife can avail herself of it. Though she would like too. I shall leave here about Dec 13th, she cannot go till Jan 20th & prefers a warm country in the winter, though she might go to Germany.

I have made no plans yet. I may go first to Frankfurt about Dec 12th. I should then go somewhere for the Christmas and perhaps come to Marburg about Jan 6 or 7th. But if you will let me know a convenient time for a visit, first if I come alone or secondly if my wife should be with me, I can begin to make some plans.

Are the German Universities closed between March 1st & April 15th? I need not go back till April 15th, but I have not yet thought of what to do during the time March 1 — April 15.

If I don't go first to Frankfurt, I should go to Hamburg & then beat Marburg rather later, perhaps in February or I wish to spend some time in Göttingen.

But on hearing from you, I shall begin to know where I am. It is very good of your wife to be willing to have us.

With kindest regards from us to you & [...] and looking forward with

great pleasure to a stay with you,

Sincerely yours

L. J. MORDELL

P.S. I shall send you my paper on modular forms in a few days.

1.7 20.11.1931, Postcard Hasse to Mordell

MARBURG,
POSTCARD 1931, NOV. 20th

DEAR PROF. MORDELL,

Please excuse my delay in answering your kind letter. I was busy on an exciting and thrilling subject. At last I was successful in proving that every normal division algebra over an algebraic number field is cyclic (of Dickson type). Moreover, this result yields the generalization of the theory of class fields to general galoisien fields (instead of only abelian fields). I was able to prove the law of decomposition (“Zerlegungssatz”) for such fields, and also the “Eindeutigkeits- und Anordnungssatz”, both in terms and by help of linear algebras. I may suppose that you, after reading my report, can realize the importance of these results — and so will be kind enough to excuse my delay.

As to your coming, we will agree with every time, except perhaps for Jan. 6th—13th. For we have invited Davenport for the second half of his Christmas vacation — or rather he has invited himself with our readily given consent — and so it would not be possible for us to accomodate you, too. I do not suppose that lectures will begin before Monday Jan. 11th. Therefore, perhaps, it will not be any loss for you, if you come later. Term will close at every University here about March 1st. This does not mean, of course, that German mathematicians disappear at all during March and the first half of April. Only big people, such as Courant, go to Arosa then for “Ski”. Perhaps it would be good to delay your Marburg visit until March or April and to come then *with* your wife, though we had to renounce then your lecture. I repeat, however, that every other time which agrees with your plans will be convenient to us likewise. Please let me know of your plans about Marburg as soon as they are fixed.

With kindest regards to you and Mrs. Mordell,

sincerely yours,

H. HASSE

1.8 14.12.1931, Mordell to Hasse

DEPARTMENT OF
MATHEMATICS,
THE UNIVERSITY,
MANCHESTER,
14/12/31

DEAR PROF. HASSE,

I have persuaded my wife to come with me to Germany and we hope to be in Marburg about the end of February or the beginning of March. We should like to stay two or three weeks if it is convenient for you. We shall look forward with the greatest pleasure to our stay. Please do not let us interfere with any plans you are making as it will be easy for us to make different arrangements.

I was very interested in your discovery, and I think I shall read nothing else at Marburg except your papers.

You may also be interested in knowing that I have made further progress with congruences. The cubic congruence $f(x, y) \equiv 0$ has in general $p + \mathcal{O}\left(p^{\frac{2}{3}}\right)$ solutions. Also $y^m \equiv a_1x^n + \dots + a_{n+1}$ has in general $p + \mathcal{O}\left[p^{\varphi(m, n)}\right]$ solutions where

$$\begin{aligned}\varphi(m, n) &= \frac{2}{3} && \text{if } n = 4, m = 2 \\ &= \frac{7}{8} && \text{if } n = 6, m = 2 \\ &= \frac{5}{6} && \text{if } n = 4, m = 4 \\ &= \frac{3}{4} && \text{if } n = 3 \quad (m \neq 2 \text{ included above}) \\ &= \frac{1}{2} && \text{if } n = 3, m = 3\end{aligned}$$

Davenport has also found results of this kind; and I saw him three days ago. With best wishes from my wife & self

Sincerely yours

L. J. MORDELL

P.S. Wedderburn is a candidate for admission into the Royal Society. Has your opinion of him (which was very favourable last year) increased this year ?

1.9 21.12.1931, Postcard Hasse to Mordell

MARBURG,
POSTCARD DEC. 21st, 1931

DEAR PROF. MORDELL,

We are very much delighted with your decision,^{§)} and look forward with great pleasure to the time of your being here. The time you propose suits us very well. We hope we shall be able to make you and your wife feel comfortable here.

There is no reason why my opinion about Wedderburn should have been either impaired or improved during the last year. I think him without any question worthy of the election to the R. S.

Good many wishes for a merry Christmas and a happy New Year to you and your wife.

Very sincerely yours,

PROF. DR. HELMUT HASSE
MARBURG/LAHN
Weißenburgstr. 22

^{§)} and in particular that of your wife

1.10 25.01.1932, Hasse to Mordell

MARBURG-LAHN, DEN

25. JANUAR 1932

LIEBER HERR MORDELL !

Beiliegend mit bestem Dank Ihr Rohmanuskript ¹ zurück. Ich habe die Einzelheiten mit grossem Interesse zur Kenntnis genommen. Die Durchführung der Methode im Falle der reinen Exponentialsummen war mir ja bereits sehr geläufig. In den hier von Ihnen behandelten Fällen kommen noch einige interessante Schlüsse hinzu, die sich auf die Kongruenzen in mehreren Variablen am Schluß des Verfahrens beziehen. Wäre es übrigens nicht zweckmäßig, in der Veröffentlichung doch zuerst mit einer noch offen gelassenen Zahl k zu potenzieren und diese erst am Schluß bestmöglich zu bestimmen. Der Leser würde dann den eigentlichen Kern Ihrer Methode viel besser durchschauen, während das Ganze so ein wenig wie Zauberei aussieht.

Es tat mir sehr leid, daß die Tage in Göttingen ² nicht Gelegenheit zu einer etwas ruhigeren Unterhaltung für uns gaben. Bitte nehmen Sie es mir nicht übel, wenn ich Ihnen dort nicht mehr Zeit widmen konnte. Aber Sie werden verstehen, daß bei solch' einer Zusammenkunft vieles mit den einzelnen auswärtigen und Göttinger Mathematikern zu besprechen ist, und daß dabei auf jeden einzelnen naturgemäß nur eine beschränkte Zeit entfallen kann. Umsomehr freue ich mich auf die Tage Ihres Besuchs hier in Marburg. Mir ist wie gesagt die von Ihnen vorgeschlagene Zeit sehr recht. Meine Frau freut sich ebenfalls sehr. Vielleicht würde es noch ein klein wenig besser passen, wenn Sie etwas später kämen und dann entsprechend länger blieben. Es ist dann auch schon mehr frühlingsmäßig hier in Marburg, und wir könnten gelegentlich schöne Spaziergänge in die Berge und Wälder ringsum machen. Aber, wenn es Ihnen besser paßt, schon um den 15. März herum

¹Es handelt sich um das Manuskript zu Mordells Arbeit [Mor:1933] in der Mathematischen Zeitschrift. Die Arbeit trägt das Eingangsdatum vom 25. Februar 1932. Möglicherweise hatte Mordell das Manuskript Hasse während ihres Zusammentreffens in Göttingen gegeben. Mordell hielt sich in der ersten Hälfte des Jahres 1932 in Deutschland auf. Er besuchte Marburg im März, und er wohnte dort bei Hasses.

²Zum 70. Geburtstag von Hilbert am 23. Januar fanden in Göttingen einige Feierlichkeiten statt.

hier einzutreffen, ist es uns ebenfalls durchaus recht. Auf jeden Fall rechnen wir damit, daß Sie das Osterfest bei uns verbringen. Wenn Ihre Pläne feststehen, darf ich um eine Mitteilung bitten. Wahrscheinlich sehen wir uns ja übrigens in 14 Tagen oder 3 Wochen wieder in Göttingen, wenn Artin dort über Klassenkörper vorträgt.³ Dann können wir vielleicht mit etwas mehr innerer und äußerer Ruhe über alle uns interessierenden Fragen sprechen.

Mit vielen Grüßen, auch an Ihre verehrte Gattin,

stets Ihr

HASSE

³Die berühmt gewordenen Göttinger Vorträge von Artin über Klassenkörpertheorie fanden vom 29.2.–2.3.1932 statt.

1.11 14.05.1932, Postcard Hasse to Mordell

BAD SOODEN
ALLENDORF,
POSTCARD 14. 5. 32

DEAR MORDELLS,

Thanks awfully for your kind letter and the priceless books. I have already finished *The Secret Adversary* and fairly pushed on with *The Good Companions*. We are out of term for Whitsun vacation (10 days) and spending the time in this little health resort. It's the place my parents live at. They couldn't put up with us, however, for also my brother's family has been coming here. Therefore we are staying at a boardinghouse. We hope you have recovered from your nasty flu and are considering your next journey to Germany. What about Frobenius' congruences and all that ? I have finished off the Siegel reviews ¹ and have been tempering a little with hypercomplex numbers. Kindest regards from my wife, Juttalein and self to very good companions,

H. Hasse

¹Hasse had written a review of Siegel's big paper on Diophantine Approximations [Si:1929]. He had read this paper with Mordell during Mordell's stay in Marburg. Later (next January) Hasse claimed in a letter to Mordell that their common discussion of Siegel's paper had given Hasse the clue for his proof of the Riemannsch hypothesis for elliptic curves. – Hasse's review appeared in the *Zentralblatt*

1.12 15.06.1932, Hasse to Mordell

MARBURG, JUNE 15th, 1932
WEISSENBURGSTR. 22

DEAR PROF. MORDELL,

Thanks awfully for your kind letter. Presumably your term is over by now, whereas our's is still in full swing. Thus I was not able to get myself at the writing desk until to-night. Please do not mind the delay.

It is really kind of you to invite us for a stay with you in the Fueters' country cottage. We are sure it would be a jolly good time for all of us, even when there are certain restrictions as to servants etc. Only we simply cannot follow your kind invitation for two quite different reasons. Firstly, we intend to knock about all over Switzerland or some of the adjacent nice districts with Davenport's car *before* the congress, as we have got no time for this *after* it. And secondly, there is the rather delicate matter of my relations to Fueter. You know that I once reviewed his book on Complex Multiplication pretty bad. He still seems to bear a grudge against me from that review, though we do not know each other personally as yet. Perhaps the personal meeting on the occasion of the congress will mend this crack. At any rate, I simply cannot go and live in his country cottage under the present conditions, even not when I am to be *your* guest. I think you will understand this. We hope there will be soon an opportunity of staying with you for a time under better circumstances, say in your beloved and "besooted" Manchester. Let us look forward to this, and to the impending meeting at the congress, of course.

I have sent you my review on Siegel's paper a few days ago. Perhaps you are interested in it. At any rate, there you have the fruit of our joint toiling. Otherwise I have not done very much. Term was and still is too busy.

A week ago I had a long and frightfully sad letter from Wilton (S. A.). As you know his wife was suffering from a serious heart complaint the last years. Now he writes she is dying. I wonder whether he will stick to his position out there after all that.

I have been reading two books of J. Conrad recently (Lord Jim, Shadow line). I did not like the first one very much whereas the second one seems not so bad to me. I have finished The Good Companions. This I find mighty

good. I really love it, in particular the funny (and serious, all at once) character of Mr. Oakroyd.

Kindest regards to both of you.

Very sincerely yours,

H. HASSE

Note on June 16th: Just got your reprints. Thanks awfully.

20. Juni

Ihr lieben Mordells —

leider blieb durch meine Schuld dieser Brief bis heute liegen, da ich im Haushalt sehr beschäftigt war. Wir hatten selbst 2× Gäste bei uns zum großen Abendessen, 10 und 8 Personen. Einige andere Abende waren wir eingeladen u. s. w. Allerherzlichst möchte ich Ihnen, liebe Frau Mordell, für den Brief aus Schottland danken und für das reizende seidene Tüchlein. Es ist mir eine liebe Erinnerung an unser Zusammensein. Schade, daß aus Ihrem schönen Plan nichts werden kann, es war so nett ausgedacht — aber hoffen wir auf später. — Diese Woche kommt Davenport wieder zu uns, wenn uns die Sonne weiter so gut ist, haben wir eine herrliche Zeit mit dem Wagen vor uns. Juttalein sprach noch oft von Tante Mordell und schickt ihr liebe Küßchen. Auf frohes Wiedersehen in Zürich

Ihre C. Hasse.

1.13 06.03.1933, Hasse to Mordell

MARBURG-LAHN,
DEN 6. 3. 1933

DEAR PROF. MORDELL,

I have succeeded recently in proving that the number of solutions of $y^2 \equiv f_4(x) \pmod{p}$ is $p + a$ term which is less than or equal to $2\sqrt{p}$. Moreover, the same holds for any Galois-field instead of rational Galois-field mod. p , that is, the analogue to Riemann's hypothesis is true for the corresponding Artin Zetafunction.

It is a curious fact that the leading idea of my proof may be considered as the fruit from our reading Siegel's great paper last year, or rather of my learning your method in the elliptic case. For, as there the *equation* $y^2 = f_4(x)$ is treated by uniformizing it through elliptic functions, so I now treat the *congruence* $y^2 \equiv f_4(x) \pmod{p}$ by uniformizing it the same way.

My leading principle is: Every solution of the congruence in any Galois-field (GF) may be considered as arising from a solution of the corresponding equation in algebraic numbers, namely by such algebraic numbers in whose field p has a prime divisor p with the given GF as a system of residues.

This principle is sort of analogous to Siegel's principle by which every algebraic relation between values of power series goes back to an algebraic relation between the power series themselves.

As to the details of my proof, I assume first w. l. o. g. that $f(x)$ is in the Weierstrass normal form:

$$f(x) \equiv 4x^3 - a_2x - a_3,$$

also that the discriminant

$$a_2^3 - 27a_3^2 \equiv 1 \pmod{p}$$

The latter may be obtained by multiplying x and y with a constant t , and it implies passing to a suitably chosen GF instead of the rational GF (of degree 1, 2, 3, or 6).

I uniformize the corresponding equation by

$$x = \pi(u) = \frac{\wp(u)}{\sqrt[6]{\Delta}}, \quad y = \bar{\pi}(u) = \frac{\wp'(u)}{\sqrt[4]{\Delta}}, \\ a_2 = \gamma_2 = \frac{g_2}{\sqrt[3]{\Delta}}, \quad a_3 = \gamma_3 = \frac{g_3}{\sqrt{\Delta}},$$

$$\Delta = g_2^3 - 27g_3^2.$$

First I prove that there is always a system α_1, α_2 of periods for which the values of the modular functions γ_2, γ_3 are congruent to $a_2, a_3 \pmod{P}$, where P is a prime divisor of p . This follows from the theory of the singular moduli. α_1, α_2 are chosen as a basis of a Ringideal in an imaginary quadratic field in which p has a prime divisor p of degree 1, and P is a prime divisor of p in the Ringklassenkörper generated by the values γ_2, γ_3 . By proper choice of α_1, α_2 one can manage that the residue system mod. P is arbitrarily large, but also that it is exactly the field *generated* by a_2, a_3 . Let q denote the number of elements of this GF. Then by the law of decomposition in the Ringklassenkörper $q = \varpi \varpi'$ where ϖ, ϖ' are integers of the quadratic field belonging moreover to the corresponding Ring.

I consider now the Teilpunkte of the divisors $\varpi \pm 1$ of the parallelogramm α_1, α_2 . The corresponding values of $\pi(u)$ generate a certain Strahlklassenkörper, in which P splits a prime divisor P of relative degree 1, that is with the same GF(q) as residue system. The main point of my proof is to show that the system of all those Teilwerte of $\pi(u)$ forms twice a complete set of residues mod. P , that is twice the GF(q). This follows from certain discriminants and resultants not being divisible by p .

The corresponding Teilwerte of $\bar{\pi}(u)$ also generate a certain Strahlklassenkörper. The congruence value of such a Teilwert will be in the GF(q) if and only if P decomposes in prime divisors \bar{P} of degree 1 in this last Strahlklassenkörper. To those Teilpunkte, and only to them, correspond solutions of the given congruence in the GF(q). By the law of decomposition in the Strahlklassenkörper one can determine explicitly for which Teilpunkte this is the case and for which not. Thus, counting up the Teilpunkte, the proposition follows at once. *)

There is of course much detail which I could not give here, and my proof will be pretty long and difficult in print. On the other hand I am very glad that this is so. For it shows that the theory of singular moduli and singular values of elliptic functions, and even the general Klassenkörpertheorie which I need for certain conclusions, are far from being very abstract castles built

*) It follows even $q + \mathcal{O}(1)$ for the number of solutions, where $\mathcal{O}(1)$ can be given explicitly. I have not worked out the details as far as to give it here. — This result holds in the GF's for which my uniformization is possible (see top of backside of sheet 1) [Page 23, "I uniformize..."]. For the rational GF, one obtains $p + \mathcal{O}(\sqrt{p})$ with $|\mathcal{O}(\sqrt{p})| \leq 2\sqrt{p}$ by the argument with the zeros of Artin's ζ -function.

into the air, but concrete enough to yield non-trivial results on rational numbers only. I know that you are one of the very few exceptions who have realized that since long. But the overwhelming lot of English, American, and even German mathematicians do not think so.

Perhaps my argument will become clearer to you when I expose it in the trivial case of $y^2 \equiv x^2 - 1 \pmod{p}$, which is uniformised by

$$x = \cos 2\pi u, \quad y = i \sin 2\pi u.$$

Here the first point in my above argument, the representation of the coefficients a_2, a_3 by singular moduli, falls out, because there is no parameter in the problem. For any divisor n , the Teilwerte $\cos \frac{2\pi\nu}{n}$ ($\nu = 0, \dots, n-1$) generate the real subfield of the n^{th} cyclotomic field. If n divides $p \pm 1$, p splits into prime divisors p of degree 1 in that field. Take the equation with all those $p-1 + p+1 = 2p$ Teilwerte as roots. By a very easy calculation one finds its left hand side $\equiv (x^p - x)^2 \pmod{p}$. This means that those Teilwerte form twice a complete set of residues mod. p . Now a single $i \sin \frac{2\pi\nu}{p \pm 1}$ generates the full cyclotomic field of exponent $n = \frac{p \pm 1}{(\nu, p \pm 1)}$, whereas a single $\cos \frac{2\pi\nu}{p \pm 1}$ generates the real subfield of this cyclotomic field. Only if $p \equiv +1 \pmod{n}$, p splits in that full field in prime divisors \bar{p} of degree 1. Hence only to those cases correspond rational solutions of the given congruence. For the denominator $p+1$, $p \equiv +1 \pmod{n}$ arises only from $\nu \equiv 0, \frac{p+1}{2} \pmod{p+1}$. To those values correspond the trivial solutions $x \equiv \pm 1 \pmod{p}$, $y \equiv 0 \pmod{p}$. For the denominator $p-1$, every ν fulfills the condition. This means 2 solutions for every x , except only one solution when $y \equiv 0$ arises, that is for $x \equiv \pm 1 \pmod{p}$, or $\nu \equiv 0, \frac{p-1}{2} \pmod{p-1}$. Counting up the solutions in y for any of our $2p$ values of ν , we get $2 + 2(p-3) + 2 = 2(p-1)$ solutions. Since the $2p$ values of x form twice a complete set of residues, the actual number of solutions arises as $p-1$, as is well known.

All my argument with the elliptic functions is quite analogous to this simple consideration. The main point is, that the law of decomposition, here in the cyclotomic fields, there in the Strahlklassenkörpern, gives an exact criterion for the solvability.

Now I have told you enough for the time being. I hope you will not be bored by all this stuff. I think with great pleasure of the jolly time nearly a year ago when you were with us and we were reading Siegel together. When are you coming again? Compared with the Gold Coast, Marburg is a "Katzensprung" from Manchester.

Kindest regards to Mrs. Mordell and to yourself,

yours,

H. HASSE

P.S. Obviously the general congruence $f(x, y)$ may be treated the same way, “only” with the “slight” generalisation of the elliptic functions into abelian functions quite analogous to Siegel! Do it!

1.14 09.03.1933, Mordell to Hasse

DEPARTMENT OF
MATHEMATICS,
THE UNIVERSITY,
MANCHESTER,
9/3/33

DEAR HASSE,

I have been meaning to write to you the last few days to see whether there is any chance of you coming to England. I seem to remember that you had some idea of coming this Easter holiday and I hope we shall have the pleasure of welcoming you & Mrs Hasse before long.

I was exceedingly interested in your mathematical news and was very glad to hear that you had completely knocked the bottom out of $y^2 \equiv f_4(x) \pmod{p}$. It is a wonderful achievement and I shall look forward with the greatest interest to seeing your paper in print. I hope you will make it as easy as possible for the reader to understand, with exact reference to all the theorems on Klassenkörpertheorie etc., for as this is the first case of any exact results for zeros of Zetafns. on $\text{Re} = \frac{1}{2}$, the paper is sure to attract an enormous amount of attention. What a tremendous vindication (for those who need it or have not appreciated the K. k. theory) that the proof should depend upon such a comparatively high brow theory. I feel rather relieved to think I did not spend too much time on further results of this kind with my method, and very pleased that my old paper should have supplied even an amount ε of usefulness. We must read another paper some other time.

You may like to hear that I have been revisiting my old paper on $\int_{-\infty}^{\infty} \frac{e^{ax^2+bx} dx}{e^{cx+d}}$, and that it is being typed. I hope to send it to you for Crelle in a few weeks time. I am still trying to read Dickson ... but it is dull reading after van der Waerden. I am also reading a little about Lebesgue integration but hope to get down to the Theory of Numbers again before long.

With best wishes from all of us to all of you

Sincerely yours

L. J. MORDELL

P.S. I think the results for $y^2 \equiv f_n(x)$ etc. should follow without infinite difficulty, but the zeta fn. theory will not be so simple now. Obviously you are now the one to try it.

1.15 06.05.1933, Postcard Hasse to Mordell

MARBURG,
POSTCARD 6. 5. 33

DEAR PROF. MORDELL,

Half a year ago our Seminar asked you by a sort of circulatory letter to all Mathematical Seminars and all mathematicians interested in theory of numbers, whether you would take a copy of my typewritten and hectographed lecture on Klassenkörpertheorie. It gives an exposition of the subject in full detail, including the analytical part which is not in my former Report, and also contains quite a number of new proofs and results due to Artin, Herbrand, and myself that have been developed since then. The price is 8.- Rm netto + the expenses for postage and cover. As I got no answer to our letter from you, I suppose that either that letter did not reach you at all or you mislaid it and forgot answering it. Of course, it is far from me wangling you into buying the copy. But I think you may be interested in it and want to have it either for yourself or for your Seminar or both.

If this is the case, please let me know soon. The copies are just being sent off.

Davenport has just left us for Göttingen. I am afraid he will be disappointed, what with several great people there no longer lecturing and all that.

Kindest regards to you and Mrs. Mordell, also from my wife,

yours,

H. HASSE

1.16 20.06.1933, Fragment of a Letter, Hasse to Mordell

WEISSENBURGSTRASSE 22

20. 6. 33

DEAR PROF. MORDELL,

I was greatly pleased to receive your two letters of June 13th and 14th. I immediately wrote to Baer, who has taken a refuge in Austria at present, and got his answer this morning. He is very much interested in the possibility which you announce. I enclose a list of his papers. As you will see from it, he has written quite a few things for his age. He was born in 1902, married a few years ago a very charming lady, and got a son in due course who may be 3 years now. He is of jewish race, I suppose 100%, but Protestant of religion. His papers as a whole represent in my opinion a fairly large field of [...].

[...] who in my opinion is at any rate more universal and cleverer than A. Brauer. But both Brauers are not in the least gifted with that active mathematical spirit which makes Baer such a helpful assistant where mathematical education is concerned. From this point of view, I should certainly prefer Baer to both of them. I know of no other man who is concerned by the new regulations here and comes virtually into consideration for your kind offer.

Baer's present address is: Innsbruck-Igls, Pension Wiesenhof. Perhaps you write to him personally, if you decide on making arrangements with him. I should be greatly pleased if you did.

1.17 20.10.1933, Hasse to Mordell

MARBURG,
20. 10. 33

DEAR PROF. MORDELL,

It is very kind of you to think of an English translation of my *Ausarbeitung*. On principle I am quite willing to consent. My only scruples are that the whole thing is not quite consistent in itself on account of its being compiled by several of my pupils in turn and only very roughly supervised by myself. Perhaps Baer could do a bit more than only translating, in particular in the first Chapter. I should also like to have some sections inserted which I actually brought in my lecture but dropped for the *Ausarbeitung* in order to keep a moderate extent. I have very detailed notes for those sections. It would be easy for Baer to make the thing round with the help of them. We had, of course, to agree about the terms.

I am about to find a purely algebraic proof for my theorem on elliptic congruences. You will hear more about it when I have got through.

Would you mind handing the enclosed letter to Mahler ?

Kindest regards to all of you.

Sincerely yours,

H. HASSE

Bemerkungen:

1. Betr. Mahler: Dieser hatte in 1933 und 1934 eine Arbeit im Crelleschen Journal. Vgl. Korrespondenz Hasse - Mahler.

1.18 01.11.1933, Hasse to Mordell

MARBURG-LAHN,
DEN 1. Nov. 1933 DEAR PROF. MORDELL,

I received your kind letter of Oct. 26th and am looking forward with great pleasure to having my work printed in England. It seems opportune to me to send you the manuscript of my proper lecture. You will receive it in a few days. I should like to have §§ 1–7 entirely revised, following closely my own notes. For, the Ausarbeitung has been drawn up by two students without a wide experience up to § 7, and has a lot of roughnesses on account of it in those §§. From § 8 on, my assistant Dr. Franz has made the Ausarbeitung with looking at my own notes. It would be good, however, to revise also the whole rest on the occasion of the translation by comparing with my manuscript.

Further suggestions:

- (a) The „Anhang” may be worked into § 15.*)
- (b) Teil III of the Ausarbeitung shall proceed up to § 21 only, and be concluded by adding § 30 of my manuscript.
- (c) As in my manuscript, a new Teil IV shall be formed, headline: “Theorie des Normenrestsymbols und des Potenzrestsymbols.” It shall contain §§ 22–29 of my manuscript, then §§ 22, 23 of the Ausarbeitung, where in § 23 the beginning up to Satz 170 may be derived immediately from the theory in § 23 of the manuscript.

I think Baer will be able to do all this. If anything be not clear, I am ready to help.

Kindest regards, sincerely yours,

*) In § 8 of the Ausarbeitung Dirichlet’s theorem of the existence etc. of units is required. This may be assumed as known from the elementary theory of algebraic numbers at this place. The Anhang contains in particular also a proof of *that* theorem, but gives far more, and its point lies in this generalisation.

H. HASSE

p. t. o.

P.S. 1 I should like to have the manuscript back when the thing is finished

P.S. 2 A student of mine, Rosenthal, will be coming to Manchester for the next term. I think he will be there already in a few weeks. He is a very good man. I should be glad, if you could give him any possible advantage. I very much regret his going abroad as a consequence of his being of Jewish descent, but I could only back him up on this decision.

Bemerkungen:

1. Läßt sich irgendetwas über den Verbleib von Rosenthal herausfinden?

1.19 15.03.1934, Hasse to Mordell

MARBURG-LAHN,
DEN 15. 3. 34 DEAR PROF. MORDELL,

I greatly regret that your kind intention of providing an English translation of my work on Klassenkörpertheorie had to be given up. I thank you very much for all the trouble you have taken about this matter. If I find a clever young mathematician willing to help me with the detail work, I think I shall publish a textbook on the whole matter within the next few years.

I should like to know whether Mr. Rosenthal has succeeded in getting a Stipendium, and how he is getting on in Manchester. I do not see whether you sent me the form concerning the Bishop Harvey Scholarship on *his* behalf. If so, what am I supposed to do ? I have already written two recommendations on his behalf on request from English Boards or Associations.

Friends of ours in Halle a. d. Saale asked us about the possibility of having a young Englishman of 15–18 years for a couple of weeks with them and sending their own son, 17 years of age, to England before or afterwards. The young men were to share the family life in order to improve in the languages, and to get general knowledge about life in a foreign country. Our friend is a physician and medical professor at Halle. His son still visits the school there. The time chosen had to be in accordance with his holidays. Perhaps you will be able to suggest somebody suitable, or your friends, the Griffins, will know a family in his parish who are willing to join in such a procedure. Otherwise would you be as kind as to suggest a suitable way for finding such an opportunity ?

With the very best wishes from all of us to all of you

Sincerely yours,

H. HASSE

1.20 09.01.1936, Hasse to Mordell

GÖTTINGEN, DEN
9. JANUAR 1936.

DEAR MORDELL,

I have applied to the Ministry for the permission to give some lectures in Manchester. I hope, I shall get the permission. I shall let you know in due course.

I don't think I shall take the route Rotterdam–Hull, for Flushing–Harwich seems to save time, particularly if Davenport is going to meet me at the boat and drive me to Manchester as he promised to do.

Thanks very much for your suggestion of bringing Klein with me : I don't think I shall need him. I would only do it, if you really cared for his presence.

I am very sorry I have no copy of my paper in Annalen 104 left. I very much regret not having sent you a copy at the time.

What would you like me lecturing upon ?

Sincerely,

Y o u r s ,

H. HASSE

Bemerkungen:

Wer ist Klein ?

Chapter 2

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