Die Korrespondenz
Helmut Hasse – L. E. Dickson

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^{tk} Hasse an Dickson 11.5.35–17.10.35

^{tk} Dickson an Hasse 3.4.35–21.6.35

^{tk} Weiteres Material zu Hasse–Dickson

t – fertig transkribiert, k – nach Tippfehlern durchgesehen

Version vom 10.10.2006
Letztmalig geändert am 10.10.2006

Quelltext: hasdix_061010.tex
übersetzt am 24. August 2020

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Kapitel 1

Die Korrespondenz Hasse–Dickson

1.1 03.04.1935, Dickson an Hasse

The University of Chicago
Department of Mathematics

April 3, 1935

Professor H. Hasse
Director of Mathematical Institute
Göttingen, Germany

Dear Professor Hasse:

From Mr. Comrie I learn that you are interested in my tables of cyclotomy and have proposals to make for certain extensions such as polynomial congruences. I have no doubt that your suggested extensions would be very useful and would be glad to know of your project in more detail. This only from curiosity, since I am willing to say in advance that I would be glad to see any tables you are interested in published with my own. I like your proposal to have the additional tables computed in Göttingen as I am not in a position to undertake further tables here. In brief I would be glad to see you undertake such new tables as, in your judgement, would be useful.

My own short tables for cyclotomy are really systematic extensions by an assistant of a table for a few special primes which I used to check various formulas in several papers now being published on cyclotomy. When reprints come to me, I shall send copies to you hoping that you would be interested to undertake the further development of this theory.

Sincerely yours,

L. E. Dickson

Professor of Mathematics.

LED:J
CC to L. J. Comrie

Do you know if Jacobi's Canon Arithmeticus is sufficiently rare to make a reprint desirable?

1.2 11.05.1935, Hasse an Dickson

11. Mai 1935

Prof. L. E. D i c k s o n,
University of Chicago
C h i c a g o (Ill.)

Dear Prof. Dickson,

Please excuse me for not answering earlier to your kind letter of April, 3rd►. I had to change a couple of letters with my friend, Mr. Sadler (the assistant of Dr. Comrie), about the whole matter first.

Unfortunately, it turned out that I had been to optimistic when I told Dr. Comrie and Sadler in March that it would be possible to have some computational work on cyclotomy done here in Göttingen. There are several difficulties with this that cannot be overcome at present.

Now Sadler informs me that the Cunningham bequest enables them to have not alone the publication but also the computation of new tables done by means of this bequest. Thus it would be possible that I give some indication, more or less detailed, for the computation of further tables, preferably on cyclotomy, to the English Nautical Almanac Office, and they do the actual calculation with the help of their splendid machines under my direction. I am going to write to Sadler with this in view.

As to the subject of the new tables, I originally thought of tables giving the exponents $e \bmod p$ for all residues $a \bmod p$, arranged according to both the order of a and of e. One could easily do this from Jacobi's Canon. I discovered however that tables in this line have already been printed, namely by Reuschle (Programmabhandl. Königl. Gymnas. Stuttgart 1856, Neue zahlentheoretische Tabellen). Although Reuschle gives the values of e for $a = 2, 3, 5, 6, 10$ only, and for larger p even only for $a = 10$, the numerical material represented in these tables would be quite sufficient for the immediate applications I can think of. I should not propose printing new tables of this kind therefore.

There is another possibility which struck me immediately on being shown your tables in Greenwich. You will presumably know the short tables Artin gives in his dissertation (Mathem. Zeitschr. **19**, p. 232–3). It would be of great value for the continuation of Davenport's and mine recent theoretical

investigations to have a set of tables of this kind for other than only quadratic (hyperelliptical) fields. Tables of this kind should select a few standard types of algebraic function fields mod p, preferably for some small fixed primes p, but complete as to all possible genera (Geschlecht) within an ample limit, and give (1) the class–number, (2) the structure or the class–group, and (3) the number of prime divisors of degree 1. The latter can be easily calculated from your tables, for it is essentially the number of solutions of a binary diophantine congruence mod p, and your tables give the "elements" for calculating the value of a given polynomial mod p for all possible values of the variables.

Finally one could think of modernising the well–known tables of Reuschle (Tafeln komplexer Primzahlen). Reuschles tables are of no great value for our modern purposes. One is much less interested in representations of the single prime ideals in a cyclotomic field by cyclotomic numbers, than one is in knowing the fundamental invariants of the single cyclotomic fields (for both prime and composite exponent of the root of unity), I mean again the class–number, and the structure of the Abelian class–group, and perhaps also the class–numbers and class–groups of the sub–fields, at least of the largest real sub–field (second factor of the class–number). It is such tables that I have in mind preferably. For quadratic fields there is a very nice sample of what I mean in the Appendix to Sommer's Zahlentheorie (Teubner 1907), which I find much better for the modern use than the great number of similar tables giving essentially the same by means of the periods of quadratic binary forms.

I am going to deliberate with Dr. Comrie and Sadler about those last two of my proposals. I thought you would be interested to learn in detail what I have in mind.

As to reprinting Jacobi's Canon, I think it a very good idea. The Canon is in a way independent of time, i. e., it is as modern to–day as it was when it was published. One very often needs it for arithmetical calculations of all possible kinds.

I am looking forward to seeing the reprints of the papers on cyclotomy you indicated in your letter▶ to me.

Sincerely yours,

H. Hasse

1.3 21.06.1935, Dickson an Hasse, Postkarte

POSTKARTE

Dear Hasse:

I have written to Mr. Sadeler approving heartily you proposed tables.

Am just starting on a vacation for [...].

Best greetings,

L E Dickson

1.4 17.10.1935, Hasse an Dickson

Prof. Dr. H. Hasse 17. Oktober 1935.

Herrn
Prof. Dr. Dickson
The University of Chicago
Chicago (Ill.) USA.

Dear Professor Dickson:

I have just received your "Researches on Waring's Problem". I thank you very much indeed for letting me have it.

I have not heard anything definite yet about the project of printing mathematical tables by the British Association.

Kindest regards,

sincerely,

Yours,

H. Hasse

Kapitel 2

Weiteres Material zu Hasse–Dickson

2.1 04.03.1935, Dickson an das Committee on Math. Tables

COPY of letter received from Professor L. E. Dickson, Department of Mathematics, University of Chicago, addressed to the B. A. Committee on Mathematical Tables, and dated March 4, 1935.

I beg to offer for your consideration:

TABLES FOR CYCLOTOMY AND TRINOMIAL CONGRUENCES, giving for each prime $p < 500$ all solutions of $1 + g^m \equiv g^n \pmod{p}$, where g is a chosen primitive root of p.

Such a table would print in 35 pages, of length about 7 inches and width to permit two columns of $2 \cdot 4$ inches each.

A brief table (of 2 pp.) to $\mu \leqq 103$ was published by Jacobi, Jour. für Math., Vol. **30** (1846), pp. 181–2.

It gives at once the information desired in Cyclotomy (Study of the "periods" of roots of unity).

It also solves other trinomial congruences (after expressing the coefficients and unknowns as powers of g).

The tables are computed by use of a corrected copy of Jacobi's Canon Arithmeticus, which prints 2 columns of present size tables per page. Complete and convincing checks were made.

The single table for a given p serves in Cyclotomy for every "e" which divides $p - 1$, a great saving!

Respectfully,

signed L. E. DICKSON.

Sample

$p = 29 \quad (g = 10)$

	˙0	1	2	3	4	5	6	7	8	9
	11	23	3	17	8	26	24	9	13	15
1	20	27	19	25	⋆	12	7	16	10	6
2	5	2	18	21	4	14	1	22		

For the tabular value m (found by heading and marginal 1, 2), the body of the table gives the value of n for which

$$1 + g^m \equiv g^n \pmod{29}$$

Examples

$$1 + g^6 \equiv g^{24}$$
$$1 + g^{16} \equiv g^7$$

The $\star$ denotes there is no answer for $1+g^{14} \equiv g^n \pmod{29}$ (since left member is divisible by 29)

The MS is being typed by the most expert typist I have ever known. It will be prepared in perfect form for reproduction by photography. But for so short a table, printing from type may be preferred.

L. E. D.

The tables increase in length with p, since there are entries for each $m = 0, 1, --, p-2$. Total length of tables for $p = 5, --, 500$ is 35 pages.

Kapitel 3

Register