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Advanced Geometry II–Symmetric spaces

Exercises 5–Isometry groups– Not to be handed in May 2, 2019

5.1. Let M be a Riemannian symmetric space, let $G = \mathrm{Iso}^0(M)$ be the connected component of the identity of the group of isometries of M and let $\omega \in \Omega^k(M)^G$ be a G-invariant differential form. Show that ω is closed.

5.2. Let M be a non-compact symmetric space. Show that the group $\mathrm{Iso}(M)$ endowed with the compact open topology is non-compact. Is the same true for a non-compact locally symmetric space?

5.3. Let (M, g) be a symmetric space and $c : \mathbb{R} \to M$ be a geodesic. Given a point $x \in M$ we denote by σ_x the geodesic symmetry at x. Let τ_c be the map

$$\begin{array}{rccc} \tau_c : & \mathbb{R} & \to & M \\ & t & \mapsto & \sigma_{c(t)} \circ \sigma_{c(t/2)} \end{array}$$

Show that τ_c is a one parameter subgroup, namely that τ_c is continuous and $\tau_c(s + t) = \tau_c(s)\tau_c(t)$.

5.4. Let M be a complete, connected Riemanian manifold and endow its group of isometries $G = \mathrm{Iso}(M)$ with the compact open topology. Let $H < G$ be a closed subgroup. Show that for every $m \in M$ the orbit $H \cdot m$ is closed.