University Bonn — SS 2018/19	Maria Beatrice Pozzetti
Advanced Geometry II–Symmetric spaces	
Exercises 3–Locally symmetric spaces	April 24, 2019

3.1. Find two Riemannian manifolds M, N and a local diffeomorphism $f : M \to N$ such that

(a) for every $p \in M$ and for every $v \in T_pM$, $|df_p(v)| \geq |v|$ but f is not a covering map.

(b) M complete, there exists $p \in M$ such that for every $v \in T_pM$, $|df_p(v)| \geq |v|$ but f is not a covering map.

3.2. Let (M, g_M), (N, g_N) be two Riemannian manifolds with parallel curvature tensors. Assume that (N, g_N) is complete. Given $m \in M$ and $n \in N$, assume that there exists a linear isometry $\phi : T_mM \to T_nN$ preserving the Riemann curvature tensor at m, i.e. such that for all $u, v, w \in T_mM$,

$$R^N_n(\phi(u), \phi(v))\phi(w) = \phi(R^M_m(u, v)w).$$

Prove that for every normal neighborhood U of m there exists a normal neighborhood V of n and a local isometry $f : U \to V$ such that $f(m) = n$ and $D_mf = \phi$.

3.3. Construct a complete metric on the cylinder $\mathbb{S}^1 \times (0, 1)$ which is locally symmetric but not symmetric. *Hint:* consider the isomety $A \in \mathrm{PO}(1, 2)$ with axis a and such that $a(m_1) = m_2$. What is $\mathbb{H}^2/\langle A\rangle$? What can you say about the geodesic reflection at $[q]$?