MATHEMATISCHES INSTITUT

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

SYMPOSIUM
GEOMETRIC DYNAMIC DAYS
BOCHUM-DORTMUND-HEIDELBERG-KÖLN-MÜNSTER

FRIDAY 27 - SATURDAY 28 OCTOBER 2017

All talks will take place at the University of Heidelberg, in the Maths building (Mathematikon) on the 5th floor
Mathematikon, INF 205, 69120 Heidelberg, Konferenzraum, 5. OG

FRIDAY

15:00 - 16:00 LAI-SANG YOUNG (Courant Institute)

Dynamics of periodically kicked oscillators

Nearly 100 years ago, van der Pol and van der Mark observed that irregularities developed when certain electric circuits with stable oscillations were periodically forced.
Their work stimulated many analytical studies, from Cartwright & Littlewood (1945), Levinson (1949) to Levi (1981) and Haiduc (2009). Much was learned along the way, but a complete analytical understanding of the forced van der Pol oscillator has remained illusive. In this talk, I will discuss a related model that is more amenable to analysis: Consider an arbitrary dynamical system with a limit cycle. To this system, periodic “kicks” (or forcings turned on for short durations) are interspersed with longer relaxation times during which the system is allowed to restore itself. To demonstrate the dynamical richness of these models, I will use an especially simple example, the linear shear flow in 2D, in which one can see clearly how the dynamical picture is controlled by a quantity proportional to shear and kick amplitude and inversely proportional to damping. Increasing this quantity gradually, one observes first invariant curves, then the breaking of invariant curves (a dissipative version of KAM), followed by the development of horseshoes and sinks, and eventually “strange attractors” with observable chaos.

17:00 - 18:00 VADIM KALOSHIN (University of Maryland)

Can you hear the shape of a drum and deformational spectral rigidity of planar domains

M. Kac popularized the question “Can you hear the shape of a drum?”. Mathematically, consider a bounded planar domain Ω and the associated Dirichlet problem

$$\Delta u + \lambda^2 u = 0,\ u|\partial\Omega = 0.$$

The set of ’s such that this equation has a solution, denoted $L(\Omega)$, is called the Laplace spectrum of Ω. Does the Laplace spectrum determine Ω? In general, the answer is negative. Consider the billiard problem inside Ω. Call the length spectrum the closure of the set of perimeters of all periodic orbits of the billiard. Due to deep properties of the wave trace function, generically, the Laplace spectrum determines the length spectrum. We show that any generic axis symmetric planar domain with sufficiently smooth boundary is dynamically spectrally rigid, i.e. can’t be deformed without changing the length spectrum. This partially answers a question of P. Sarnak. This is a joint work with J. De Simoi, A. Figalli and Q. Wei.

SATURDAY

10:00 - 11:00 VADIM KALOSHIN (University of Maryland)

Birkhoff Conjecture for convex planar billiards

G.D. Birkhoff introduced a mathematical billiard inside of a convex domain as the motion of a massless particle with elastic reflection at the boundary. A theorem of Poncelet says that the billiard inside an ellipse is integrable, in the sense that the neighborhood of the boundary is foliated by smooth closed curves and each billiard orbit near the boundary is tangent to one and only one such curve (in this particular case, a confocal ellipse). A famous conjecture by Birkhoff claims that ellipses are the only domains with this property. We show a local version of this conjecture - namely, that a small perturbation of an ellipse has this property only if it is itself an ellipse. This is based on several papers with A. Avila, J. De Simoi, G. Huang, D. Sorrentino.

11:30 - 12:30 LAI-SANG YOUNG (Courant Institute)

Shear-induced chaos

There are two parts to this talk. In the first part, I will explain the idea of SRB measures for chaotic attractors as the analog of Liouville measure for Hamiltonian systems.
SRB measures were discovered for Axiom A attractors by Sinai, Ruelle and Bowen in the 1970s. Even though their role is taken for granted in physics, proving rigorously the existence of these invariant measures outside of the Axiom A category poses serious analytical challenges. The main examples so far have come from attractors that are only mildly chaotic, in the sense that they have only a single direction of instability. An example of such an attractor is the periodically kicked linear shear flow. In the second half of this lecture, I will describe a few other examples of shear-induced chaos, a phenomenon that occurs easily in the periodic kicking of general limit cycles, and of systems undergoing Hopf bifurcations, in ODEs as well as in some PDEs.

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Organisator:
Prof. Dr. Peter Albers