

Young Researcher's Workshop on Positivity in Lie Groups

IWH Heidelberg

March 15th - 19th, 2021

The aim of this seminar is to understand the notion of total positivity and Θ -positivity and how it relates to Higher Teichmüller theory. Θ -positivity is a notion of positivity in semisimple Lie groups and was recently introduced by Guichard and Wienhard in [GW18] as a generalization of Lusztig's total positivity. The workshop is aimed at graduate students in the field and no previous knowledge is assumed. Some familiarity with Lie groups or representations of surface groups is helpful, but not necessary. Every participant will play an active role in the seminar – either by preparing a talk, or by organising an exercise session to consolidate what we have learned.

Working in representation theory, one often comes across the concept of “positivity”. It appears in different contexts, for example the positive reals, the order on the circle, total positivity for matrices, positivity of triples in flag varieties and the Maslov index. An especially important role is played by Lusztig's total positivity in the context of split real Lie groups. The new notion of Θ -positivity generalizes this to other types of semisimple Lie groups and, in particular, includes the notion of positivity for Lie groups of Hermitian type.

We start with recalling some important definitions from Lie theory, including the notions of split real Lie groups and Lie groups of Hermitian type from above. Then we introduce the known concepts of positivity with a special focus on Lusztig's positivity. This prepares us to define Θ -positivity for semisimple Lie groups. Our main motivation for studying positivity lies in its close relation to representation theory. Thus, we will define the character variety of representations of a surface group into a semisimple Lie group and see for which classes of Lie groups we can define so-called higher Teichmüller spaces. The two known higher Teichmüller spaces are given by maximal representations and by Hitchin representations - both of which can in fact be characterized by their positive structures. Guichard and Wienhard define Θ -positive representations and conjecture that they give a new class of higher Teichmüller spaces.

References

- [GW18] Olivier Guichard and Anna Wienhard. Positivity and higher Teichmüller theory. In *European Congress of Mathematics*, pages 289–310. Eur. Math. Soc., Zürich, 2018.