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Kleine AG:
Construction of modular Galois representations

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Overview

The aim of this Kleine AG is to present the following theorem due to Deligne.

Theorem: *Let $f \in S_k(\Gamma_1(N), \chi)$ be a normalized eigenform of weight $k \geq 2$. Let $f = \sum a_n q^n$ its Fourier expansion and define $K_f := \mathbb{Q}(a_1, a_2, ..) \subseteq \mathbb{C}$. Then K_f is a number field and if λ is a place of K_f dividing the prime $\ell \in \mathbb{Z}$, there exists a continuous two-dimensional representation*

$$\rho_{f,\lambda} : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathrm{GL}_2(K_{f,\lambda})$$

which is unramified at all primes $p \nmid \ell N$ and satisfies

$$\det(X - \rho_{f,\lambda}(\mathrm{Frob}_p)) = X^2 - a_p X + p^{k-1}\chi(p)$$

for each arithmetic Frobenius Frob_p at p. In particular,

$$\mathrm{Tr}(\rho_{f,\lambda}(\mathrm{Frob}_p)) = a_p.$$

So basically, for suitable modular forms there is associated a Galois representation with prescribed traces. This emphasizes once more the arithmetic significance of modular forms.

The proof of the above theorem will heavily use geometry and cohomology of modular curves. The constructions and proofs in the theorem are involved and unfortunately not clearly outlined in our references,[1] so we present a short summary here (see also `http://vbrt.org/writings/l-adic-talk.pdf` for another one).

As a first and easy reduction it suffices (by taking the contragredient representation) to construct a two-dimensional $K_{f,\lambda}$-representation $\mathcal{V}_{f,\lambda}$ of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ unramified for $p \nmid \ell N$ and satisfying

$$\det(1 - F_p X \mid \mathcal{V}_{f,\lambda}) = 1 - a_p X + p^{k-1}\chi(p)X^2$$

[1] [Del71] deals with the case $N = 1$ to capture the discriminant and [Con09] presents some arguments only in the weight two case.

for each *geometric* Frobenius $F_p = \mathrm{Frob}_p^{-1}$ at p.

The construction of $\mathcal{V}_{f,\lambda}$ proceeds in two steps. The first step uses étale cohomology of a modular curve to introduce a representation $\mathcal{V}_\ell$ having our final representation $\mathcal{V}_{f,\lambda}$ as a quotient. Namely, let $a\colon Y_1(N) \longrightarrow \mathrm{Spec}(\mathbb{Q})$ be the modular curve over $\mathbb{Q}$ of level $\Gamma_1(N)$ with universal elliptic curve $f\colon E \longrightarrow Y_1(N)$. Now define

$$\mathcal{V}_\ell := \mathrm{Im}(\mathrm{H}^1_{\mathrm{c}}(Y_1(N) \otimes_{\mathbb{Q}} \overline{\mathbb{Q}}, \mathrm{Sym}^{k-2}(\mathrm{R}^1 f_* \mathbb{Q}_\ell)) \longrightarrow \mathrm{H}^1(Y_1(N) \otimes_{\mathbb{Q}} \overline{\mathbb{Q}}, \mathrm{Sym}^{k-2}(\mathrm{R}^1 f_* \mathbb{Q}_\ell)))$$

as the image of the cohomology with compact support of the local system $\mathrm{Sym}^{k-2}(\mathrm{R}^1 f_* \mathbb{Q}_\ell)$ in the usual cohomology (talk 1). By construction, this $\mathbb{Q}_\ell$-vector space carries a $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$-action. The second step uses the[2] Hecke algebra $\mathcal{T}$ away from ℓN. As we will see $\mathcal{V}_\ell$ is actually a module[3] over the algebra $\mathcal{T} \otimes_{\mathbb{Q}} \mathbb{Q}_\ell$ with $\mathcal{T}$-action defined over $\mathbb{Q}$ (talk 2). Hence the $\mathcal{T}$-action commutes with the $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$-action. Moreover, the algebra $\mathcal{T}$ surjects onto the field K_f by sending a Hecke operator T_p to a_p (talk 3). Using the decomposition

$$K_f \otimes_{\mathbb{Q}} \mathbb{Q}_\ell \cong \prod_{\lambda' \mid \ell} K_{f,\lambda'}$$

we can conclude that $\mathcal{T} \otimes_{\mathbb{Q}} \mathbb{Q}_\ell$ surjects onto $K_{f,\lambda}$ as well. Finally, we set

$$\mathcal{V}_{f,\lambda} := K_{f,\lambda} \otimes_{\mathcal{T} \otimes_{\mathbb{Q}} \mathbb{Q}_\ell} \mathcal{V}_\ell$$

which turns out to be the representation we were looking for.

To prove Deligne's theorem we have to do a lot of work, namely we first have to investigate the $\mathcal{T} \otimes_{\mathbb{Q}} \mathbb{Q}_\ell$-module $\mathcal{V}_\ell$ and then to relate the action of a geometric Frobenius F_p at p to the action of $\mathcal{T}$. At first, known comparison theorems between étale and singular cohomology imply that $\mathcal{V}_\ell$ arises by tensoring the $\mathbb{Q}$-vectorspace

$$\mathcal{V} := \mathrm{Im}(\mathrm{H}^1_{\mathrm{c}}(Y_1(N)^{\mathrm{an}}, \mathrm{Sym}^{k-2}(\mathrm{R}^1 f^{\mathrm{an}}_* \mathbb{Q})) \longrightarrow \mathrm{H}^1(Y_1(N)^{\mathrm{an}}, \mathrm{Sym}^{k-2}(\mathrm{R}^1 f^{\mathrm{an}}_* \mathbb{Q})))$$

with $\mathbb{Q}_\ell$ (talk 1). Here $(-)^{\mathrm{an}}$ denotes the analytification functor sending a variety over $\mathbb{Q}$ to its $\mathbb{C}$-valued points endowed with the classical topology. Like $\mathcal{V}_\ell$, the space $\mathcal{V}$ carries a $\mathcal{T}$-action (talk 2), and this is identified with the $\mathcal{T}$-action on $\mathcal{V}_\ell$ after tensoring with $\mathbb{Q}_\ell$. The structure of $\mathcal{V}$ as a $\mathcal{T}$-module can now be read off from the $\mathcal{T}$-equivariant Eichler-Shimura isomorphism

$$\mathrm{sh}\colon S_k(\Gamma_1(N)) \oplus \overline{S_k(\Gamma_1(N))} \cong \mathcal{V} \otimes_{\mathbb{Q}} \mathbb{C}$$

using classical theory (talk 3 and 5). At this point we understand the $\mathcal{T}$-module $\mathcal{V}$ rather explicitly and we can turn our interest towards the Galois action. The action of a geometric Frobenius F_p on $\mathcal{V}_\ell$ can fortunately be analyzed by means of the congruence relation

$$1 - T_p X + \langle p \rangle p^{k-1} X^2 = (1 - F_p X)(1 - \langle p \rangle p^{k-1} F_p^{-1})$$

[2] Note that there is not "the" Hecke algebra. Hecke operators are acting on various spaces, and different actions may give different Hecke algebras.

[3] free of rank 2 over $\mathcal{T} \otimes_{\mathbb{Q}} \mathbb{Q}_\ell / \mathrm{Ann}(\mathcal{V}_\ell)$

which should be understood as an equality in $\mathrm{End}_{\mathcal{T}}(\mathcal{V}_\ell)[X]$ (talk 4). Here $\langle p\rangle \in \mathrm{End}_{\mathcal{T}}(\mathcal{V}_\ell)$ denotes the diamond operator for p acting on $\mathcal{V}_\ell$ (talk 2). Finally, our last ingredient comes into play. Namely, there exists a (modified) Poincaré duality pairing

$$[\cdot,\cdot]\colon \mathcal{V}_\ell \otimes \mathcal{V}_\ell \longrightarrow \mathbb{Q}_\ell$$

which is $\mathcal{T}$-equivariant and for which F_p and $\langle p\rangle p^{k-1}F_p^{-1}$ are adjoint (talk 5). Comparing characteristic polynomials (over $\mathcal{T}$) of adjoints (over $\mathbb{Q}_\ell$) yields then the equality

$$\det(1 - T_pX + \langle p\rangle p^{k-1}X^2 \mid S_k(\Gamma_1(N)))^2 = \det(1 - F_pX \mid \mathcal{V}_\ell)^2$$

from which our theorem follows easily by tensoring with $K_{f,\lambda}$.

For simplicity, we will assume $N \geq 5$ throughout. The reason for this is that for $N \leq 4$ the moduli functors for elliptic curves with $\Gamma_1(N)$ structures are not representable by schemes, i.e. the modular curves do not exist. This can be circumvented by using some ad-hoc arguments [Con09], but we do not want to care about this.

Caution! The k we use will always be the weight of our modular form f. In some of the references, modular forms of weight $k+2$ are considered, so some k's in the references may be off by 2 compared to ours.

The talks

Each talk should take 45 minutes.

Talk 1 (Jonathan Zachhuber)**: Modular curves and modular forms.** The first talk will introduce level structures and modular curves over $\mathbb{Q}$ together with their analytic counterparts over $\mathbb{C}$. The spaces $\mathcal{V}$ and $\mathcal{V}_\ell$ will be defined in this talk.

- Define generalized elliptic curves [DR73, Déf. I.1.12]. Introduce $\Gamma_1(N)$-level structures and state representability of the moduli functors, e.g. following [DR73]. More explicitely: a point of exact order N, also called a $\Gamma_1(N)$-level structure, on a generalized elliptic curve $E \longrightarrow S$ is a monomorphism

$$\underline{\mathbb{Z}/N\mathbb{Z}}_{/S} \hookrightarrow E[N]$$

  such that the image meets each irreducible component in each geometric fiber. The functor that associates to a $\mathbb{Z}[1/N]$-scheme S the set of isomorphism classes of pairs (E,ϕ), where E is a generalized elliptic curve over S and ϕ is a point of exact order N on E, is representable by a proper smooth curve $X_1(N)$ over $\mathbb{Z}[1/N]$.[4]

[4] A representing object is defined as a Deligne-Mumford stack in [DR73, Déf. IV.3.3, Constr. IV.4.14] (putting $H = \Gamma_{00}(N)$ in their notation). That it is in fact a scheme follows from the discussion in [DR73, §VI.2.1] together with [KM85, Cor. 2.7.3] resp. [Con09, Lem. 4.2.1.1] and the fact that the only automorphism of a Néron polygon fixing a chosen point of exact order N is the identity. This last statement is easy to see using the description of automorphisms of Néron polygons in [DR73, §II.1.9].

- Define $Y_1(N)$ as the open subset of $X_1(N)$ parametrizing smooth curves with level structure.
- Also state that the analytifications $Y_1(N)^{\mathrm{an}}$ and $X_1(N)^{\mathrm{an}}$ represent the analogous moduli functors for analytic spaces over $\mathbb{C}$ (see [Con09, Thm. 4.2.6.2]) and that they can be realized as quotients of the upper half plane $\mathbb{H}$ resp. the extended upper half plane $\mathbb{H}^* = \mathbb{H} \cup \mathbb{P}^1(\mathbb{Q})$ by the group $\Gamma_1(N)$ ([Con09, Thm. 2.2.2.1]; note that there $X_1(N)$ is defined as $\mathbb{H}^*/\Gamma_1(N)$).
- Define $\omega = f_*\Omega^1_{E/X_1(N)^{\mathrm{an}}}$ for $f\colon E \longrightarrow X_1(N)^{\mathrm{an}}$ being the universal generalized elliptic curve over $X_1(N)^{\mathrm{an}}$ and define the space of cusp forms $S_k(\Gamma_1(N))$ as

$$\mathrm{H}^0(X_1(N)^{\mathrm{an}}, \omega^{\otimes(k-2)} \otimes \Omega^1_{X_1(N)^{\mathrm{an}}}).$$

  State that it recovers the classical definition as functions on the upper half plane if we view $X_1(N)^{\mathrm{an}}$ as a quotient of the upper half plane (this is proved in [Con09, Lem. 1.5.7.2, Thm. 1.5.7.1], but need not be presented in detail).
- Define

$$\mathcal{V} = \widetilde{\mathrm{H}}^1(Y_1(N)^{\mathrm{an}}, \operatorname{Sym}^{k-2} \mathrm{R}^1 f_*\mathbb{Q}),$$
$$\mathcal{V}_\ell = \widetilde{\mathrm{H}}^1_{\text{ét}}(Y_1(N) \otimes_{\mathbb{Z}[1/N]} \overline{\mathbb{Q}}, \operatorname{Sym}^{k-2} \mathrm{R}^1 f_*\mathbb{Q}_\ell)$$

  and the sheaf $\mathcal{F} = \mathrm{R}^1\widetilde{a}(\operatorname{Sym}^{k-2} \mathrm{R}^1 f_*\mathbb{Q}_\ell)$. Here, we write $\widetilde{\mathrm{H}}$ for the image of compactly supported cohomology in usual cohomology. Similarly, by $\mathrm{R}^1\widetilde{a}$ we mean the image of $\mathrm{R}^1 a_!$ in $\mathrm{R}^1 a_*$. Explain why $\mathcal{F}$ is a local system on $\operatorname{Spec} \mathbb{Z}[1/N\ell]$ unramified outside $N\ell$ with generic fiber $\mathcal{V}_\ell$ and why $\mathcal{V} \otimes_{\mathbb{Q}} \mathbb{Q}_\ell \cong \mathcal{V}_\ell$ [Con09, Thm. 5.2.8.1, §5.2.9].

Since this talk contains mainly definitions and statements, the speaker is encouraged to present some more details or even full proofs of whatever he likes.

Talk 2 (Konrad Fischer)**: Hecke operators and eigenforms.** The important action of the Hecke and diamond operators which has not been further described in the outline above will be presented in this talk. Both also have classical definitions as operators acting on modular forms.

- State the representability of the $Y_1(N, p)$ moduli functor [Con09, Thm. 4.2.7.1].
- Define the endomorphisms T_p and $\langle p \rangle$ of the sheaf $\mathcal{F}$ and the vector space $\mathcal{V}$ from talk 1. Follow [Con09, §2.3.1], where the definition is given for $\mathcal{V}$. The definition for $\mathcal{F}$ is similar (see [Del71, (4.5)], where it is given for full level structures). Also define T_n for general n by the relations in [Con09, top of p. 146].
- Define the Hecke algebra $\mathcal{T}$ away from N as a non-commutative polynomial ring over $\mathbb{Q}$ generated by symbols T_p, $p \nmid N$, and $\langle n \rangle$, $n \in (\mathbb{Z}/N\mathbb{Z})^\times$.

- Present the classical definition of Hecke and diamond operators [DS05, §5.2]. Define eigenforms as simultaneous eigenvectors for all T_p and $\langle p \rangle$ for all primes p, and define the nebentype of an eigenform as the character of $(\mathbb{Z}/N\mathbb{Z})^\times$ giving the eigenvalues of the $\langle d \rangle$ for $d \in (\mathbb{Z}/N\mathbb{Z})^\times$.
- Introduce the Petersson scalar product and say how the adjoints of the Hecke and diamond operators T_p and $\langle p \rangle$ for $p \nmid N$ with respect to this scalar product look like [DS05, §§5.4–5.5], additionally [Miy89, Thm. 4.5.4 (1)].

Talk 3 (Stephan Neupert): **Analytic theory and the Eichler-Shimura isomorphism.** The Eichler-Shimura isomorphism introduced in this talk provides a comparison of the geometric and analytic definitions of the last talk. Moreover, the structure of the space of modular forms under the Hecke algebra will partially be presented in this talk.

- Define the Eichler-Shimura map following [Con09, §1.7] (with $\Gamma = \Gamma_1(N)$). You may start with the case $k = 2$ (which resembles Hodge theory) as a motivation for the general case.
- State the Eichler-Shimura isomorphism [Con09, Thm. 1.7.1.1] and its Hecke equivariance [Con09, Thm. 2.3.2.1].
- Define $T_1(N) \subset \mathrm{End}_{\mathbb{Q}}(\mathcal{V})$ as the image of $\mathcal{T}$ under the action map $\mathcal{T} \longrightarrow \mathrm{End}_{\mathbb{Q}}(\mathcal{V})$.[5] State that $T_1(N)$ is commutative [Con09, Thm. 2.3.4.1].
- Recall the Fourier expansion of modular forms. Mention that $a_1(T_n f) = a_n(f)$, where $a_n(\cdot)$ means the n-th Fourier coefficient of a modular form (this follows from [Shi71, (3.5.12)]). Deduce that the pairing $S_k(\Gamma_1(N)) \times (T_1(N) \otimes_{\mathbb{Q}} \mathbb{C}) \longrightarrow \mathbb{C}$ defined during the proof of [Con09, Lem. 2.4.1.1] is perfect. Describe the bijection between algebra homomorphisms $T_1(N) \longrightarrow \mathbb{C}$ and eigenforms (away from N) in $S_k(\Gamma_1(N))$ [Con09, p. 173]. Conclude that K_f is a quotient of $T_1(N)$.

Talk 4 (Andreas Mihatsch): **The congruence relation.** This talk will prove the congruence relation

$$T_p = F_p + \langle p \rangle p^{k-1} F_p^{-1}.$$

The proof will use an explicit description of the reduction of the modular curve $Y_1(N,p)$ at p.

- Recall the structure of the p-torsion in elliptic curves [DR73, §I.6].
- Describe the geometry of the reduction $Y_1(N,p) \otimes_{\mathbb{Z}[1/N]} \mathbb{F}_p$ [Con09, Thm. 4.2.8.1].
- Describe shortly the action of Frobenius on $\mathrm{R}^1\widetilde{a}(\mathrm{Sym}^k(\mathrm{R}^1 f_{N*}(\mathbb{Q}_\ell)))$ [Con09, Lem. 5.3.2.2].

[5] So $T_1(N) = \mathbf{T}_1(N) \otimes_{\mathbb{Z}} \mathbb{Q}$ with $\mathbf{T}_1(N)$ defined as in [Con09, §2.3.4].

- Prove the congruence relation $T_p = F_p + \langle p \rangle p^{k-1} F_p^{-1}$ following [Con09, Thm. 5.3.3.1] and [Del71, Lem. 4.6]. Consulting [Del71, Prop. 4.8] might be helpful although Deligne discusses $\Gamma(N)$-level structures instead of $\Gamma_1(N)$-level structures.

The remaining time (if any) can be used for sketching the proof of [Con09, Thm. 4.2.8.1] at least for the ordinary locus. It can be enlightening to present Deligne's proof [Del71, §4.3] instead although it considers $\Gamma(N)$-level structures.

Talk 5 (Lars Kühne): **Properties of $\mathcal{V}_{f,\lambda}$.** The last talk finishes the proof of our main theorem by establishing the desired properties of $\mathcal{V}_{f,\lambda}$. The final ingredient, a pairing $[\cdot,\cdot]_\ell \colon \mathcal{V}_\ell \otimes \mathcal{V}_\ell \longrightarrow \mathbb{Q}_\ell$, will be defined.

- Define the pairing $(\cdot,\cdot)\colon \mathcal{V} \times \mathcal{V} \longrightarrow \mathbb{Q}(-k+1)$ and its ℓ-adic counterpart $(\cdot,\cdot)_\ell \colon \mathcal{V}_\ell \times \mathcal{V}_\ell \longrightarrow \mathbb{Q}_\ell(-k+1)$ [Del71, (3.20)], [Con09, §5.2.11, (2.3.3.4)]. Explain shortly why $(\cdot,\cdot) \otimes_\mathbb{Q} \mathbb{Q}_\ell = (\cdot,\cdot)_\ell$. State how $(\cdot,\cdot) \otimes_\mathbb{Q} \mathbb{C}$ is related to the Petersson scalar product [Con09, Thm. 2.3.3.1].
- Introduce the operator w_ζ in the classical as well as in the geometric setting [Con09, §2.3.6]. State that they agree under the Eichler-Shimura isomorphism [Con09, Lem. 2.3.7.1].
- Define the pairing $[\cdot,\cdot] = (\cdot, w_\zeta \cdot)$ and prove that the Hecke and diamond operators are self-adjoint with respect to this pairing [Con09, Thm. 2.3.7.3] and hence also with respect to $[\cdot,\cdot]_\ell$. The self-adjointness follows from the calculation of the adjoints of the Hecke operators with respect to the Petersson scalar product from talk 2, the easy relation $w_\zeta \langle n \rangle = \langle n \rangle^{-1} w_\zeta$ and [Miy89, Thm. 4.5.5].
- Prove [Con09, Lem. 2.4.1.1] saying that $T_1(N)$ is Gorenstein and $\mathcal{V}$ is free of rank 2 over $T_1(N)$. Note that in talk 3 we already saw the duality between $S_k(\Gamma_1(N))$ and $T_1(N) \otimes_\mathbb{Q} \mathbb{C}$.
- Prove that $w_\zeta^{-1} F_p w_\zeta = \langle p \rangle F_p$ [Con09, Thm. 5.3.3.1]. You need [Con09, Thm. 4.2.8.4] for this. It follows that F_p and $\langle p \rangle p^{k-1} F_p^{-1}$ are adjoint with respect to $[\cdot,\cdot]_\ell$.

Now present the final argument. Unfortunately, its description in [Con09, Thm. 4.1.3.1 resp. §3.3] deals only with the weight 2 case in detail while [Del71, proof of Thm. 5.6] takes care only of the case $N = 1$ where w_ζ and $\langle p \rangle$ act trivially. Therefore we list the necessary arguments:

- $\det(1 - F_p X \mid \mathcal{V}_\ell) = \det(1 - \langle p \rangle p^{k-1} F_p^{-1} X \mid \mathcal{V}_\ell)$ as polynomials with coefficients in $T_1(N) \otimes_\mathbb{Q} \mathbb{Q}_\ell$, viewing $\mathcal{V}_\ell$ as a $T_1(N) \otimes_\mathbb{Q} \mathbb{Q}_\ell$-module. You need [Con09, Lem. 3.3.2.1] and your previous results about $[\cdot,\cdot]_\ell$ for this.
- $\det(1 - T_p X + p^{k-1} \langle p \rangle X^2 \mid \mathcal{V}_\ell) = \det(1 - T_p X + p^{k-1} \langle p \rangle X^2 \mid S_k(\Gamma_1(N)))^2$ (because of the Eichler-Shimura isomorphism)

- $\det(1 - F_p X \mid \mathcal{V}_\ell)^2 = \det(1 - T_p X + p^{k-1} \langle p \rangle X^2 \mid S_k(\Gamma_1(N)))^2$ (after the congruence relation from talk 4 and the previous steps)
- $\det(1 - F_p X \mid \mathcal{V}_{f,\lambda}) = 1 - a_p X + p^{k-1} \chi(p) X^2$ by tensoring with $K_{f,\lambda}$ over $T_1(N) \otimes_{\mathbb{Q}} \mathbb{Q}_\ell$. Recall that $\mathcal{V}_{f,\lambda} = K_{f,\lambda} \otimes_{T_1(N) \otimes_{\mathbb{Q}} \mathbb{Q}_\ell} \mathcal{V}_\ell$.

References

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