Seiberg-Witten Invariants

Contents

1. Week 1: Introduction

Physical Origin and Historical Comments

Donaldson theory = top. twisted $4d$ $\mathcal{N} = 2$ SYM [Wi88]

$4d$ $\mathcal{N} = 2$ $SU(N)$ SYM in IR limit (strong coupling) $\overset{\text{S-duality}}{\longleftrightarrow}$ abelian gauge theory of *monopoles* (weakly coupled) [SW94a, SW94b]

Supersymmetric localization in this dual theory at solutions to the Seiberg-Witten equations [Wi94]; physical arguments suggest that counting solutions gives "topological" invariants, aka *Seiberg-Witten invariants*.

$\Rightarrow$ Witten Conjecture: Donaldson invariants $\simeq$ Seiberg-Witten invariants.

Review eg. in [Le97, Ma99]

2. Week 2-5: Definition of Seiberg-Witten Invariants

Main Reference for this block will be [Mo01]. We assume familiarity with differential geometry, see [Mo01][Ch 1] for a summary of the most relevant notions.

2.1. Week 2

Spin geometry (on four-manifolds)
Assigned Reading:

- [Tau1][Ch.2.1-2.5] (for the first reading)
- [Mo01][Ch 2.1-2.4](for more details)

2.2. Week 3

Spin geometry (on four-manifolds)
Assigned Reading:

- For many Facts on Spin$^{\mathbb{C}}$ structure have a look on [Ma99][Ch. 2.1-2.6]
- *Spin* connection:
  1. [Tau1][Ch.2.6] (for the first reading)
  2. [Spn10][Lecture 6] for more details.
- *Dirac* operator
  1. [Tau1][Ch.2.7] (for the first reading)
  2. [Mo01][2.6](for more details on the Dirac operator)

- Bachelorthesis of Simon for a precise defition of a *Dirac* oparator.

Topics for discussion:

- Why is the operator defined in [Tau1][Ch.2.7 def. 2.13] a *Dirac* operator?

2.3. Week 4

Dirac operators, the Seiberg-Witten equations, moduli space of solutions.
Assigned Reading:

- [Tau1][3.1-3.4]
- [Mo01][Ch 3.3-3.4] More details on moduli space of solutions; proof ideas.
- [Ma99][Ch.4.1] for more Details of the moduli space of solutions
- [Ma99][Ch. 3.1-3.3] for a background on the *Seiberg-Witten* functional. Solutions of the *Seiberg-Witten* equations are critical points of the functional. And the gauge group of this "field theory".

2.4. Week 5

The definition of Seiberg-Witten invariants.
Assigned Reading:

- [Tau1][3.4]from the of the proof to the begin of 3.5.
- [Tau1][3.5-3.6] for the definition of the Seiberg-Witten invariants and properties of the Seiber-Witten invariants.
- Discussion of [Mo01][Ch 3.7, Thm] and [Tau1][Ch.4.1]

3. Week 6-8: Applications in Symplectic Geometry

3.1. Week 6

canonical spin^C structure, statement of Theorem 4.1, reexpressing the SW-eqns
Assigned Reading: [Tau1][Chapter 4.1-4.3]

3.2. Week 7

Weitzenböck Formulas, Estimates, Uniqueness
Assigned Reading: [Tau1][Chapter 4.3-4.6]

3.3. Week 8

Relation to Pseudoholomorphic Curves, Gromov-Invariants
Assigned Reading:[Tau1][Chapter 5]

4. Week 9 (25 June): Donaldson theory and the Witten conjecture

Donaldson Invariants, Gromov-Invariants, Witten's Conjecture Assigned Reading: [Sal99][Chapter 7.5, page 262-264]

5. Week 10 - 12: Physical Origin of Witten's Conjecture

5.1. Week 10 (16 July)

Overview, SUSY review, Low-energy effective action of $\mathcal{N} = 2$ SUSY SU(2) Yang-Mills theory, S-Duality
Assigned Reading: [Bilal96][sec. 1-4]

5.2. Week 11 (23 July)

Monodromies, Seiberg-Witten's Solution of SYM
Assigned Reading: [Bilal96][sec. 5-6]

5.3. Week 12 (30 July)

From Instantons to Monopoles
Assigned Reading: ?

6. Other Directions and Further Reading

6.1. Generalizations of SW-invariants

6.1.1. Families and Wall-Crossing

6.1.2. Bauer-Furuta Invariants

these are strictly stronger invariants than Seiberg-Witten [Bau1]

6.2. Monopole Floer-Theory

Main reference: [KM07] ($\simeq$ 4 Weeks) Review: [Ma99]

6.3. SW/Monopole-Floer (HM) = Heegard-Floer (HF)

The main statement of the five papers are the isomorphism beetween Seiberg-Witten-Floer-Theory and Heegard-Floer-Theory.See for reference [Kut1]. So first we have to define both Floer-Theorys xD

References

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Inventiones mathematicae 2004 https://doi.org/10.1007/s00222-003-0288-5

[Tau1] Michael Hutchings and Clifford Henry Taubes: An introduction to the Seiberg-Witten equations on symplectic manifolds
https://math.berkeley.edu/~hutching/pub/tn.pdf

[Kut1] Cagatay Kuthluhan, Yi-Jen Lee, and Clifford Taubes: Heegaard Floer Homology and Seiberg–Witten Floer Homology
https://arxiv.org/pdf/1007.1979.pdf

[Mo01] John Douglas Moore: Lecture Notes on Seiberg-Witten Invariants
http://web.math.ucsb.edu/~moore/seibergwittenrev2edition.pdf

[KM07] Kronheimer, Mrowka: Monopoles and Three-Manifolds
Cambridge University Press, 2007

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[Wi88] E. Witten, "Topological quantum field theory,"
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[SW94a] N. Seiberg and E. Witten, "Electric-Magnetic Duality, Monopole Condensation, And Confinement In N=2 Supersymmetric Yang-Mills Theory,"
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[SW94b] N. Seiberg and E. Witten, "Monopoles, duality and chiral symmetry breaking in N=2 supersymmetric QCD,"
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[SW96] N. Seiberg and E. Witten, "Gauge Dynamics And Compactification To Three Dimensions,"

[Le97] W. Lerche, "Introduction to Seiberg-Witten Theory and its Stringy Origin,"
Fortschritte Phys Phys, vol. 45, no. 3–4, pp. 293–340, 1997.

[Spn10] José Figueroa-O'Farrill:G course on Spin Geometry https://empg.maths.ed.ac.uk/Activities/Spin/

[Sal99] Dietmar Salamon:SPIN GEOMETRY AND SEIBERG-WITTEN INVARIANTS
https://people.math.ethz.ch/~salamon/PREPRINTS/witsei.pdf

[Bilal96] Adel Bilal
DUALITY IN N=2 SUSY SU(2) YANG-MILLS THEORY:A pedagogical introduction to the work of Seiberg and Witten
https://arxiv.org/pdf/hep-th/9601007.pdf