

Geometric Structures and Representation Varieties

19 - 22 February 2018

We will bring together more experienced and young researchers who are interested in the deformation theory of geometric structures on manifolds and representation varieties. The schedule will leave ample room for discussion and scientific interaction.

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Monday, 19 February 2018

09:30 – 10:30 Volume variation of representations of three-manifolds in $SL(N, \mathbb{C})$

Joan Porti (Universitat Autònoma de Barcelona)

Given an orientable, hyperbolic three-manifold of finite volume, several authors have considered the volume of a representation of its fundamental group in $SL(N, \mathbb{C})$. In this talk I will define the volume of a representation by using the symmetric space $SL(N, \mathbb{C})/SU(N)$ and the Cartan-Killing form on the Lie algebra. I plan to overview its main properties and my goal is to talk about a differential formula for the variation of the volume, along variations of the representation. This is joint work with Wolfgang Pitsch.

10:30 – 11:00 Coffee

11:00 – 12:00 Anosov representations and eigenvalue gaps

Fanny Kassel (Institut des Hautes Études Scientifiques)

Anosov representations into $SL(n, \mathbb{R})$ are representations of word hyperbolic groups which play an important role in higher Teichmüller theory. Kapovich-Leeb-Porti characterized them in terms of singular value gaps of matrices. I will explain that a similar characterization holds in terms of eigenvalue gaps. This is joint work with Rafael Potrie.

12:00 – 14:30 Lunch

14:30 – 15:30 Non-arithmetic lattices and hypergeometric monodromy

John Parker (Durham University)

Classical work HA Schwarz says that any triangle group is the monodromy group of a hypergeometric differential equation. Deligne and Mostow gave a list of all lattices that are monodromy groups of hypergeometric equations in several variables. An important consequence of their work was the construction on non-arithmetic lattices in $SU(2,1)$ and $SU(3,1)$. For many years, all examples of such non-arithmetic lattices were on the Deligne-Mostow list. More recently, Deraux, Paupert and I gave new examples of non-arithmetic lattices in $SU(2,1)$. In this talk I will show that all currently known non-arithmetic lattices in $SU(2,1)$ are (commensurable to) monodromy groups of higher order hypergeometric equations in one variable.

15:30 – 16:00 Coffee

16:00 – 17:00 On the equivalence between Q-conditions and primitive stability

Binbin Xu (Université du Luxembourg)

The group $\text{Out}(F_2)$ of outer-automorphisms of F_2 the rank 2 free group acts naturally on the $\text{PSL}(2, \mathbb{C})$ -character variety of F_2 . To study the dynamical property of $\text{Out}(F_2)$ -action, Bowditch's Q-condition and the primitive stable condition on a representation from F_2 to $\text{PSL}(2, \mathbb{C})$ have been introduced by Bowditch (generalized by Tan-Wong-Zhang) and by Minsky, respectively. Each one of them can characterize an open subset of the character variety on which $\text{Out}(F_2)$ acts properly discontinuously. These two open sets are both candidates for the maximal domain of discontinuity for the $\text{Out}(F_2)$ -action. In a joint work with Jaejeong Lee, we show that these two conditions are equivalent to each other.

Tuesday, 20 February 2018

09:30 – 10:30 Metrics induced on convex hulls of quasicircles

Jean-Marc Schlenker (Université du Luxembourg)

We consider the "universal" version of results and conjectures concerning the induced metrics on the boundary of the convex cores of quasifuchsian manifolds, as well as constant curvature surfaces in those manifolds. Given a quasisymmetric homeomorphism u of the circle, there is a quasicircle C in $\mathbb{C}P^1$ (resp. $\mathbb{R}P^1 \times \mathbb{R}P^1$) such that the gluing map between the two connected components of the boundary of the convex hull of C in H^3 (resp. AdS^3) is determined by u . C is conjectured to be unique. Similar results apply to the induced metrics and third fundamental forms on the boundary of convex domains in H^3 (resp. AdS^3) with constant curvature and asymptotic boundary a quasicircle. Joint work with Francesco Bonsante, Jeff Danciger and Sara Maloni.

10:30 – 11:00 Coffee

11:00 – 12:00 Tau function and flat connections

Vladimir Fock (Université de Strasbourg)

Tau function of Sato is a tool mainly used in the theory integrable systems to describe solutions in terms of certain determinants. It is usually a subject of very technical studies and not widely known by non-specialists. The aim of the talk is to give an alternative (and so far conjectural) definition of a tau-function as a certain limiting case of a generating function for a Lagrangian subvariety of the space of connections on a Riemann surface.

12:00 – 14:30 Lunch

14:30 – 15:30 Hilbert distance, beyond convexity

Antonin Guilloux (Université Paris VI)

With a mild reformulation of Hilbert distance on convex projective sets, I will explain how to define a similar object in other settings. It may open a path to new applications. I will present this reformulation, as well as first applications in dimension 1, real or complex. This is a joint work with E. Falbel and P. Will.

15:30 – 16:00 Coffee

16:00 – 17:00 The co-Minkowski space and an asymmetric norm on the Teichmüller space

Thierry Barbot (Université d'Avignon)

W.P. Thurston has defined two asymmetric norms on the Teichmüller space. The most famous is the one related to the minimization problem of the Lipschitz constant of a map between two hyperbolic surfaces. In the same paper, he also defined another one, dual to the first one in some meaning, which is defined as the length of measured geodesic laminations, once identified in the correct way tangent vectors with measured geodesic laminations. I will show that this construction can be generalized to some asymmetric Finsler norm on $H^1(G, \mathbb{R}^{1,n})$ where G is a cocompact lattice of $SO(1, n)$. I will also comment on a useful tool related to this: the co-Minkowski space, i.e. the space of spacelike hyperplanes in the Minkowski space, and the fact that the involved representations of G are Anosov relatively to the boundary of the co-Minkowski space. This is a work in collaboration with F. Fillastre.

Wednesday, 21 February 2018

09:30 – 10:30 Teeny-weeny representations and subalgebras containing regular elements

Olivier Guichard (Université de Strasbourg)

Fixed points of the \mathbb{C}^* -action on the space of Higgs bundles correspond to representations whose Zariski closures contain a specific regular element. This situation leads to the objects mentioned in the title, and we give a classification of them and show in turn a rigidity phenomenon. In particular, this applies to Brian Collier's representations into split orthogonal groups. Joint with Clément Guérin.

10:30 – 11:00 Coffee

11:00 – 12:00 Liouville current, intersection and pressure metric revisited

Andrés Sambarino (Université Paris VI)

The purpose of the talk is to explain a construction of a Liouville current for a representation in the Hitchin component. Bonahon's intersection between two such currents can be computed. We will also explain the relation with a pressure metric. This is joint work with D. Canary, M. Bridgeman and F. Labourie.

12:00 – 13:00 Strip coordinates for hyperbolic surfaces with spikes

François Guéritaud (Université Lille 1)

This will be a report on work of my student Pallavi Panda concerning the deformation space of complete finite-area hyperbolic surfaces with geodesic, possibly noncompact boundary. The deformations that lengthen all measured laminations are naturally parametrized, via a so-called strip map, by an appropriate arc complex, generalizing the compact-boundary case.

13:00 – Free afternoon

Thursday, 22 February 2018

09:30 – 10:30 Limit sets of Anosov representations

Daniel Monclair (Université Paris-Sud)

Given a real semi-simple Lie group G and a parabolic subgroup P (i.e. G/P is compact), P -Anosov representations in G are characterized by their "nice" limit sets in G/P . Depending on the pair (G,P) , these limit sets can be very different. In rank one (quasi-Fuchsian groups), they are fractal objects. For some examples in higher rank (such as Hitchin representations), they are more regular, and can even be C^1 submanifolds of G/P . We will describe the case of some Anosov representations in $SO(n,2)$, called quasi-Fuchsian, which are holonomies of some special Lorentzian manifolds. We will see that the limit sets are Lipschitz submanifolds of G/P , but not C^1 submanifolds. We will also discuss "how fractal" these objects are by looking at a modified version of the Hausdorff dimension. This is joint work with Olivier Glorieux.

10:30 – 11:00

Coffee

11:00 – 12:00

Lorentzian Quasi-Fuchsian group which are not lattice

Ludovic Marquis (Université de Rennes 1)

A (hyperbolic) $(d + 1)$ -quasi-fuchsian group is a convex cocompact subgroup of isometries of the real hyperbolic space of dimension $d + 1$ whose limit set is a sphere of dimension $d - 1$. A Lorentzian Quasi-Fuchsian group is the analogue of the former group but for isometries of the Anti de Sitter space, where the expression convex-cocompact should be understood as defined by Danciger-Guéritaуд-Kassel, or equivalently as a subgroup that preserves an acausal sphere in the boundary of the Anti de Sitter space and acts properly and cocompactly on the convex hull of this sphere in the Anti de Sitter space. We will build examples of Lorentzian quasi-Fuchsian groups which are not lattice in small dimension $d = 4, 5, 6$, hence in particular not obtained by deforming fuchsian subgroups. This is a joint work with Gye-Seon Lee.

12:00 – 14:30

Lunch

14:30 – 15:30

Mapping Class Group actions on some relative character varieties

Frederic Palesi (Aix-Marseille Université)

The Teichmüller space of a surface (possibly with boundaries) can be embedded in the relative $\mathrm{PGL}(2, \mathbb{R})$ character variety as a connected component. The mapping class group acts properly discontinuously on that connected component, but the dynamic of the action is much more mysterious on the rest of the character variety. In this talk we construct open domains of discontinuity for the mapping class group action on relative $\mathrm{SL}(2, \mathbb{C})$ character varieties of some surfaces of low complexity, using methods introduced by Bowditch in the one-punctured torus case. We apply these methods to get informations on the dynamical decomposition of the relative $\mathrm{PGL}(2, \mathbb{R})$ character varieties, in particular to construct domains where the action is ergodic.

15:30 – 16:00

Coffee

16:00 – 17:00

Similarity Manifolds after Mickaël Kourganoff

Ghani Zeghib (Ecole Normale Supérieure de Lyon)

A similarity structure on a manifold consists in giving a Riemannian metric on a neighborhood of each point such that the metrics on the intersection of two neighborhoods are homothetic, i.e. proportional by means of a locally constant function. The basic example is that of the cone over a Riemannian manifold which gives in the case of the standard sphere a (flat) Hopf manifold. The question is whether there exist more complicated examples?