RUPRECHT-KARLS-UNIVERSITÄT HEIDELBERG

Mathematisches Institut

Vorlesung Differentialgeometrie II
Heidelberg, 10.1.2017

Exercise sheet 8

Happy New Year!

Not to be handed in, will be discussed on January 18th

Exercise 1.

(a) Let N be a surface of genus 2 and let $M = N \times S^1$ be the product manifold. Show that M does not carry a metric of negative curvature.

(b) Let $T^m = S^1 \times \ldots \times S^1$ be the m-torus with $m \geq 2$. Show that T^m does not carry a metric of negative curvature.

Exercise 2. Let M be a compact Riemannian manifold with negative sectional curvature $K < 0$. Show that the fundamental group $\pi_1(M)$ is not abelian.
Hint: Let $\widetilde{M}$ be the universal cover of M and $\mathrm{Deck}(\widetilde{M})$ the group of deck transformations. Show that if M is a complete Riemannian manifold with $K \leq 0$ and there exists a geodesic invariant under the elements of $\mathrm{Deck}(\widetilde{M})$, then M is not compact.

Exercise 3. In this exercise we want to prove a refinement of Preissman's Theorem. We need a definition first:

Definition. A group G is called *solvable* if there is a sequence of subgroups

$$\{1\} = G_0 < G_1 < \ldots < G_{k-1} < G_k = G$$

such that

- $G_{j-1} \trianglelefteq G_j$ is normal
- G_j / G_{j-1} is abelian

for all $j = 1, \ldots, k$.

Let M be a compact Riemannian manifold with $K < 0$ and let H be a non-trivial solvable subgroup of $\pi_1(M)$. Show that

(a) H is infinite cyclic,

(b) $\pi_1(M)$ does not have a cyclic subgroup of finite index.