RUPRECHT-KARLS-UNIVERSITÄT HEIDELBERG

Mathematisches Institut

Vorlesung Differentialgeometrie II

Heidelberg, 22.11.2016

Exercise sheet 5 - short

Submanifolds and constant curvature

Not to be handed in, will be discussed on November 30th

Exercise 1. Suppose M, N are smooth manifolds and $f\colon M \to N$ a smooth map. Prove that the *graph*

$$\Gamma(f) = \{(x, y) \in M \times N \mid y = f(x)\} \subset M \times N$$

is a closed submanifold of $M \times N$.

Exercise 2. Let $p, q \geq 1$ be integers.

a) Do the manifolds $S^q \times S^p$ admit constant curvature metrics?
Hint/Remark: To prove that some spaces are not diffeomorphic (or even homeomorphic), you can use results from algebraic topology.

b) Do the manifolds $\mathbb{C}P^q$ admit constant curvature metrics?
Remark: You might need to use some slightly more advanced algebraic topology for this exercise.

Exercise 3. Let M be a Riemannian manifold and $\Gamma \subset \mathrm{Isom}(M)$ a set of isometries. Prove that every connected component of the fixed point set

$$\mathrm{Fix}(\Gamma) = \{x \in M \mid \phi(x) = x \ \forall \phi \in \Gamma\}$$

is a totally geodesic submanifold.