Computing generalized Seifert matrices for closures of colored braids

Seifert matrices have been a foundational tool in knot theory ever since they were introduced in the 1930's. They are not link invariants – in fact, the definition of a Seifert matrix for a link L depends heavily on the choice of a Seifert surface S for L and of a $\mathbb{Z}$-basis of $\mathrm{H}_1(S)$. Nevertheless, Seifert matrices provide a flexible method for computing important link invariants, most notably the Alexander polynomial. In 2006, Collins described a convenient algorithm for producing a Seifert matrix for a link given as the closure of a braid (and it is well-known that every link is isotopic to a braid closure). This algorithm has been implemented as a computer program shortly thereafter.

This story has a generalization to μ-colored links, that is, links whose components come with a partition into μ "colors" (where $\mu \in \mathbb{N}_{\geq 1}$). The role of the Seifert surface is now played by a "clasp-complex", which is a collection of μ Seifert surfaces with certain restrictions on how they intersect. In this setting, the Alexander polynomial has μ variables, and for computing it one needs a family of 2^μ "generalized Seifert matrices". In my talk, I will introduce these notions and describe an algorithm for computing a family of generalized Seifert matrices for a colored link given as the closure of a colored braid. This algorithm was devised in joint work with Stefan Friedl and Chinmaya Kausik. If time permits, I will also showcase a computer implementation of our algorithm due to Kausik.