An overview of Sigma-invariants

The features of a group being finitely generated or finitely presented are well-known to be, respectively, the $n = 1$ and $n = 2$ cases of the property F_n: a group G is said to be of type F_n if it admits a $\mathrm{K}(G, 1)$ with finite n-skeleton. Towards the end of the last century, the question of when such finiteness conditions descend to subgroups of G led to the discovery of the homotopical Sigma-invariants $\Sigma^n(G)$ and their homological counterparts $\Sigma^n(G; A)$ (for A a $\mathbb{Z}G$-module). Intuitively, $\Sigma^n(G)$ can be thought of as the set of group homomorphims $\chi\colon G \to \mathbb{R}$ for which G is "of type F_n in the direction of χ", with the classical property F_n being equivalent to $0 \in \Sigma^n(G)$.

In my talk I will give an introduction to the theory of Sigma-invariants, with particular focus on Σ^1, mentioning some of their applications in group theory and topology. If time permits I will also say a few words on the more recent efforts to extend these notions to locally compact groups.