Higher Teichmüller theory via Higgs bundles

This seminar is intended to give an overview on recent developments in higher Teichmüller theory using Higgs bundles. The moduli space of $G$-Higgs bundles on a surface is homeomorphic to the $G$-character variety by the celebrated Non-Abelian-Hodge Theory. This allows the study of topological properties of character varieties via Higgs bundles. In the recent years, this was used to establish the classification of components of geometrically rich representations, so-called higher Teichmüller components.

Location INF 205, Seminarraum 4   Time Friday, 9:15-11:45.
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This collection of topics is thought as a guide through the seminar. Not all talks in this list are necessary. Concerning the individual talks the bullet points should be rather understood as an inspiration than strict guidelines.

1. Introduction to Holomorphic Vector Bundles (Arnaud)
   - topological $\mathbb{C}$-vector bundles on a surface, classification by degree
   - holom. vector bundles, stability, the moduli space of holomorphic vector bundles

1 talk, references: [Sch15], [NS65], [Don83].

2. Introduction to Higgs bundles and Non-Abelian Hodge (Levin)
   - $\text{GL}(n, \mathbb{C})$-Higgs bundles, stability, moduli space.
Abelian Hodge theory $G = \text{GL}(1, \mathbb{C})$.

NAH: Hitchin equation, from Higgs bundles to flat connections, harmonic maps to symmetric spaces, from flat connections to Higgs bundles.

The uniformizing $\text{SL}(2, \mathbb{C})$-Higgs bundle

2 talks, references: [GX08], [Wen16], [Hit87].

3. Higgs for real Lie groups (Jonas)
   
   - Holomorphic principal $G$-bundles, vector bundles associated by representations
   
   - Definition of $G$-Higgs bundle, Examples, associated bundles under standard representation of matrix Lie groups
   
   - stability conditions, Hitchin equation.

1 talk, references: [Col19], [GGR09].

4. Lie theory of $\text{SL}(2, \mathbb{C})$ embeddings (Oscar)
   
   - representation theory of $\text{SL}(2, \mathbb{C})$
   
   - Jacobson-Morozov theorem

0-1 talk, references: [FH91], [Kna02], [Col19].

5. The Hitchin component - the first higher Teichmüller component
   
   - Hitchin map, properties
   
   - split real forms, principal embedding
   
   - constructing section, reality

1 talk, references: [Hit92], [Col19].

6. Geometric Properties of Hitchin representations
   
   - Anosov property
7. Higgs bundles for hermitian Lie groups
   · Examples $G = \text{Sp}(2n, \mathbb{R}), \text{SU}(p, q)$, Toledo number, Milnor wood inequality
   · Morse theory, component count
   · Cayley correspondence
2 talks, references: [BGG06], [BGR17].

8. Geometric Properties of maximal representations
   · Anosov property
   · discrete, faithful
   · positivity
1 talk, references: [BIW14] [Poz].

9. Higher Teichmüller components in $\text{SO}(p, q)$-character varieties
   · topological invariants, $\mathbb{C}^*$-fixed points
   · local minima of Hitchin function, exotic components
1 talk, references: [Col19], [Apa+19].

10. Positive Anosov representations
   · notion of positivity, examples: Hitchin representations, maximal representations
   · Classification, conjecture: closeness property
1 talk, references: [GW18].

11. Magical nilpotents and higher Teichmüller components
notion of magical nilpotent, Classification

Cayley correspondence

References


[Poz] Beatrice Pozzetti. “Notes on Maximal Representations”. In: ().
