Workshop

"Iwasawa Theory and Related Topics"

May 14–18, 2018 • University of Heidelberg

Titles and abstracts

D. Benois (Université de Bordeaux) – On p-adic height parings

In this talk we will discuss different constructions of the p-adic height pairing for families of p-adic representations using the theory of (φ,Γ)-modules. We prove the compatibility of our approach with the p-adic height pairing constructed by Nekovar in the semistable case.

B. Cais (University of Arizona) – Growth of p-parts of motivic class groups in $\mathbb{Z}_p$-towers of function fields

Let $\{X_n\}$ be a $\mathbb{Z}_p$-tower of smooth projective curves over a perfect field k of characteristic p that totally ramifies over a finite, nonempty set of points of X_0 and is unramified elsewhere. In analogy with the case of number fields, Mazur and Wiles studied the growth of the p-parts of the class groups $\mathrm{Jac}(X_n)[p^\infty](\bar{k})$ as n varies, and proved that these naturally fit together to yield a module that is finite and free over the Iwasawa algebra. We introduce a novel perspective by proposing to study growth of the full p-divisible group $G_n := \mathrm{Jac}(X_n)[p^\infty]$, which may be thought of as the p-primary part of the *motivic class group* $\mathrm{Jac}(X_n)$. One has a canonical decomposition $G_n = G_n^{\mathrm{et}} \times G_n^{\mathrm{m}} \times G_n^{\mathrm{ll}}$ of G into its etale, multiplicative, and local-local components, as well as an equality $G_n(\bar{k}) = G_n^{\mathrm{et}}(\bar{k})$.

Thus, the work of Mazur and Wiles captures the etale part of G_n, so also (since Jacobians are principally polarized) the multiplicative part: both of these p-divisible subgroups satisfy the expected structural and control theorems in the limit. In contrast, the local-local components G_n^{ll} are far more mysterious (they can not be captured by $\bar{k}$ points), and indeed the tower they form has no analogue in the number field setting. This talk will survey this circle of ideas, and will present new results and conjectures on the behavior of the local-local part of the tower $\{G_n\}$.

M. Fütterer (Universität Heidelberg) – Canonical interpolation formulas for p-adic L-functions of families of modular forms

Fukaya and Kato derived from the ETNC a conjectural interpolation formula for p-adic L-functions of a wide class of motives. We apply this to Hida families, for which a p-adic L-function has been constructed by Kitagawa. Using families of p-adic Eichler-Shimura isomorphisms, we show how Kitagawa's construction can be modified to yield a p-adic

L-function having exactly the conjectured interpolation behaviour. We also indicate a possible generalization to overconvergent families.

H. Hida (University of California, LA) – Galois deformation ring associated to a real quadratic field

For almost all primes split in a real quadratic field, we describe how to determine the isomorphism class of the universal ordinary deformation ring of a 2-dimensional induced representation from the Galois group over the real field. As an application, we can write down the adjoint Selmer group of the universal deformation explicitly.

M.-L. Hsieh (Academia Sinica, Taipei) – Generalized Kato classes and elliptic curves of rank two

Generalized Kato classes are the p-adic Abel-Jacobi image of diagonal cycles in the triple product of modular curves. These classes have been introduced and studied extensively by Darmon and Rotger. Moreover, they also conjecture that one should be able to construct explicit non-trivial elements in the Selmer group of elliptic curves of the Mordell-Weil rank 2 by using generalized Kato classes. In this talk, we will give a criterion for the non-vanishing of generalized Kato classes associated with elliptic curves of rank two and provide numerical examples of non-trivial generalized Kato classes.This is a joint work with Francesc Castella.

M. Kakde (King's College London) – Higher Chern classes in Iwasawa theory

The main conjecture of Iwasawa theory can be phrased as saying that the p-adic L-function is the first Chern class of an Iwasawa module. The first Chern class is concerned with codimension one behaviour of the module. Higher Chern classes are related to higher codimension behaviour of the module. For an imaginary quadratic field in which a prime prime p splits we proved that two Katz p-adic L-functions give the second Chern class of an Iwasawa module constructed from an inverse limit of p-parts of ideal class group of abelian extensions of the imaginary quadratic field. In this talk I will briefly recall this result and then present a generalisation to CM fields. In the more general situation two appropriately chosen Katz p-adic L-functions of a CM field describe the second Chern class of modules constructed from top exterior powers, over Iwasawa algebra, of Galois groups of maximal abelian pro-p extension unramified outside a set of primes above p. In special cases I will describe how these modules can be related to particular subquotients of higher terms in the lower central series of the maximal pro-p extension unramified outside p. This is a joint work with Frauke Bleher, Ted Chinburg, Ralph Greenberg, Romyar Sharifi and Martin Taylor.

M. Kim (Merton College, University of Oxford) – Non-abelian reciprocity, period maps, and Diophantine geometry

This lecture will survey recent developments in the use of fundamental groups, non-abelian cohomology, and non-abelian period isomorphisms to the construction of non-abelian reciprocity maps and their applications to Diophantine geometry.

G. Kings (Universität Regensburg) – Motivic Eisenstein classes for Hilbert modular varieties and their p-adic interpolation

Harder has constructed the Eisenstein cohomology for GL_2 over a totally real field by analytic means and Graf, in his 2015 Regensburg thesis, has given a purely topological construction of this Eisenstein cohomology. In this talk I want to explain that Graf's construction is in fact motivic. Moreover, the images of these motivic Eisenstein classes in étale cohomology admit a p-adic interpolation as in our earlier work for the modular curve, which had applications to the proof of explicit reciprocity laws. Finally, I want to mention work in progress, which identifies the syntomic realization of these Eisenstein classes in terms of Katz Eisenstein measure.

M. Kurihara (Keio University, Yokohama) – Gereralized Stark elements and their integral properties

I will talk on certain elements in cohomology groups, corresponding to the zeta-values, as a generalization of Rubin-Stark elements. I discuss their integral properties and their properties describing the Galois module structure of certain cohomology groups, ideal class groups, and Selmer groups. This is joint work with D. Burns and T. Sano.

A. Lei (Université Laval, Québec) – Towards a rank-two Euler system via Wach modules

Given two modular forms f and g that are non-ordinary at p, Perrin-Riou conjectured the existence of an integral rank-two Euler system for the Rankin-Selberg convolution $f \otimes g$. Using Beilinson-Flach elements, Loeffler and Zerbes constructed four non-integral rank-one Euler systems for $f \otimes g$. In this talk, we will study the theory of Wach modules attached to the p-adic realization T_p of $f \otimes g$. In particular, I will explain how this allows us to convert the rank-one Euler systems of Loeffler and Zerbes into certain elements in the wedge square of the first Galois cohomology of T_p over the p-power cyclotomic extensions. This gives evidence towards the conjecture of Perrin-Riou on the existence of a rank-two Euler system. This is joint work with Büyükboduk, Loeffler and Venkat.

K. Nakamura (Saga University) – Euler system for rank two universal deformation

This is a work in progress. Using a modified version of Fukaya-Kato's zeta elements associated to modular symbols and Emerton's work on complete cohomology of modular

curves, we construct Euler systems for rank two universal deformations, which interpolates Kato's Euler systems for Galois representations associated to Hecke eigen cusp newforms.

B. Palvannan (University of Pennsylvania) – Codimension two cycles in Iwasawa theory and elliptic curves with supersingular reduction

A recent result of Bleher, Chinburg, Greenberg, Kakde, Pappas, Sharifi and Taylor has initiated the topic of higher codimension Iwasawa theory. As a generalization of the classical Iwasawa main conjecture, they prove a relationship between analytic objects (a pair of Katz's 2-variable p-adic L-functions) and algebraic objects (two "everywhere unramified" Iwasawa modules) involving codimension two cycles in an Iwasawa algebra. The talk will discuss an analogous result by considering the restriction to an imaginary quadratic field K of an elliptic curve E, defined over $\mathbb{Q}$, with good supersingular reduction at p. This is joint work with Antonio Lei.

R. Pollack (Boston University) – μ-invariants of modular forms in Hida families

In this talk we study the variation of μ-invariants in Hida families. We show these invariants are unbounded and in some cases directly related to p-adic L-functions of Dirichlet characters. In the famous case of $X_0(11)$ and $p = 5$ which has μ-invariant 1, we give a simple and explicit formula for how this μ-invariant varies in the Hida family attached to this elliptic curve.

C. Popescu (University of California, San Diego) – On a refinement of a conjecture of Gross

I will discuss an integral refinement of a conjecture of Gross on special values of global and p-adic L-functions. Links to recent work of Dasgupta-Kakde-Ventullo on a rational version of the conjecture will be made.

R. Sharifi (University of California, LA) – The augmentation filtration in Iwasawa theory

We will discuss isomorphisms between graded quotients of augmentation filtrations on second Iwasawa cohomology groups and cokernels of certain connecting maps, and settings in which the values of these maps can be computed via Massey products.This is joint work in progress with Josh Lam, Yuan Liu, Preston Wake, and Jiuya Wang that began at this year's Arizona Winter School and which extends long-unpublished work. We will give some motivation for the the study of these objects in the course of the talk.

F. Sprung (Arizona State University) – Recent developments in the two-variable Iwasawa theory for elliptic curves at supersingular primes

We give an overview of some recent results, but concretely focus on the role played by a map from Euler systems to p-adic L-functions, the Coleman maps. In the supersingular case, we need a pair of Coleman maps (unlike in the ordinary case, in which there is a single map), the sharp/flat Coleman maps. We describe two recent works in which our sharp/flat Coleman maps play a crucial role: 1) One is the proof of the (one-variable) main conjecture, which relies on one inclusion of the two-variable main conjecture. An essential ingredient here is the *kernel* of the sharp/flat Coleman map. 2) The other result is joint work in progress with A. Lei, in which we give a first estimate of the Mordell-Weil rank of an elliptic curve in finite layers of $\mathbb{Z}_p^2$ extensions. Here, an essential ingredient is an analysis of the *image* of the sharp/flat Coleman map.

V. Vatsal (University of British Columbia, Vancouver) – Test vectors for some ramified representations of $\mathrm{GL}(2)$

We give an explicit construction of test vectors for T-equivariant linear functionals on representations of $\mathrm{GL}(2)$ of a local field, in certain cases when the representation of the torus and $\mathrm{GL}(2)$ are both ramified. Of particular interest is the case when the both representations are supercuspidal; we solve this problem in the case of depth zero supercupsidals of $\mathrm{GL}(2)$ over $\mathbb{Q}_p$. Our method is based on modular representation theory, and reveals some interesting features related to the Langlands correspondence in characteristic p. We show in particular that the test vector problem has an obstruction in characteristic p beyond the root number criterion of Waldspurger and Tunnell, and exhibits an unexpected dichotomy.

C. Wang-Erickson (Imperial College London) – On a question of Greenberg

We present some joint work with Francesc Castella related to the following question of Ralph Greenberg. Let f be a classical p-ordinary cuspidal eigenform of weight at least two. It is well-understood that the associated irreducible 2-dimensional p-adic representation of the absolute Galois group of $\mathbb{Q}$ becomes reducible after restriction to a decomposition group at p. Greenberg asked for a characterization of those f such that this reducible representation splits. The following characterization has been proposed, with supporting evidence provided by Ghate and Vatsal: such f have complex multiplication.