Opers and the Hitchin component

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Given a compact Riemann surface $C$ and a holomorphic quadratic differential on $C$, there are two different but very similar-looking families of differential equations on $C$ which one can construct. One is a family of opers on $C$, also known as holomorphic Schrödinger operators. The other is a family of first-order differential equations which appear in the study of Hitchin’s integrable system and its relation to Teichmüller space. Recently Gaiotto conjectured a precise relation between these two families. I will describe a proof of this conjecture, as well as an extension involving a general simple Lie group $G$ (the original case is $G = SU(2)$) and some other possible extensions. This is joint work with Olivia Dumitrescu, Laura Fredrickson, Georgios Kydonakis, Rafe Mazzeo, and Motohico Mulase.