Deformation theory for higher genus CMC surfaces

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We consider compact surfaces of constant mean curvature (CMC) in the 3-sphere. These surfaces are characterized by the harmonicity of their Gauss maps. While equivariant harmonic maps into hyperbolic 3-space lead to Hitchins self-duality equations and play a very prominent role in modern geometry, their analogues into positively curved space, namely harmonic maps from compact Riemann surfaces into the round 3-sphere, are not well-understood. Besides the trivial case of harmonic spheres only harmonic tori have allowed for a profound gauge theoretic description: They can be parametrized in terms of (algebraic-geometric) spectral data due to the work of Hitchin (and Pinkall-Sterling). In this talk I will describe how CMC surfaces of higher genus in the 3-sphere are determined by special holomorphic curves into the moduli space of flat SL(2,C)-connections. Under the assumption of certain discrete symmetries, irreducible connections on higher genus surfaces are determined by flat line bundle connections. This observation enables us to define a generalization of the spectral curve theory for symmetric higher genus CMC surfaces. In a recent preprint (joint work with L. Heller and N. Schmitt) we have introduced a flow on the spectral data from CMC tori towards higher genus CMC surfaces. This (generalized Whitham) flow can be used to study deformations of symmetric CMC surfaces of higher genus and to obtain a clearer picture of the moduli space of CMC surfaces of higher genus.