SPP 2026
GEOMETRY AT INFINITY

UNIVERSITÄT
HEIDELBERG
Zukunft. Seit 1386.

Workshop on Bounded Cohomology

Mathematikon · Heidelberg
July 29th - August 2nd 2019

Speakers

Jonas Beyrer (Universität Heidelberg)
Caterina Campagnolo (KIT)
Carlos De la Cruz Mengual (ETH Zürich)
James Farre (University of Utah)
Elia Fioravanti (University of Oxford)
Roberto Frigerio (Università di Pisa)
Anton Hase (Technion)
Nicolaus Heuer (University of Oxford)
Joel Louwsma (Niagara University)
Michał Marcinkowski (Polish Academy of Sciences)
Marco Moraschini (Universität Regensburg)
Hester Pieters (Weizmann Institute of Science)
Roman Sauer (KIT)
Alessandro Sisto (ETH Zürich)
Genkai Zhang (Chalmers University)

Organizers

Tobias Hartnick (JLU Gießen)
Andreas Ott (Universität Heidelberg)

Monday, July 29th

9:00 Registration

9:30 Roberto Frigerio (Università di Pisa):
  ℓ^1-homology and amenable covers

Let X be a topological space admitting an amenable cover of multiplicity k. We show that, for every $n \geq k$ and every n-dimensional singular homology class α on X, the image of α in the ℓ^1-homology of X vanishes. This strenghtens previous results by Gromov and Ivanov, which, under the same hypotheses, proved that the ℓ^1-seminorm of α vanishes.

10:30 Coffee break

11:00 Michał Marcinkowski (Polish Academy of Sciences):
  Bounded cohomology of transformation groups

Let M be a finite-volume Riemannian manifold and let μ be the measure induced by its volume form. Denote by G_M the group of all μ-preserving homeomorphisms of M isotopic to the identity. We show how to construct classes in the bounded cohomology of G_M. As an application we show that, under certain conditions on $\pi_1(M)$, the third bounded cohomology of G_M (and some of its subgroups) is highly nontrivial. This is joint work with Michael Brandenbursky.

12:00 Lunch break

13:00 Informal discussions

15:00 Coffee break

15:30 Anton Hase (Technion):
   Dynamics of $\mathrm{Out}(F_n)$ *on* $H_b^2(F_n)$

  The $\mathrm{Out}(G)$-action on the group cohomology $H^k(G)$ of a group G is an important object of study in group theory. On the contrary, almost nothing is known about the corresponding $\mathrm{Out}(G)$-action on the bounded group cohomology $H_b^k(G)$. For the free group F_n and $k = 2$, this action is given by the action of $\mathrm{Out}(F_n)$ on quasimorphisms $Q(F_n)$. In this talk, we will study the action of $\mathrm{Out}(F_n)$ on counting quasimorphisms, which form an invariant, dense subspace of $Q(F_n)$. There the dynamics of the unipotent elements in $\mathrm{Out}(F_n)$ will be presented concretely and visualized. In particular we will explicitly characterize all invariant finite-dimensional subspaces.

16:30 Coffee break

17:00 Caterina Campagnolo (KIT):
   Alternating cup products in degree 4

In a joint project with Tobias Hartnick and Hester Pieters, we study the alternated cup-product of a 2-cocycle with itself. Its norm can be computed by restricting to cocycles defined on a finite set of points. These turn out to have a very concrete description as linear combinations of certain orientation cocycles. As an application, we obtain estimates for the Gromov norm of the square of the Kähler class for Hermitian symmetric spaces of tube type.

Tuesday, July 30th

9:00 Coffee

9:30 James Farre (University of Utah):
Hyperbolic volume and dense actions of discrete groups

Many examples of classes in bounded cohomology come from geometric constructions. Any isometric action of a discrete group G on hyperbolic n-space defines a class in bounded cohomology that measures the hyperbolic volume of geodesic simplices whose vertices are contained in an orbit $G.x$. We will focus in low dimensions and study when this volume class can tell dense representations from one another. Moving to higher dimensions, we explain a sufficient condition, phrased in terms of the discrete subgroups of $\mathrm{Isom}(\mathbb{H}^n)$ for the volume class of a dense action to yield a non-zero class in bounded cohomology.

10:30 Coffee break

11:00 Nicolaus Heuer (University of Oxford):
Values of simplicial volume

Simplicial volume was first introduced by Gromov to study the minimal volume of manifolds. Since then it has emerged as an active research field with a wide range of applications. However, not much is known about the actual set of simplicial volumes for orientable closed connected manifolds of a fixed dimension d: Only for $d = 2, 3$ this set is known by explicit calculations and using Geometrization.

We show that for d larger than 3, the set of simplicial volumes is dense in $\mathbb{R}^+$. For $d = 4$, we obtain that every non-negative rational number may be realised as the simplicial volume of some 4-manifold. We show these statements using stable commutator length.

This is joint work with Clara Löh (Universität Regensburg).

12:00 Lunch break

13:00 Informal discussions

15:00 Coffee break

15:30 Roman Sauer (KIT):
Macroscopic scalar curvature and simplicial volume

We prove a generalization of Gromov's main inequality between volume and simplicial volume where the lower Ricci curvature bound is replaced by a macroscopic scalar curvature bound. This generalization also extends the more recent volume theorem of Guth from hyperbolic manifolds to arbitrary Riemannian manifolds. As an introduction we discuss the concept of macroscopic scalar curvature in general. The talk is based on joint work with Sabine Braun.

16:30 Coffee break

17:00 Carlos De la Cruz Mengual (ETH Zürich):
On the degree-three bounded cohomology of the complex symplectic group

In the first part of this talk, we will explain how spectral sequences can be used to produce bounded-cohomological stability theorems along infinite ascending sequences of groups, such as the classical families of simple Lie groups, along the lines of a method devised for ordinary group homology by Quillen. Building on a stability theorem for the family of the complex symplectic groups by De la Cruz and Hartnick, we will then sketch an argument that computes the continuous bounded cohomology in degree three for the groups in said family.

Wednesday, July 31st

9:00 Coffee

9:30 Jonas Beyrer (Universität Heidelberg):
Marked length spectrum rigidity for actions on CAT(0) cube complexes

Given an action on a metric space one can associate to each element of the group its translation length. This gives a function of the group to the reals called the marked length spectrum. Adding requirements for space and action, it is a natural question if the marked length spectrum already uniquely defines space and action. In this talk we want to show that this is the case when considering CAT(0) cube complexes (under some natural assumptions). The main tool to prove this will be a boundary rigidity for those space using cross ratios. Joint work with Elia Fioravanti.

10:30 Coffee break

11:00 Elia Fioravanti (University of Oxford)
Superrigidity of actions on finite rank median spaces

Median spaces simultaneously generalise real trees, CAT(0) cube complexes and L^1 spaces. They arise in many limiting procedures and, in particular, as asymptotic cones of hierarchically hyperbolic groups. These include mapping class groups, right-angled Artin/Coxeter groups and most 3-manifold groups.

We prove a superrigidity result for actions of irreducible lattices in products of locally compact groups on median spaces of finite topological dimension. This extends work of Chatterji-Fernós-Iozzi on CAT(0) cube complexes. While these authors make essential use of bounded cohomology and Burger-Monod superrigidity, we instead rely on reduced cohomology and build on the work of Shalom. Our techniques also exploit some intriguing similarities between horofunction compactifications of median spaces and Satake compactifications of symmetric spaces. For lattices in products of rank-one simple Lie groups, we obtain a sharp contrast between the behaviour of actions on finite- and infinite-dimensional median spaces: they can be proper and cocompact on the latter, but always have a global fixed point on the former. As another corollary of our results, random groups at low density do not satisfy Shalom's property H_{FD}.

12:00 Lunch break

14:00 Hike

Meeting point: In front of the main entrance of Mathematikon
Route: Philosopher's trail, old town, castle (optional)

18:30 Conference dinner

Venue: Alt-Heidelberger Brauhaus Vetter
Steingasse 9 (old town)

Thursday, August 1st

9:00 Coffee

9:30 Genkai Zhang (Chalmers University):
*Convexity and plurisubharmonicity of energy functions
on Teichmüller space*

We consider the energy functional $E(u)$ of harmonic maps $u : (M, g) \to S$ from a Riemannian manifold M to a Riemann surface S of genus g as well as harmonic maps $u : S \to M$. The energy $E(u)$ can be defined as a function on the Teichmüller space $\mathcal{T}$ of S as the hyperbolic metric of S varies. We prove Weil-Petersson geodesic convexity and plurisubharmonicity of E on $\mathcal{T}$. (Joint work with I. Kim and Xueyuan Wan.)

10:30 Coffee break

11:00 Alessandro Sisto (ETH Zürich):
Bounded cohomology of acylindrically hyperbolic groups

Acylindrically hyperbolic groups form a vast collection of groups including non-elementary hyperbolic and relatively hyperbolic groups, mapping class groups, $\mathrm{Out}(F_n)$, many groups acting on CAT(0) cube complexes, and many others.
I will present a few results that describe their bounded cohomology and, time permitting, discuss an unexpected connection with hyperbolic 3-manifolds.

12:00 Lunch break

13:00 Informal discussions

15:00 Coffee break

15:30 Marco Moraschini (Universität Regensburg):
Ideal simplicial volume of manifolds with boundary

Simplicial volume is a homotopy invariant of compact manifold, which measures the complexity of a manifold in terms of real singular chains. It was introduced by Gromov in 1982 in his seminal paper "Volume and Bounded cohomology" and it can be studied via bounded cohomology.

The aim of this talk is to define the ideal simplicial volume, which is a variation of the ordinary one for compact manifolds with boundary. The main difference between the two invariants is that the ideal simplicial volume measures the minimal size of ideal triangulations of M "with real coefficients". Indeed, in this new framework ideal simplices are allowed to appear in representatives of the fundamental class. We will investigate the main properties of the ideal simplicial volume. One remarkable feature is that it agrees with the ordinary simplicial volume for any compact manifolds with amenable boundary components.

If we have enough time, we will present the precise computation of this new invariant for an infinite family of hyperbolic 3-manifolds with geodesic boundary, whose classical simplicial volume is not known.

This is joint work with Roberto Frigerio.

16:30 Coffee break

17:00 *Problem Session*

We will discuss possible future directions in bounded cohomology and related areas.

Friday, August 2nd

9:00 Coffee

9:30 Hester Pieters (Weizmann Institute):
The volume cocycle of the complex hyperbolic plane

The Gromov norm is a way of assigning a numerical invariant to a cohomology class. One of its important applications is that it gives a-priori bounds on characteristic numbers such as the simplicial volume and the Toledo invariant. However, to this day it has only been calculated for very few cohomology classes. I will give estimates for the Gromov norm of the volume class of the complex hyperbolic plane.

10:30 Coffee break

11:00 Joel Louwsma (Niagara University):
Quasimorphisms on groups that act on trees and
stable commutator length in Baumslag-Solitar groups

We construct efficient quasimorphisms on groups that act on trees and show that their defect is at most 6. A consequence is that every suitable element of a group that acts on a tree must have stable commutator length at least $1/12$; in Baumslag-Solitar groups, no element can have stable commutator length between 0 and $1/12$. We show how to compute the stable commutator length of certain types of elements of Baumslag-Solitar groups, and these calculations show that the bound of $1/12$ is the best that can be achieved in this generality. We also characterize which of these elements of Baumslag-Solitar groups admit extremal surfaces. This is joint work with Matt Clay and Max Forester. Many of these results have been generalized by Lvzhou Chen and/or Nicolaus Heuer, and we indicate some of these generalizations.

12:00 End of the conference

Thank you all for coming!

Conference Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00	Registration	Coffee	Coffee	Coffee	Coffee
9:30	Roberto Frigerio *ℓ^1-homology and amenable covers*	James Farre *Hyperbolic volume and dense actions of discrete groups*	Jonas Beyrer *Marked length spectrum rigidity for actions on CAT(0) cube complexes*	Genkai Zhang *Convexity and pluri-subharmonicity of energy functions on Teichmüller space*	Hester Pieters *The Gromov norm for degree 4 classes*
10:30	Coffee	Coffee	Coffee	Coffee	Coffee
11:00	Michał Marcinkowski *Bounded cohomology of transformation groups*	Nicolaus Heuer *Values of simplicial volume*	Elia Fioravanti *Superrigidity of actions on finite rank median spaces*	Alessandro Sisto *Bounded cohomology of acylindrically hyperbolic groups*	Joel Louwsma *Quasimorphisms on groups acting on trees and scl on Baumslag-Solitar groups*
12:00	Lunch	Lunch	Lunch	Lunch	Lunch
13:00	Informal Discussions	Informal Discussions	**14:00** Hike	Informal Discussions	
15:00	Coffee	Coffee	Philosopher's trail, Old Town, Castle	Coffee	
15:30	Anton Hase *Dynamics of Out(F_n) on $H_b^2(F_n; \mathbb{R})$*	Roman Sauer *Macroscopic scalar curvature and simplicial volume*		Marco Moraschini *Ideal simplicial volume of manifolds with boundary*	
16:30	Coffee	Coffee		Coffee	
17:00	Caterina Campagnolo *Alternating cup products in degree 4*	Carlos De la Cruz Mengual *Measured Stiefel complexes and bounded cohomology*	**18:30** Conference dinner	Problem Session	