SYLLABUS FOR THE STUDENT SEMINAR: THE ARITHMETICS OF THE HYPERBOLIC PLANE

Talk 1: Hyperbolic geometry

Speaker: Valentino Delle Rose

Give an introduction to Hyperbolic geometry: discuss the Poincaré model, the upperhalf plane model, geodesics, isometries. Explain that hyperbolic geometry is an example of a non-Euclidean geometry [Schwartz [6], Chapter 10 p. 111–125].

Talk 2: The Farey diagram

Speaker: Oskar Riedler

Construct the Farey diagram, prove the determinant rule for edges [Hatcher [2], Chapter 1 and page 33–35, Series [7] Pages 2–6].

Talk 3: Continued franctions and cutting sequences

Speaker: Clemens Fruböse

Define continued fractions, prove that rationals have finite continued fractions, and that every irrational number has a unique expression as an infinite continued fraction. Show how continued fractions can be read off Farey Diagrams and introduce the notion of cutting sequences. Prove Lemma 3.1 in Series' survey [Hatcher [2], pages 26–33, Series [7], pages 7–12].

Talk 5: Existence of dense geodesics

Speaker: Menelaos Zikidis (?)

Introduce the modular surface, and use continued fractions and cutting sequences to show that there exists a dense geodesic in the modular surface [Series [7], page 15].

Talk 7: Quadratic forms

Speaker: Ferdinand Vanmaele

The toponograph (dual tree for Farey graph) can be used for describing graphically the values attained by a quadratic form. Prove arithmetic progression theorem. Discuss the applications to continued fraction expansion. Prove that $Q(x, y) = x^2 - 2y^2$ has the same positive and negative values, but $Q(x, y) = x^2 - 3y^2$ doesn't [Hatcher [2], Chapter 4].

Date: May 3, 2017.

Talk 8: Classification of Quadratic forms

Speaker: Manuel Hoff

Define parabolic, elliptic and hyperbolic quadratic forms. Introduce the discriminant to distinguish between them. Prove that the separatrix line for hyperbolic forms is infinite in both directions, and that, for an hyperbolic form $Q(x, y)$, if $Q(x, y) = n$ has one integer solution, then it has infinitely many integer solutions. Time permitting, define equivalence of quadratic forms, and prove that up to equivalence there are just a finite number of quadratic forms with given discriminant, except in the special case when the discriminant is zero. Define the class number [Hatcher [2], Chapter 5]

Talk 9: A Theorem of Fermat

Speaker: Tetyana Lemeshko

State the Theorem of Fermat from page 91, and explain how one can use the toponograph to guess that it might be true. Prove the criterion for representability (page 98) and Fermat's theorem [Hatcher [2], Chapter 6]

Talk 10*: Quadratic number fields and discrete subgroups of $\mathrm{PSL}(2, \mathbb{R})$

Speaker: Menelaos Zikidis (?)

[*This talk requires some basic knowledge in number theory, like some understanding of number fields and their rings of integers*] Define quaternion algebras, and orders. Discuss the relation with quadratic forms. Time permitting show how this can be used to produce examples of hyperbolic surfaces [Maclachlan-Reid [4] Definition 2.1.1, Section 2.2, Theorem 2.3.1, Katok [3] Pages 113–120].

Talk : Continued fractions and Lagrange's theorem

Define continued fractions, prove that rationals have finite continued fractions, and that every irrational number has a unique expression as an infinite continued fraction. Discuss Lagrange's theorem: the irrational numbers whose continued fraction expansion are eventually periodic are precisely quadratic irrationals [Hatcher [2], pages 26–33 and 37–44].

Talk : Farey Tassellation and circle packings

Construct the Ford circle packing using the Farey tasselation, describe the corresponding horocircles in the once punctured torus. Show that two Ford circles are either tangent or disjoint, and give a criterion on (p, q), (r, s) that guarantees when $C[p, q]$ and $C[r, s]$ are tangent. Define badly approximable numbers and deduce that every real number [Bonahon [1] pages 207–217].

References

[1] F. Bonahon, *Low-Dimansional Geometry*, A.M.S. Student Library series, Volume 49.
[2] A. Hatcher, *Topology of Numbers*, https://www.math.cornell.edu/~hatcher/#TN.
[3] S. Katok, *Fuchsian groups*.
[4] C. Maclachlan, A. Reid *The arithmetic of Hyperbolic 3-Manifolds*.
[5] D.W. Morris, *Introduction to Arithmetic Groups*, Deductive Press.
[6] R. E. Schwartz, *Mostly Surfaces*, A.M.S. Student Library series, Volume 60. https://www.math.brown.edu/~res/Papers/surfacebook.pdf
[7] C. Series, *Continued Fractions and Hyperbolic Geometry* http://homepages.warwick.ac.uk/~masbb/HypGeomandCntdFractions-2.pdf.
[8] C. Ulcigrai, *Cutting sequences, regular polygons and Veech groups*, https://people.maths.bris.ac.uk/~maxcu/SlidesOctagonCuttSeq.pdf