Übungsblatt 10

Symmetric spaces

Exercise 1. Consider the Lie algebra $\mathfrak{g} = \mathfrak{su}(p, q)$.

(a) What is the complexified Lie algebra?

(b) Construct a symmetric space M such that the Lie algebra of $\mathrm{Isom}_0(M)$ is $\mathfrak{g}$.

(c) Construct the compact dual of M.

Exercise 2. We proved that if G is a compact Lie group with a bi-invariant metric then forall $X, Y, Z \in \mathfrak{g} = T_e G$ it holds

$$R(X, Y)Z = \frac{1}{4}[[X, Y], Z].$$

On the other hand If X is a symmetric space forall $X, Y, Z \in \mathfrak{p} = T_e X$ it holds

$$R(X, Y)Z = [[X, Y], Z].$$

Check that these two results are compatible.

Exercise 3. Consider the symmetric space S_n from Sheet 8, Exercise 1. Find a 2-plane in the tangent at one point where the sectional curvature is maximal and a 2-plane where the sectional curvature is minimal.