RUPRECHT-KARLS-UNIVERSITÄT HEIDELBERG

Mathematisches Institut

Vorlesung Differentialgeometrie II
Heidelberg, 11.01.2018

Übungsblatt 8

Spaces of non-compact type

To hand in by January 24, 14:00.

Exercise 1. (Parallel sets in the symmetric space for $SL(n, \mathbb{R})$)

(a) Show that, in a symmetric space two geodesics σ and τ are parallel then there exists $g \in G$, $C \in \mathbb{R}$ such that $g(\sigma) = \tau$ and for every $t \in \mathbb{R}$, $d(g\sigma(t), \sigma(t)) < C$.

(b) Let σ_1 be the geodesic

$$\begin{pmatrix} e^{nt} & & & \\ & e^{(n-2)t} & & \\ & & \ddots & \\ & & & e^{-nt} \end{pmatrix}$$

in the symmetric space associated to $SL(n, \mathbb{R})$. Describe the elements g of G such that $d(g\sigma_1(t), \sigma_1(t)) < C$ for t large.

(c) What is the parallel set of σ_1?

(d) Let σ_2 be the geodesic

$$\begin{pmatrix} e^{(n-1)t} & & & \\ & e^{-t} & & \\ & & \ddots & \\ & & & e^{-t} \end{pmatrix}$$

Describe the elements g of G such that $d(g\sigma_2(t), \sigma_2(t)) < C$ for $t \to +\infty$. What are the elements g of G such that $d(g\sigma_2(t), \sigma_2(t)) < K$ for $t \to -\infty$?

(e) Describe the parallel set of σ_2.

(f) Deduce that there is no element $g \in G$ such that $g(\sigma_1) = \sigma_2$.

Exercise 2. (Flat subspaces) Consider the Lie algebras $\mathfrak{o}(p, q), \mathfrak{sl}(n, \mathbb{C}), \mathfrak{sp}(2n, \mathbb{R})$. In each case choose a Cartan decomposition $\mathfrak{g} = \mathfrak{p} + \mathfrak{k}$ and find a maximal abelian subalgebra of $\mathfrak{p}$. What is the rank of the associated symmetric space?

Exercise 3. (Flat strip theorem) Show that the hypothesis that the geodesics are biinfinite is crucial in the flat strip theorem by exhibiting two geodesic rays in a Hadamard manifold that stay at bounded distance but do not bound a flat strip.