RUPRECHT-KARLS-UNIVERSITÄT HEIDELBERG

Mathematisches Institut

Vorlesung Differentialgeometrie I

Heidelberg, 18.12.2012

Exercise sheet 10

Immersions

To hand in by January 8, 14:00

Exercise 1. Consider again the catenoid and helicoid of exercise sheet 5 and 7, §1.

(a) Compute the second fundamental form of the images of the two immersions.

(b) Compute the sectional curvature, the scalar curvature and the Ricci curvature of the images of the two immersions.

(c) Prove that the two Riemannian manifolds $(\mathbb{R}^2, g_1)$ and $(\mathbb{R}^2, g_2)$ are isometric. (Hint: apply the following change of variables to the Helicoid: $(s,t) \to (\sinh s, t)$).

Exercise 2. Let M be a 2-dimensional manifold, and $f : M \to \mathbb{R}^3$ be an immersion, with second fundamental form S. Let $c : (-1,1) \to M$ be a smooth curve, and denote by $\alpha(t)$ the angle between $(f \circ c)''(t)$ and the normal vector to f. Prove that

$$\|(f \circ c)''(t)\| \cos \alpha(t) = \|S_{c(t)}(\dot{c}(t), \dot{c}(t))\|.$$

(The left hand side of this equality, divided by $\|(f \circ c)'(t)\|^2$ is called the normal curvature of the curve c, and this result says that it only depends on the direction of $\dot{c}$).

Exercise 3. Let M be a compact 2-dimensional submanifold of $\mathbb{R}^2$. Prove that there exists a point of M with strictly positive sectional curvature.

Exercise 4. Let (M, g) be a pseudo-Riemannian manifold.

(a) Let N be a submanifold of M that is the intersection of two totally geodesic submanifold. Prove that N is also totally geodesic.

(b) Let $F : (M, g) \to (M, g)$ an isometry. Show that the connected components of the fixed point set of F are totally geodesic submanifolds.