Asymptotic geometry of groups and spaces
RTG 2229

Monday February 20th

09:15 - 09:30	**Welcome**
09:30 - 10:30	**Gabriela Weitze-Schmithüsen** Congruence group problems for Veech groups
10:30 - 11:15	coffee and tea
11:15 - 12:15	**Jean Raimbault** Some questions and fewer answers about hyperbolic manifolds
12:15 - 14:00	Lunch
14:00 - 15:00	**Tsachik Gelander** Torsion in negative curvature
15:00 - 15:45	coffee and tea
15:45 - 16:45	**Fanny Kassel** Convex cocompactness in real projective geometry
evening	**Reception**

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Tuesday February 21st

09:30 - 10:30	**Olivier Guichard** (Generalized) Satake compactifications and compactifications of locally symmetric spaces associated with Anosov representations
10:30 - 11:15	coffee and tea
11:15 - 12:15	**Jean Lécureux** Projectivities and lattices in exotic affine buildings
12:15 - 14:00	Lunch
14:00 - 15:00	**Nicolas Tholozan** Volume of compact pseudo-Riemannian Clifford–Klein forms
15:00 - 15:45	coffee and tea
15:45 - 16:45	**Chris Judge** On the rigidity of cusp forms

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Wednesday February 22nd

09:30 - 10:30	**Joan Porti** Reidemeister torsion for sequences of hyperbolic three-manifolds
10:30 - 11:15	coffee and tea
11:15 - 12:15	**Frédéric Paulin** On the ergodic theory of group actions on trees
12:15 - 14:00	Lunch
	free afternoon

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Thursday February 23rd

09:30 - 10:30	**Corina Ciobotaru** Affine buildings, group actions and various compactifications
10:30 - 11:15	coffee and tea
11:15 - 12:15	**Giuseppe Martone** Positively ratioed representations and their properties
12:15 - 14:00	Lunch
14:00 - 15:00	**Arielle Leitner** Generalized cusps on convex projective manifolds
15:00 - 15:45	coffee and tea
15:45 - 16:45	**Anne Parreau** Vectorial metric, Finsler geometries and convexity in symmetric spaces and affine buildings.

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Abstracts

Corina Ciobotaru
Affine buildings, group actions and various compactifications

Affine buildings were introduced by Bruhat and Tits in order to study semi-simple algebraic groups oven non-Archimedean local fields. They are the analog of the symmetric spaces associated with semi-simple real Lie groups. Giving old and new results, I discuss various compactifications that can be considered for affine buildings and important properties and decompositions of groups acting on those.

Tsachik Gelander
Torsion in negative curvature

A classical theorem of Gromov states that the Betti numbers, i.e. the size of the free part of the homology groups, of negatively curved manifolds are bounded by the volume. We extend this theorem to the torsion part of the homology in all dimensions $d > 3$. From Gromov's work it is known that in dimension 3 the size of torsion homology cannot be bounded in terms of the volume. In dimension 4 we give a somewhat precise estimate for the number of negatively curved manifolds of finite volume, up to homotopy, and in dimension $d > 4$ up to homeomorphism. These results are based on an effective simplicial thick-thin decomposition which is of independent interest.

A joint work with Uri Bader and Roman Sauer.

Olivier Guichard
(Generalized) Satake compactifications and compactifications of locally symmetric spaces associated with Anosov representations

Satake compactifications of a Riemannian symmetric space are constructed from the data of an irreducible representation of its isometry group. By allowing instead any representation, one abtains a wider class of compactifications that can be called “generalized Satake compactifications”. We will work out the basic properties of those compactifications (local structure, orbits, desingularisation, etc.) as well as a (tentative) description of an universal object. We will also explain how to use these compactifications in order to give compactifications of locally symmetric spaces associated with Anosov representations.

This is a joint work with Fanny Kassel and Anna Wienhard.

Chris Judge
On the rigidity of cusp forms

Selberg developed his trace formula to show that the Neumann Laplacian of the hyperbolic triangle with angles $(0, \frac{\pi}{2}, \frac{\pi}{3})$ has infinitely many nonconstant eigenfunctions Later, Phillips and Sarnak conjectured that the Neumann Laplacian of the hyperbolic triangle with angles $(0, \frac{\pi}{2}, \frac{\pi}{n})$ has nonconstant eigenfunctions if and only if n=3,4, or 6. In joint work with Luc Hillairet, we show that the Neumann Laplacian triangle $(0, \frac{\pi}{2}, \alpha)$ has no nonconstant eigenfunctions for all but a countable collection of α lying between 0 and 1. In this talk we will review the spectral theory of the Laplacian on hyperbolic surfaces with cusps, and we will describe how the conjecture is related to representation theory and number theory.

Fanny Kassel

Convex cocompactness in real projective geometry

I will discuss a notion of convex cocompactness for discrete groups preserving a properly convex open domain in real projective space. For hyperbolic groups, this notion is equivalent to being the image of a projective Anosov representation; we use it to construct new examples of Anosov representations. For nonhyperbolic groups, the notion covers Benoist's examples of divisible convex sets which are not strictly convex, as well as their deformations inside larger projective spaces. This is joint work with J. Danciger and F. Guéritaud.

Jean Lécureux

Projectivities and lattices in exotic affine buildings

Let X be a building of type $\tilde{A}_2$. Assume that X has a cocompact lattice G. We are interested in this talk in "exotic" examples, mainly of buildings which are not associated to $SL(3, k)$, where k is a local field. Then G does not admit any infinite linear representations. After recalling some basic facts on affine buildings and their boundaries, I will talk about an important tool in the proof of this theorem, namely, the projectivity group of the building. This is based on a joint work with Uri Bader and Pierre-Emmanuel Caprace.

Arielle Leitner

Generalized cusps on convex projective manifolds

A convex projective manifold $C = \Omega/\Gamma$ is the quotient of convex subset of projective space, Ω, by a discrete group of projective transformations $\Gamma \subset PGL(n+1, R)$. If the manifold is not closed, the convex projective structure can be deformed to give a new convex projective structure if the ends of the manifold are "generalized cusps". A generalized cusp in dimension 3 is a convex projective manifold that is the product of a ray and a torus. The holonomy centralizes a 1-parameter subgroup of $PGL(n, R)$. I have shown: A generalized cusp on a properly convex projective 3-dimensional manifold is projectively equivalent to one of 4 possible cusps. For a generalized cusp $C = \Omega/\Gamma$ in dimension n, we require that ∂C is compact and strictly convex (contains no line segment) and that there is a diffeomorphism $h : [0, \infty) \times \partial C \mapsto C$. Together with Sam Ballas and Daryl Cooper we have classified generalized cusps in dimension n, and explored new geometries arising from such cusps. We show the holonomy of a generalized cusp is a lattice in one of a family of Lie groups $G(\lambda)$ parameterized by a point$\lambda = (\lambda_1, \ldots, \lambda_n) \in R^n$. More generally a maximal-rank cusp in a hyperbolic n-orbifold is determined by the similarity class of lattice in $Isom(E^{n-1})$.

Giuseppe Martone

Positively ratioed representations and their properties

For a hyperbolic metric on a surface, a classical construction of Bonahon describes the length of closed geodesics in terms of their geometric intersection number with a certain geodesic current. We generalize this picture to higher rank Lie groups. Namely, we associate geodesic currents to Anosov representations that satisfy an additional positivity property. We use this tool to relate the entropy and the systole length of such a positively ratioed representation. This is joint work with Tengren Zhang.

Anne Parreau

Vectorial metric, Finsler geometries and convexity in symmetric spaces and affine buildings.

Finsler metrics on higher rank symmetric spaces and affine buildings may sometimes prove a better tool than the usual one. They are for example attracting more and more interest in the context of Anosov representations, higher teichmuller theory, and maximal representations. In particular, they allow to define a promising notion of weak convexity, sufficiently flexible to allow to construct many examples of weakly convex trees and weakly convex cocompact surface groups in buildings. A natural and unified setting to study Finsler geometries of higher rank spaces is to study the natural projection of the segments in a closed Weyl chamber, regarded as a universal metric with vectorial values, refining all Finsler metrics. We will show that many of the traditional basic properties of CAT(0) spaces hold for the vectorial metric, in particular convexity properties, and describe the associated Busemann boundary.

Frédéric Paulin

On the ergodic theory of group actions on trees

In this talk, I will study the variational principle and the exponential mixing properties of Gibbs measures under the discrete time geodesic flow on the quotient of a locally finite simplicial tree by a lattice. Applications will be given to error terms in equidistribution and counting problems in non-Archimedean local fields. This is joint work with Anne Broise and Jouni Parkkonen.

Joan Porti

Reidemeister torsion for sequences of hyperbolic three-manifolds

I plan to review first the definition of Reidemeister torsion and then to focus on a specific torsion for closed hyperbolic three manifolds. This is a topological invariant and I shall study its behaviour for sequences of manifolds.

Jean Raimbault

Some questions and fewer answers about hyperbolic manifolds

In this talk I will discuss the behaviour of global metric invariants of complete hyperbolic manifolds of finite volume. Much work has been devoted to estimating minima of these invariants on the set of all manifolds of a given dimension (in particular the volume and the systole), here I want to discuss the following problems :

1) Estimating how many hyperbolic manifolds are there (up to isometry) whose volume lies in an interval $[0, V]$ for large V?

2) For an arbitrary $R > 0$, how large can the R-thin part be for a "typical" manifold of large volume?

Question 1) has a rough answer given by work of Burger–Gelander–Lubotzky–Mozes part of which was refined by Gelander–Levit. I will present a new result on the subgroup growth of certain right-angled Artin groups which aims at being a toy analogue for a starting point for a more precise answer.

Question 2) has a more precise rephrasing in terms of the so-called "Benjamini–Schramm topology" on hyperbolic manifolds. A spectacular recent result of Mikolaj Fraczyk gives a complete answer for compact so-called "arithmetic congruence manifolds" in dimensions 2 and 3. Time permitting, we will explain this result which can be partially extended to higher dimensions using simple arguments on top of a more general version of Fraczyk's arguments.

This summary might be overambitious and I will likely end up discussing only one of the two questions.

Nicolas Tholozan

Volume of compact pseudo-Riemannian Clifford–Klein forms

The volume of a compact quotient of an even-dimensional hyperbolic space is essentially an integer. This statement is well-known to be false in dimension 3, where the volume is not a Chern–Weil characteristic class.
In this talk, I will explain why the compact quotients of many pseudo-riemannian symmetric spaces have rational volume even when this volume is not a Chern–Weil characteristic class. Such symmetric spaces include the odd-dimensional anti-de Sitter spaces, as well as the Lie groups $\mathrm{SO}(n, 1)$ and $\mathrm{SU}(n, 1)$ with their Killing metric, for which we obtain more precise formulas.

Gabriela Weitze-Schmithüsen

Congruence group problems for Veech groups

The classical congruence group problem asks whether each finite index subgroup of the linear group $GL(k, \mathbb{Z})$ is a congruence group, i.e. it is defined by congruence equations modulo n. This problem naturally generalises to subgroups of the automorphism group of any group G. For $G = F_2$ the problem was solved by a result of Asada in 2001. We study analogues in Veech groups of translation surfaces. The latter are surfaces obtained from polygons in the Euclidean plane whose edges are glued by translations. Their Veech groups are discrete subgroups of $SL(2, \mathbb{R})$ which carry a lot of information about the geometry of the surfaces.