ε-isomorphisms for analytic (φ_L, Γ_L)-modules over Lubin-Tate Robba rings

joint with M. Malcic, R. Steingart and M. Witzelsperger

Otmar Venjakob

Institute for Mathematics
Universität Heidelberg

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Functional equation, Artin-Verdier duality, ε-constants and Tamagawa Number Conjectures

M motive over $\mathbb{Q}$, for all prime ℓ

$$\begin{array}{ccccc}
L^*(M) & \sim & \prod_v \varepsilon_v(M) & \cdot & L^*(M^*(1)) \\
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FUKAYA-KATO'S ε/ζ-isomorphism conjecture concern ***p*-adic families** (for $v = p$, $l = p$), e.g. cyclotomic deformations.

$\overline{<\gamma>} = \Gamma := G(\mathbb{Q}_p(\mu(p))/\mathbb{Q}_p)$ cyclotomic extension ($p \neq 2$)

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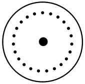

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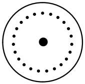

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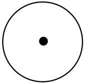
$\mathcal{R}_K^+ := \mathcal{R} \cap K[[Z]]$ analytic functions, which converge on open unit disk $\mathbb{B} := \{|Z| < 1\}$



(φ, Γ)-modules over the relative Robba-rings

$Rep_A(G_{\mathbb{Q}_p})$	category of finite projective A-modules with cts, linear $G_{\mathbb{Q}_p}$-action
$\Phi\Gamma_{\mathcal{R}_A}$	category of (φ, Γ)-modules over $\mathcal{R}_A$

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$$Rep_A(G_{\mathbb{Q}_p}) \hookrightarrow \Phi\Gamma_{\mathcal{R}_A}$$
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$\mathrm{R}\Gamma(\mathbb{Q}_p, W)$, if $M = D^\dagger_{rig}(W)$

The fundamental line

KEDLAYA-POTTHARST-XIAO: For $M \in \Phi\Gamma_{\mathcal{R}_A}$ there exists an invertible A-module $\mathcal{L}_M$ and a cts character $\delta : \mathbb{Q}_p^\times \to A^\times$ such that

$$\det_{\mathcal{R}_A}(M) \cong \mathcal{R}_A(\delta) \otimes_A \mathcal{L}_M.$$

Define the line bundle over A

$$\Delta(M) := \det_A(C^\bullet_{\varphi,\gamma}(M)) \cdot \det_{\mathcal{R}_A}(M)^{\varphi=\delta(p),\gamma=\delta(\chi_{cyc}(\gamma))}$$

and the unit-object

$$\mathbf{1}_A := \det_A(0).$$

NAKAMURA's ε-isomorphism Conjecture

For all generators $\xi \in \mathbb{Z}_p(1)$, all affinoid A and all $M \in \Phi\Gamma_{\mathcal{R}_A}$

$$\exists_1 \quad \varepsilon_{A,\xi}(M) : \mathbf{1}_A \xrightarrow{\cong} \Delta_A(M) \quad \text{satisfying:}$$

1. (base change) For any affinoid algebra B over A we have $\varepsilon_{A,\xi}(M) \otimes_A \mathrm{id}_B = \varepsilon_{B,\xi}(M\hat{\otimes}_A B)$.
2. (multiplicativity) in short exact sequences.
3. For any $a \in \Gamma$ we have $\varepsilon_{A,a\cdot\xi}(M) = \delta_{\det M}(a)\varepsilon_{A,\xi}$.
4. (duality) $\varepsilon_{A,\xi}(M^*(1))^* \text{ "}=\text{" } \varepsilon_{A,-\xi}(M)$.
5. (de Rham) If M is de Rham and defined over $\mathcal{R}_F$, $F/\mathbb{Q}_p$ finite, then $\varepsilon_{F,\xi}(M) = \varepsilon^{dR}_{F,\xi}(M)$.
6. Compatibility with FUKAYA-KATO's integral conjecture for étale M.

analogue

I.e., $\varepsilon_{A,\xi}(N)$ interpolates $\varepsilon^{dR}_{F,\xi}(N\hat{\otimes}_A F)$ for all points in $\mathrm{Sp}(A)(F)$, where

$$\varepsilon^{dR}_{F,\xi}(M) \xrightarrow{\cong} \Delta_F(M)$$

is a specific (always given) trivialisation involving

- NAKAMURA's (dual) BLOCH-KATO exponential maps $\exp_{BK}(M), \exp^*_{BK}(M)$,
- the ε-factor $\varepsilon(D_{pst}(M), \psi_\xi)$ for the character $\psi_\xi : \mathbb{Q}_p \to \mathbb{Q}_p(\mu(p))^\times$, $\psi_\xi(\frac{1}{p^n}) = \xi_n$ and
- a Γ-factor.

NAKAMURA's Theorem

Theorem (NAKAMURA)

For all $M \in \Phi\Gamma_{\mathcal{R}_A}$ of rank one (or trianguline) the conjecture is true.

Guiding Question

Is there an analogous conjecture/result for Lubin-Tate (φ_L, Γ_L)-modules over the Robba ring ?

Lubin-Tate setting versus cyclotomic setting

$L/\mathbb{Q}_p$	degree d extension of	$\mathbb{Q}_p$
o_L	integers	$\mathbb{Z}_p$
$\pi_L \in o_L$	prime element	p
$q = \lvert o_L/\pi_L o_L\rvert$	cardinality of residue field	$p = \lvert\mathbb{Z}_p/p\mathbb{Z}_p\rvert$

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$[a](Z) \in o_L[[Z]]$	giving o_L-action, $a \in o_L$	$(Z+1)^a - 1$

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$\mathcal{R}_L$	Robba ring	$\mathcal{R}_{\mathbb{Q}_p}$
φ_L	$f(Z) \mapsto f([\pi_L](Z))$	$\varphi_{\mathbb{Q}_p}$
Ψ_L	left inverse of φ_L	$\psi_{\mathbb{Q}_p}$

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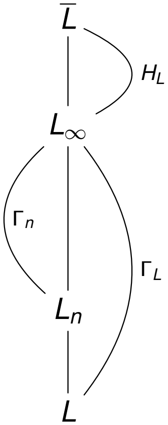

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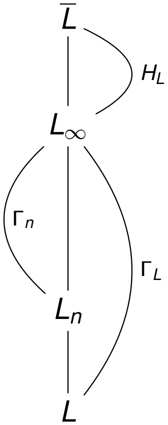
The Lubin-Tate character χ_{LT} induces isomorphism $\Gamma_L \xrightarrow{\cong} o_L^\times$ and $Lie(\Gamma_L) \xrightarrow{\cong} L$,



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$\Phi\Gamma_{\mathcal{R}_L}^{an}$ (φ_L, Γ_L)-modules with **L-linear** $Lie(\Gamma_L)$-action

Equivalence of categories (KISIN-REN/FONTAINE, BERGER)

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Without ^{an} false for $L \neq \mathbb{Q}_p$!

Problem

The theory of *analytic* (φ_L, Γ_L)-modules should be *one-dimensional*, but how to replace the operator

$$\gamma - 1$$

in the generalized Herr-complex $C^\bullet_{\varphi_L,\gamma-1}(M)$?

(since for $d > 1$ there is more than one topological generator!)

Fourier theory and Lubin-Tate isomorphism

Assume $\Omega \in K \subseteq \mathbb{C}_p$ complete (not spherically complete!):

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$$D(o_L^\times, K) \cong D(o_L, K)^{\psi=0} \xrightarrow{Mellin-transform} \mathcal{O}_K(\mathbb{B})^{\psi=0} = (\mathcal{R}^+)^{\psi=0}$$

(Topological isomorphism of $D(o_L^\times, K)$-modules)

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since $o_L = \pi_L o_L \coprod o_L^\times$ as disjoint sets.

Generalized *analytic* Herr-complex

$$D(\Gamma_n, K) \xrightarrow[\cong]{Fourier} \mathcal{O}_K(\mathfrak{X}_{\Gamma_n}) \xrightarrow[\cong]{LT-iso} \mathcal{O}_K(\mathbb{B})$$

$$Z_n \quad \longleftrightarrow \quad Z$$

Hence, we work with the Herr-complex

$$C^\bullet_{\varphi_L, Z_n}(M)$$

which amounts to using *analytic* cohomology à la Kohlhaase:

$$C^\bullet_{\varphi_L, D(\Gamma_L)}(M) := \mathrm{cone}\left(\mathrm{RHom}_{D(\Gamma_L,K)}(K, M) \xrightarrow{\varphi_L - \mathrm{id}} \mathrm{RHom}_{D(\Gamma_L,K)}(K, M)\right)$$

Work of STEINGART

$\chi := x|x| : L^\times \to K^\times \subseteq A^\times$ analytic character, $|\pi_L| = \frac{1}{q}$,
$\tilde{M} := \mathrm{Hom}_{\mathcal{R}_A}(M, \mathcal{R}_A(\chi))$

Theorem (KEDLAYA-POTTHARST-XIAO(cyc),STEINGART(LT))

For $M \in \Phi\Gamma^{an}_{\mathcal{R}_A}$ and $A \to B$ morphism of K-affinoids it holds

(i) $C^\bullet_{\varphi_L, Z_n}(M) \in D^{[0,2]}_{\mathrm{perf}}(A)$ *(even in $D^{[0,2]}_{\mathrm{perf}}(A[\Gamma_L/\Gamma_n])$),*

(ii) $C^\bullet_{\varphi_L, Z_n}(M) \otimes^{\mathbb{L}}_A B \xrightarrow{q.i.} C^\bullet_{\varphi_L, Z_n}(M \hat{\otimes}_A B)$,

If, moreover, M is trianguline, then we have

(iii) $\chi_{\mathrm{EP},n}(M) := \sum_i (-1)^i \mathrm{rk}_A H^i_{\varphi_L, Z_n}(M) = [\Gamma_L : \Gamma_n] \mathrm{rk}_{\mathcal{R}_A}(M)$,

(iv) $\mathcal{T}_\Psi(M) := [M \xrightarrow{\Psi - 1} M] \in D_{\mathrm{perf}}(D(\Gamma_L, K))$.

Replacing Tate-duality

Take $A = K$ and let $(-)^*$ denote the continuous K-dual, $Z_n^\iota = \lambda Z_n$ for $\lambda \in D(\Gamma_n, K)^\times$

Corollary

In the derived category $D(K)$ we have

$$C^\bullet_{\varphi_L, Z_n}(M)^* \cong C^\bullet_{\Psi_L, Z_n}(M)^* \cong C^\bullet_{\varphi_L, Z_n}(\tilde{M})[2],$$

e.g. we have a perfect duality pairing

$$H^1_{\varphi_L, Z_n}(M) \times H^1_{\varphi_L, Z_n}(\tilde{M}) \to H^2_{\varphi_L, Z_n}(\mathcal{R}_K(\chi)) \xrightarrow[\cong]{\mathrm{Res}} K$$

$$(\overline{(m,n)}, \overline{(f,g)}) \mapsto -\mathrm{Res}\Big(\varphi_L(g)(m) + (\lambda^\iota f)(n)\Big)$$

Proof uses that $M^{\Psi=0}$ is free over $\mathcal{R}_K(\Gamma_L)$ by SCHNEIDER-V.

Conjecture in the *analytic* LT-setting

For all generators $u \in T_\pi\mathcal{F}$, all K-affinoid A (with K a complete field extension of L^{ab}) and all $M \in \Phi\Gamma^{an}_{\mathcal{R}_A}$

$$\exists_1 \quad \varepsilon_{A,u}(M) : \mathbf{1}_A \xrightarrow{\cong} \Delta_A^{an}(M) \quad \text{satisfying:}$$

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4. (duality) $\varepsilon_{A,u}(M^*(1))^* \text{"}=\text{"} (-1)^{\dim_K H^0(M)}\Omega^{-\mathrm{rk}_{\mathcal{R}_A}(M)}\varepsilon_{A,-u}(M)$.
5. (de Rham) If M is de Rham and defined over $\mathcal{R}_F$, $K/F/L$ finite, then $\varepsilon_{K,u}(M\hat{\otimes}_F K) = \varepsilon^{dR}_{F,u}(M)_K$.
6. Sofar no integral version of conjecture (for étale M).

Although it specializes to NAKAMURA's conjecture for $L = \mathbb{Q}_p$ we have the following differences:

- $$\Delta^{an}(M) := \det_A(C^\bullet_{\varphi, D(\Gamma_L)}(M)) \cdot \det_{\mathcal{R}_A}(M)^{\varphi=\delta(p),\gamma=\delta(\chi_{LT}(\gamma))},$$

  i.e., the ε-isomorphism concerns **analytic** instead of *continuous* cohomology, e.g. $H^1_{an}(L, V) \subseteq H^1(L, V)$!
- the ε-factor $\varepsilon(D^{\mathrm{id}}_{pst}(M), \psi_u)$ is taken with respect to the character $\psi_u : L \to L(\mu(p))^\times$, $\psi_u(\frac{a}{\pi_L^n}) = \eta(a, u_n)$ and only for the id-component $D^{\mathrm{id}}_{pst}(M)$ of $D_{pst}(M)$!
- The occurrence of the power of Ω in the compatibility with duality (4) is a conceptually new phenomenon in our conjecture!

The Lubin-Tate deformation

Aim: Deform $M \in \Phi\Gamma^{an}_{\mathcal{R}_L}$ along the character variety $\mathfrak{X}_{\Gamma_L}$ à la Schneider-Teitelbaum with $\mathcal{O}_K(\mathfrak{X}_{\Gamma_L}) \cong D(\Gamma_L, K)$:

Heuristically: $\mathbf{Dfm}(M) = D(\Gamma_L, K)\hat{\otimes}_K M$, but technically difficult!

$\mathfrak{X}_{\Gamma_L} = \bigcup_n X_n$ affinoid cover , $\mathcal{R}_{\mathfrak{X}_{\Gamma_L}} : X_n \mapsto \mathcal{R}_{\mathcal{O}_{\mathfrak{X}_{\Gamma_L}}(X_n)}$ sheaf of Robba rings. Define $\mathbf{Dfm}(M)$ as a (φ_L, Γ_L)-module (sheaf) over $\mathcal{R}_{\mathfrak{X}_{\Gamma_L}}$:

$$\mathbf{Dfm}(M)(X_n) := \mathbf{Dfm_n}(M) := \mathcal{O}_{\mathfrak{X}_{\Gamma_L}}(X_n)\hat{\otimes}_L M,$$

where Γ_L acts diagonally, on the left factor via the inversion and on M via its given action. For each n, this is a (φ, Γ_L)-module M over $\mathcal{R}_{\mathcal{O}_{\mathfrak{X}_{\Gamma_L}}(X_n)}$.

The Lubin-Tate deformation

L-analytic Iwasawa cohomology of M :

$$R\Gamma_{\Psi,D(\Gamma_L,K)}(\mathfrak{X}_{\Gamma_L}, \mathbf{Dfm}(M)) := \mathbf{Rlim}\Big(C_{\Psi,D(\Gamma_L,K)}(\mathbf{Dfm}_n(M)) \Big).$$

Theorem (STEINGART)

1. $H^i_{\Psi,D(\Gamma_L,K)}(\mathfrak{X}_{\Gamma_L}, \mathbf{Dfm}(M)) \cong \varprojlim_n H^i_{\Psi,D(\Gamma_L,K)}(\mathbf{Dfm}_n(M))$
2. $R\Gamma_{\Psi,D(\Gamma_L,K)}(\mathfrak{X}_{\Gamma_L}, \mathbf{Dfm}(M)) \cong \mathcal{T}_\Psi(M) \quad in\ \mathbf{D}^b_{\mathrm{perf}}(D(\Gamma_L, K)).$

Sheaf of cohomology groups $X_n \mapsto H^i_{\Psi,D(\Gamma_L,K)}(\mathbf{Dfm}_n(M))$ gives rise to coadmissible modules over $D(\Gamma_L, K)$ with global sections given in (1).

Main result

Theorem (Local ε-conjecture for Lubin-Tate deformation of rank one modules)

$K/F/L$ finite subextension, M a rank one *analytic (φ_L, Γ_L)-module over $\mathcal{R}_F$. Then, setting $M_K := M\hat{\otimes}_F K$, there is a unique isomorphism*

$$\varepsilon_{D(\Gamma_L),u}(\mathbf{Dfm}(M_K)) : \mathbf{1}_{D(\Gamma_L)} \xrightarrow{\cong} \Delta^{an}_{\mathfrak{X}_{\Gamma_L}}(\mathbf{Dfm}(M_K)) = \det_{D(\Gamma_L)}(\mathcal{T}_\Psi(M_K))$$

inducing for every L-analytic character $\vartheta : \Gamma_L \to F^\times$ such that $M(\vartheta)$ is de Rham, the following commutative diagram

$$\begin{array}{ccc} \mathbf{1}_{D(\Gamma_L)} \otimes_{D(\Gamma_L),f_\vartheta} K(\vartheta) & \xrightarrow{\text{can}} & \mathbf{1}_K \\ \downarrow{\scriptstyle \varepsilon_{D(\Gamma_L),u}(\mathbf{Dfm}(M_K))\otimes \mathrm{id}_{K(\vartheta)}} & & \downarrow{\scriptstyle \varepsilon^{dR}_{F,u}(M(\vartheta))_K} \\ \Delta_{\mathfrak{X}_{\Gamma_L}}(\mathbf{Dfm}(M_K)) \otimes_{D(\Gamma_L),f_\vartheta} K(\vartheta) & \xrightarrow{\mathrm{sp}_\vartheta} & \Delta_K(M_K(\vartheta)), \end{array}$$

Outlook

What is the (arithmetic) meaning of this conjecture/result?

How does it relate to FUKAYA-KATO's original conjecture or to the *p*-adic local Langlands programme?

Many thanks
for your attention!