

# GAUS AG: Condensed Mathematics

AG Venjakob

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ORGANISATION: The dates are preliminary. We meet every Thursday at 11 (c.t.) in SR8.  
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We will deviate from the notes [CS21] towards the second half and study applications in representation theory of  $p$ -adic Lie groups.

## **Talk 1 (20.04.23, Milan): Condensed Sets and condensed abelian groups**

Condense Lectures 1 and 2 of [CS21] into an introductory lecture.

## **Talk 2 (27.04.23, Marlon): Cohomology**

For compact spaces  $X$ , this talk relates Ext out of free condensed abelian group  $\mathbb{Z}[X]$  to Čech cohomology.

## **Talk 3 (04.05.23, Max): Locally compact abelian groups**

Building on the previous talk, this talk computes Ext between locally compact abelian groups (viewed as condensed abelian groups), and finds that they behave exactly as one would hope.

## **Talk 4 (11.05.23, Anna): Solid abelian groups I**

This talk introduces solid abelian groups, a type of completeness condition on condensed abelian groups, and sets up Theorem 5.8, which states that solid abelian groups have extremely general closure properties, and derive well. Coordinate with the speaker of the next talk.

## **Talk 5 (25.05.23, Marvin): Solid abelian groups II**

This talk proves Theorem 5.8. Coordinate with the speaker of the previous talk.

## **Talk 6 (01.06.23, Achim): Analytic Rings**

This talk introduces the notion of analytic rings, which consist of a condensed ring together with a suitable notion of “completion” on its category of modules satisfying a condition similar to Theorem 5.8.

**Talk 7 (15.06.23, Katharina): Solid A-modules**

This talk proves that finitely generated ring together with solidification provides an example of an analytic ring, i.e. the analogue of Theorem 5.8 holds over more general finitely generated base rings.

**Talk 8 (22.06.23, Rustam): Non-archimedean condensed functional analysis I**

Cover the content of Sections 3.1-3.3 in [JC21]. See also Appendix A of [Bos21].

**Talk 9 (29.06.23, Jakob B.): Non-archimedean condensed functional analysis II**

Cover the content of Sections 3.4-3.6 with the main goal being [JC21, Lemma 3.40].

**Talk 10 (06.07.23, Otmar): Analytic Representations**

The goal of this talk is to cover Theorem 4.36 in [JC21]. To this end give a streamlined treatment of Sections 4.1-4.3. It might be sensible to restrict the consideration to  $G = G_0$  to simplify the presentation, as the extension to general  $G$  is done in the obvious way. Also highlight the relationship between the condensed analogues of the Iwasawa and distribution algebras and their classical counterparts ([JC21, Remark 4.9]).

**Talk 11 (13.07.23, Gautier): Analytic Cohomology**

The goal is to cover the comparison results from [JC21, Section 5]. The introduction to the corresponding section gives a good outline of the key ideas. The speaker can elaborate on the proofs as they see fit, ideally highlighting the interplay between the classical results and the machinery developed in the seminar.

## Literatur

- [Bos21] Guido Bosco, *On the  $p$ -adic pro-étale cohomology of drinfeld symmetric spaces*, <https://arxiv.org/abs/2110.10683>, 2021.
- [CS21] Dustin Clausen and Peter Scholze, *Lectures on condensed mathematics*, <https://www.math.uni-bonn.de/people/scholze/Condensed.pdf>, 2021.
- [JC21] Joaquín Rodríguez Jacinto and Juan Esteban Rodríguez Camargo, *Solid locally analytic representations of  $p$ -adic lie groups*, <https://arxiv.org/abs/2110.11916>, 2021.