Lenny Taelman's body of work on Drinfeld modules

Seminar in the summer semester 2015 at Universität Heidelberg

Prof Dr. Gebhard Böckle, Dr. Rudolph Perkins, Dr. Patrik Hubschmid

1 Introduction

In the 1930's, Carlitz [4] - playing on analogies with the work of Euler on the Riemann zeta values $\zeta(2)$, $\zeta(4)$, $\zeta(6), \ldots$ - constructed an explicit element $\widetilde{\pi} \in \mathbb{F}_q((1/t))(\sqrt[q-1]{-t})$ such that, for all positive integers k,

$$\frac{1}{\widetilde{\pi}^{k(q-1)}} \sum_{\substack{a \in \mathbb{F}_q[t] \\ \text{monic}}} \frac{1}{a^{k(q-1)}} \in \mathbb{F}_q(t).$$

After normalizing these fractions suitably, by a "factorial" in $\mathbb{F}_q[t]$, he was able to determine their denominators exactly, in parallel to the von-Staudt Theorem for the denominators of the classical Bernoulli numbers. Goss rediscovered these "numbers" upon introducing Eisenstein series in characteristic p, and he went on to develop a framework for studying L-functions with values in fields of positive characteristic [9, Ch. 8]. Various authors studied the arithmetic content in the numerators of Carlitz' fractions defined above, and they were partially successful in relating these numerators to the Galois module structure of class groups of extensions of the rational function field $\mathbb{F}_q(t)$ obtained by adjoining prime-torsion of the Carlitz module. Sometime later, Anderson [1] succeeded in defining a finitely generated submodule of integral points in Carlitz prime-torsion extensions of a rational function field which was directly relatable to Goss' abelian L-values

$$L(1, \chi) := \sum_{\substack{a \in \mathbb{F}_q[t] \\ \text{monic}}} \frac{\chi(a)}{a}$$

and which had the flavor of the classical cyclotomic units group; here χ is a positive characteristic valued Dirichlet character. All of these results suggest the existence of $\mathbb{F}_q[t]$-module analogs of class and unit groups to which these special L-values relate.

In a series of papers [13, 14, 15, 16], L. Taelman introduced the appropriate $\mathbb{F}_q[t]$-module analogs of class and unit groups to give a very satisfactory arithmetic interpretation of the rational zeta values of Carlitz and the abelian L-values of Goss. Further, his results culminated in a class number formula for certain positive characteristic special L-values associated to Drinfeld modules. Taelman's methods work in great generality, and already they have been shown by F. Demeslay [5] and J. Fang [7, 8] to extend into the worlds of Drinfeld modules over Tate algebras and Anderson t-modules, respectively. Obviously, such discoveries open a huge door for interesting future research.

The goal of this seminar is to dig deeply into the work of L. Taelman. Specifically, we will study his papers introducing the unit and class modules associated to a Drinfeld module, their algebraic reformulation in the case of the Carlitz module, Taelman's class number formula, and the work of Anglès-Taelman on the Galois module structure of his unit and class modules in the case of Carlitz prime torsion extensions of the rational function field. Throughout we will return our attention to the special case of the Carlitz module, which has a close analogy with the multiplicative group functor classically.

For consistency throughout the seminar, we would like to set the following notations: Let $\mathbb{F}_q$ be the finite field with q elements and characteristic p, t an indeterminate over $\mathbb{F}_q$, $A := \mathbb{F}_q[t]$, $k := \mathbb{F}_q(t)$, $k_\infty := \mathbb{F}_q((1/t))$. We will denote a finite extension of k by K, and the integral closure of A in K will be denoted by R or $\mathcal{O}_K$.

2 Talks

Talk 1 (Review and motivation of Drinfeld modules, a result of Poonen about the integral points of a Drinfeld module)**.**
Start your talk with a reminder on the algebraic and analytic theory of Drinfeld modules over a global function field (definition, uniformization by lattices, torsion points). Also mention the definition of a Drinfeld module as a functor as in [15, Definition 2] because we will work with this definition.

Explain that the torsion points of the Carlitz module generate an abelian extension of the base field (see, e.g., [9, Prop. 3.3.8]) and the analogy with cyclotomic fields, putting particular emphasis on the example of the Carlitz module as the analogue of the multiplicative group over a number field as in [16, Prop. 1]. Continue with a brief summary about Hayes's explicit class field theory over function fields (see [9, §§7.2-7.5, in particular Prop. 7.5.4] and/or [10, Part II]) which gives an explicit description of the abelian extensions of a global function field completely split at a fixed place ∞.

As a first result towards an analogue of Dirichlet's unit theorem state Poonen's Mordell-Weil theorem [12, Theorem 2] for Drinfeld modules (see also [9, §10.5]) which shows that the modules of integral and rational points of a Drinfeld A-module over a global function field is the direct sum of countably many copies of A and a finite torsion module. In the case of the Carlitz module this can be seen as a partial analogue of Dirichlet's unit theorem, but note that, in the classical case, the group of units of the ring of integers in a number field is finitely generated.
References: [9, §§4, 7, 10.5], [15, §1], [12, §§6, 7]
Date: April 15, 2015 **Speaker: Konrad Fischer**

Talk 2 (Definition of Taelman's unit and class module and a Dirichlet's unit theorem for Drinfeld modules)**.**
The goal of this talk is to introduce Taelman's unit and class modules and a regulator for Drinfeld modules. Also, Taelman's analogue of Dirichlet's unit theorem [13, Theorem 1] should be formulated and proved.

Start the talk by introducing the notation from Section 1 in [15], i.e., denote by K a finite extension of a rational function field $\mathbb{F}_q(t)$ over a finite field $\mathbb{F}_q$ with q elements, by R the integral closure of $\mathbb{F}_q[t]$ in K and by K_∞ the $\mathbb{F}_q((t^{-1}))$-vector space $K \otimes_{\mathbb{F}_q(t)} \mathbb{F}_q((t^{-1}))$. The latter vector space should be seen as the analogue of the $\mathbb{R}$-vector space $L \otimes_{\mathbb{Q}} \mathbb{R}$ for a number field L.

Then give the definition of Taelman's unit module $\ker(E(R) \to E(K_\infty)/\exp_E K_\infty)$, class module $E(K_\infty)/(E(R) + \exp_E K_\infty)$ and regulator $[R : \exp_E^{-1} E(R)] \in \mathbb{F}_q((T^{-1}))$ for a Drinfeld module E over R. Also formulate the analogue of Dirichlet's unit theorem saying that the "lattice of logarithms" $\exp_E^{-1}(E(R))$ is discrete and cocompact in K_∞ and explain why it implies that the unit module of R is finitely generated. Try to work out the analogy with the classical situation, in particular mention the exponential sequence from Section 1 of [13], and say a few words about the analogy of the quantity $[R : \exp_E^{-1} E(R)] \in \mathbb{F}_q((T^{-1}))$ with the regulator of a number field, cf. [13, Remark 4]. Finish your talk by giving the proof of the analogue of Dirichlet's unit theorem and the finiteness of the class module [13, Theorem 1].
References: [13, §§1-3, 5], [15, §1]
Date: April 22, 2015 **Speaker: David Guiraud**

Talk 3 (The case of the Carlitz module)**.**
Describe the unit module for the Carlitz module C over a ring $R \subset K$ as in talk 2 ([13, Proposition 1]). Also show that all torsion elements of the Carlitz module are in the unit module ([13, Proposition 2]). Continue to explain that the class module of the Carlitz module for the Carlitz module over $R = \mathbb{F}_q[t]$ is trivial and that its regulator is equal to $\log_C(1)$ ([13, Proposition 3]).

The goal of the second part of the talk is to work out the details of [13, Remark 2]. For this, first explain the statement of Anderson's log-algebraicity theorem for the Carlitz module [9, Theorem 10.6.2.1] saying that certain formal power series $S_m(T, z)$, $m \geq 1$, are in fact polynomials in T and z over $\mathbb{F}_q[t]$ (see also [11] for a good introduction to log-algebraicity). In particular, explain that, in the case $m = 1$, it simply means the equality $\exp_C(\log_C(Tz)) = Tz$ of formal power series in T and z and mention, for comparison, the classical equality [9, (10.6.1)] of formal power series (important note: to avoid confusion please continue to use the notation of [13] where t is a scalar in $R \subset K$. Therefore you can use, for example, T as first variable in the considered power series).

In the sequel, fix an irreducible polynomial polynomial $f \in A$ and let K be the finite extension of k obtained by adjoining the f-torsion points $C[f]$ of the Carlitz module and R the integral closure of A in K. Define Anderson's module of special points $\mathcal{L}$ by the A-span (under the Carlitz module) of 1 and all elements of the form $S_m(x, 1)$ with $x \in C[f]$ and $m \geq 1$ (compare [9, p. 398]). This module can be seen as an analogue of cyclotomic units in cyclotomic number fields. Explain why $\mathcal{L}$ is contained in the unit module U_R for the Carlitz module over R and the quotient module $U_R/\mathcal{L}$ is finite. Note that we will relate the latter quotient module to Taelman's class module in Talk 13. Conclude that the divisible closure of $\mathcal{L}$ in $C(R)$ is equal to U_R. Also discuss the analogy with cyclotomic units in number fields.
References: [9, §10.6], [13, §§4, 5], [11]
Date: April 29, 2015 **Speaker:**

Talk 4 (Algebraic reformulation of Taelman's unit and class modules for the Carlitz module)**.**
We will show that Taelman's unit and class modules for the Carlitz module can be realized purely algebraically, which has independent interest. The goal of this talk will be to prove the following result.

For a smooth projective geometrically connected curve X over a finite field $\mathbb{F}_q$ with a surjective map $X \to \mathbb{P}^1(\mathbb{F}_q) = \mathbb{P}^1$, we define, for each non-negative integer n, the sheaf $\mathcal{O}_X(n\infty)$ to be the pullback over $X \to \mathbb{P}^1$ of the sheaf $\mathcal{O}_{\mathbb{P}^1}(n\infty)$ on $\mathbb{P}^1$ of functions with poles of order at most n at the usual place $\infty \in \mathbb{P}^1$. Let $\mathcal{C}$ be defined as the cokernel of the map of sheaves $\mathcal{O}_X \otimes_{\mathbb{F}_q} A \xrightarrow{\partial} \mathcal{O}_X(\infty) \otimes_{\mathbb{F}_q} A$ given by $\partial : f \otimes a \to f \otimes ta - (tf + f^q) \otimes a$, where t is the coordinate on $\mathbb{P}^1$. Let Y be the preimage of $\mathbb{P}^1 \setminus \infty$ in X. Finally, we arrive at the statement of Taelman's result: *The zeroth cohomology group $H^0(X, \mathcal{C})$ is finitely generated as an A-module and gives rise to Taelman's unit module as the image of the restriction map $\mathcal{C}(X) \to \mathcal{C}(Y)$. The first cohomology group $H^1(X, \mathcal{C})$ is Taelman's finite class module.* The injectivity of ∂ is easy to show, and thus there is an exact sequence of sheaves on X given by

$$0 \to \mathcal{O}_X \otimes_{\mathbb{F}_q} A \xrightarrow{\partial} \mathcal{O}_X(\infty) \otimes_{\mathbb{F}_q} A \to \mathcal{C} \to 0.$$

This potentially gives a useful tool for computing (e.g. with Sage) the class and unit modules of Taelman, since the first two sheaves in the sequence above along with cohomology in exact sequences are well understood.

Cover Section 1 up through the statement of Theorem 1. Skip Sections 1.5 through 1.7. Cover Sections 2 and 3 in full detail.
References: [14, §2,3]
Date: May 6, 2015 **Speaker:**

Talk 5 (Statement of Taelman's class number formula and nuclear operators)**.**
The goal of the first part of this talk is to formulate Taelman's analogue of the class number formula for Drinfeld modules following [15, §1]. Use the notation $k = \mathbb{F}_q(t)$, K, R, K_∞ as in the previous talks. Start with the definition of the "special L-value" $\zeta(R, 1) \in 1 + T^{-1}\mathbb{F}_q[[T^{-1}]]$. Note the *formal* analogy with the Dedekind zeta function of a number field at $s = 1$.

Continue with [15, Proposition 1] which re-expresses the infinite product defining $\zeta(R, 1)$. This leads to the definition of the value $L(E/R) \in 1 + T^{-1}\mathbb{F}_q[[T^{-1}]]$ for general Drinfeld modules E over R. If it was not already done in talk 2, show the formula [15, Proposition 3] for the "index function" of two lattices in a finite-dimensional $\mathbb{F}_q((t^{-1}))$-vector space. Then formulate the class number formula [15, Theorem 1]. Discuss its correctness for the Carlitz module when $R = \mathbb{F}_q[t]$ with the results from talk 3 and point out the analogy with the classical class number formula (see [15, Remark 6]). As a remark, mention [13, Theorem 2] which gave to Taelman some evidence towards his conjectured class number formula before he proved it.

As an outlook to the coming three talks, present an overview of the proof of Taelman's class number formula in the case where the class module is trivial following the end of [15, §1].

In the remaining time, introduce normed vector spaces over fields with non-archimedean absolute values and define locally contracting endomorphisms of them. Present [15, Proposition 6 and 7] with proof and define for a compact normed $\mathbb{F}_q$-vector space V the $\mathbb{F}_q[[Z]]/Z^N$-modules $V[[Z]]/Z^N$ and the $\mathbb{F}_q[[Z]]$-module $V[[Z]]$ and nuclear endomorphisms of these modules ([15, Definition 4]).

References: [15, §§1,2], [13, Theorem 2]
Date: May 13, 2015 **Speaker:**

Talk 6 (Determinants and characteristic power series)**.**
After quickly recalling the definitions from the previous talk, present [15, Proposition 8] with proof. Then explain how it enables us to define a determinant for endomorphisms of the form $1 + \Phi$ for Φ a nuclear endomorphism of $V[[Z]]/Z^N$ resp. $V[[Z]]$ for a compact normed $\mathbb{F}_q$-vector space V. Make clear that V can be infinite-dimensional over $\mathbb{F}_q$ and that the defined determinant coincides with the usual one if V is finite-dimensional over $\mathbb{F}_q$. Also present the immediate properties [15, Proposition 9 and 10] of the defined determinant.

Continue with the definition of the characteristic power series of a locally contracting endomorphism of a compact normed $\mathbb{F}_q$-vector space and explain that it is in fact a polynomial ([15, Example 7]). Then present the properties [15, Theorem 2, Corollary 1] of characteristic power series with proof.

In the remaining time, introduce the notation from the beginning of [15, §3] and present Proposition 11 of *loc.cit.* with proof as a preparation for the statement of the trace formula in the next talk. Note that, in the statement of the trace formula, the product K_∞ of complete fields can be more general as in the previous talks.
References: [15, §§2, 3]
Date: May 20, 2015 **Speaker:**

Talk 7 (Trace formula and maps infinitely tangent to the identity)**.**
The first part of this talk should be devoted to the formulation and proof of Taelman's trace formula [15, Theorem 3] for certain nuclear endomorphisms. It will be used to express the value $L(E/R)$ in the class number formula as the determinant of an endomorphism of some compact $\mathbb{F}_q[[T^{-1}]]$-module. The proof is quite technical, try to present its structure as well as possible.

At this point, mention Anderson's trace formula [2, Thm. 1]. Taelman's trace formula is a variation of Anderson's trace formula, in fact the latter can be deduced from the former, see [15, Remark 13]. If time permits, give a sketch of this deduction.

In the rest of the talk, follow the beginning of [15, §4] to define the notion of $\mathbb{F}_q$-linear maps $\gamma : M_1 \to M_2$ *infinitely tangent to the identity* for $M_i := V/\Lambda_i \times H_i$ with V a finite-dimensional $\mathbb{F}_q((t^{-1}))$-vector space, Λ_1, Λ_2 lattices in V and H_1, H_2 finite $\mathbb{F}_q[t]$-modules. Conclude the talk by presenting [15, Proposition 12] with proof which gives important examples of $\mathbb{F}_q$-linear maps infinitely tangent to the identity.
References: [15, §§3, 4], [2, Thm. 1]
Date: May 27, 2015 **Speaker:**

Talk 8 (End of the proof of the class number formula)**.**
The goal of this talk is to put together the results from the previous talks to a proof of Taelman's class number formula. First present [15, Theorem 4] with proof. For a $\mathbb{F}_q$-linear isomorphism $\gamma : M_1 \to M_2$ infinitely tangent to the identity as in talk 7, this theorem expresses the ratio $[\Lambda_1 : \Lambda_2]|H_2|/|H_1|$ of the "volumes" of M_2 and M_1 as the determinant of $1 + \Delta_\gamma$ for some nuclear endomorphism Δ_γ of $M_1[[T^{-1}]]$ depending on γ. Before going through the proof explain [15, Remark 16] saying that the above ratio of "volumes" lies in fact in $\mathbb{F}_q[[T^{-1}]]^\times$.

Conclude the talk with a clear presentation of the proof of Taelman's class number formula using his trace formula and the theorem from the first part of this talk. Follow closely the exposition in [15, §5]. You should reserve about 30 minutes for this important proof.
References: [15, §§4, 5]
Date: June 3, 2015 **Speaker:**

Talk 9 (Galois equivariant class number formula for Carlitz prime torsion extensions)**.**
The goal here is to understand the Galois module structure of Taelman's unit and class modules in the case of Carlitz prime torsion extensions of the rational function field. The main tool is a Galois equivariant class number formula (ECNF) whose proof follows similar lines to what we have already

done. Corollaries of this ECNF are particularly similar to classical results for unit and class groups of cyclotomic number fields over the rational numbers.

Begin by setting notation: Let $\mathbb{F}_q$ be the finite field with q elements and $A := \mathbb{F}_q[t], k := \mathbb{F}_q(t), k_\infty := \mathbb{F}_q((1/t))$. Fix a monic irreducible (*i.e. a prime*) $P \in A$ and set K as the Carlitz P-torsion extension of k, $\mathcal{O}_K$ as the integral closure of A in K, and $\Delta := \mathrm{Gal}(K/k) \cong (A/PA)^\times$ (recall this isomorphism from Talk 1, if it was proved there, otherwise sketch the proof via the action of Galois on Carlitz P-torsion!). Cover the results about principal ideal rings from [3, §3].

Next move to Section 6 of *op. cit.*. Give Prop. 6.2 and its proof. Then use Cor. 6.3 and its proof as the definition of the Galois equivariant L-value $L(1, \Delta)$. Finish with 2.7 and the statement of Theorem A.

References: [3, §§3 and 6]
Date: June 10, 2015 **Speaker:**

Talk 10 (ECNF and Anderson's module of special points for the Carlitz module)**.**
The goal of this talk is to finish the proof of the equivariant class number formula [3, Theorem A]. We will then apply this to understanding the Galois module structure of "Anderson's lattice of logarithms" $\mathfrak{M}$, which is defined as the A-span of the elements

$$\mathfrak{L}_m := \sum_{\sigma \in \Delta} \sigma(\lambda)^m \sum_{a \in A_{+,\sigma}} \frac{1}{a} \in K_\infty,$$

where λ is a fixed generator of Carlitz P-torsion and $A_{+,\sigma}$ is the set of monic elements which map to σ in A/PA. We will observe that the module $\mathfrak{M}$ gives rise to Anderson's module of special points $\mathcal{L}$, as introduced in Talk 3 above, via the Carlitz exponential function $\exp_C$; indeed, one has $\exp_C \mathfrak{M} = \mathcal{L}$. After the ECNF, the main result here is that inside $K \otimes_k k_\infty$ (here the notation is as in the previous talk) we have

$$\mathfrak{M} = \mathrm{Fitt}_{A[\Delta]} H(\mathcal{O}_K) \cdot U(\mathcal{O}_K),$$

where $\mathrm{Fitt}_{A[\Delta]} H(\mathcal{O}_K)$ is the Fitting ideal of the $A[\Delta]$-module $H(\mathcal{O}_K)$. Beware the change in notation from previous papers for the class and unit modules over $\mathcal{O}_K$.

Recall the definition of $L(1, \Delta)$ (as given in the previous talk) and the statement of Theorem A in [3]. Then finish the proof of Theorem A, beginning in Section 6.4. and finishing Section 6.

Cover Section 7 of *op. cit.* giving the details in 7.5–7.8. Stress the connection with Talk 3.

References: [3, §§6.4–7]
Date: June 17, 2015 **Speaker:**

Talk 11 (Gauss-Thakur Sums, Generalized Bernoulli-Carlitz numbers and the 'odd' part of Taelman's class module)**.**
The goal here is to review various well-known facts about D. Thakur's Gauss sums for Carlitz torsion extensions which will be used in our analysis of the odd part of Taelman's class module. One primary use for these Gauss-Thakur sums is the explicit construction of a generator for the cyclic Galois module $\mathcal{O}_K$, see Theorem 5.5 and the remarks following in [3]. For an odd character $\chi : \Delta \to (A/PA)^\times$, the product of the Gauss-Thakur sum and Goss abelian L-value for χ will turn out to be a rational (in $(A/PA) \otimes_{\mathbb{F}_q} k$) multiple of a fixed choice of fundamental period of the Carlitz module. This rational element will essentially be our generalized Bernoulli-Carlitz number for χ, and the essential content of Theorem B is that the generalized Bernoulli-Carlitz number for χ generates the fitting ideal in $(A/PA) \otimes_{\mathbb{F}_q} A$ of the χ-part of the Galois module $(A/PA) \otimes_{\mathbb{F}_q} H(\mathcal{O}_K)$, where $H(\mathcal{O}_K)$ is Taelman's class module for K coming from the Carlitz module; here a small modification must be made when χ extends to a ring homomorphism on A.

Recall well-known results on Gauss-Thakur sums from [3, §5]. Define the notion of an odd character as in 2.9 of *loc.cit.*, and state Theorem B. Finish by stating and proving Prop. 8.2.

References: [3, 2.9, §5, 8.7, 8.2]
Date: June 24, 2015 **Speaker:**

Talk 12 (The Fitting ideals of the odd part of Taelman's class module and Carlitz von-Staudt)**.**
The goal today is to finish the proof of Theorem B in [3] that "the generalized Bernoulli-Carlitz number associated to an odd character χ generates the fitting ideal of the χ-part of $(A/PA) \otimes_{\mathbb{F}_q} H(\mathcal{O}_K)$." We will then move toward the statement and proof of Taelman's Herbrand-Ribet Theorem [3, Theorem C] by introducing the Carlitz factorials $\{\Pi(n)\}_{n\geq 0} \subset A$ and Bernoulli-Carlitz numbers $\{BC_n\}_{n\geq 0} \subset k$ (see [3, 8.9–8.11] and [9, 9.1–9.2]), which are function field analogs of factorials and Bernoulli numbers in the classical setting over the integers. For mathematical culture, we will state without proof Carlitz' von-Staudt Theorem [9, Theorem 9.2.2], which gives a complete description of the denominators of the Bernoulli-Carlitz numbers. We shall see in the next talk that the P-divisibility of the numerators of Bernoull-Carlitz numbers BC_n, with $n < |P|_\infty$, is directly related to the vanishing and non-vanishing of the isotypic components of the odd part of Taelman's class module for Carlitz P-torsion extensions. It is interesting to note here that, despite all of the similarities between the classical Bernoulli numbers and the function field Bernoulli-Carlitz numbers, there is no known analog of the Kummer congruences for the Bernoulli-Carlitz numbers.

Continue in Section 8.3 of [3], and finish the proof of Theorem B.

Close by defining Carlitz factorials and Bernoulli-Carlitz numbers and state Carlitz von-Staudt Theorem, as in [9, Theorem 9.2.2]. If time permits, state Theorem 8.15 (Taelman's Herbrand-Ribet Theorem).
References: [3, 8.3–8.11], [9, 9.1–9.2]
Date: July 1, 2015 **Speaker:**

Talk 13 (Taelman's Herbrand-Ribet Theorem and the even part of Taelman's class module)**.**
Our goal here is to prove Taelman's Herbrand-Ribet Theorem which relates the vanishing and non-vanishing of the isotypic components of the odd part of his class module for a Carlitz P-torsion extension to the P-divisibility of the numerators of the Bernoulli-Carlitz numbers. The key here is a congruence (mod P) between the generalized Bernoulli-Carlitz number associated to an odd character χ and a Bernoulli-Carlitz number BC_n, with n depending on χ and in the range $n < |P|_\infty$. We shall finish our investigations of Taelman's class module for Carlitz P-torsion extensions by relating the $A[\Delta]$-Fitting ideal of Taelman's unit module modulo Anderson's special units module to the $A[\Delta]$-Fitting ideal of the even part of Taelman's class module.

Cover Section 8.12 of [3], and finish with the proof of Theorem 9.3.
References: [3, §8.12 - §9]
Date: July 8, 2015 **Speaker:**

Talk 14 (Motivic interpretations: Reformulating the class and unit modules in terms of shtukas)**.**
The goal of this final talk will be to relate Taelman's class and unit modules for the Carlitz module to Yoneda extension groups of certain shtukas. This tightens the analogy between the Carlitz module and the Tate motive, Taelman's unit module and the group of units of a number field, and Taelman's class module and the class group of a number field from a motivic point of view.

After introducing shtukas, and in particular, the unit and Carlitz shtukas, as in [14, §1.5], the definition of Yoneda extension groups in the category of shtukas should be given. An appropriate reference for this is the Stacks Project [17], §13.27 which builds on 13.9 and 13.11.3. Note that for an object a in an abelian category, they write $a[i]$ for the associated cochain complex with a in the *minus* i-th position and 0's elsewhere. Recall Theorem 1 from [14] and state Theorem 2 giving the proof from Section 4. Hypercohomology groups must also be defined, and a reference here is the appendix of [6].
References: [14, §§1.5, 4], [17, §13]
Date: July 15, 2015 **Speaker:**

References

[1] Anderson, G. W. (1996), *Log-Algebraicity of Twisted A-Harmonic Series and Special Values of L-Series in Characteristic p*, J. Number Theory **60**, pp. 165-209.

[2] Anderson, G. W. (2000), *An elementary approach to L-functions mod p*, J. Number Theory **80**, pp. 291–303.

[3] Anglès, B. and Taelman, L. (2015), *Arithmetic of characteristic p special L-values*, Proc. London Math. Soc. **110** (4), pp. 1000-1032.

[4] Carlitz, L. (1935) *On certain functions connected with polynomials in a Galois field*, Duke Math. J. **1**, pp. 137-168.

[5] Demeslay, F. (2015), *A class formula for L-series in positive characteristic*, arXiv:1412.3704.

[6] Esnault, E. and Viehweg, E. (1992), *Lectures on Vanishing Theorems*, Birkhäuser Verlag, Basel.

[7] Fang, J. (2015), *Special L-values of abelian t-modules*, J. Number Theory **147**, pp. 300-325.

[8] Fang, J. (2015), *Equivariant special L-values of abelian t-modules*, arXiv:1503.07243.

[9] Goss, D. (1996), *Basic structures of function field arithmetic*, Springer-Verlag, New York.

[10] Hayes, D. R. (1992), *A brief introduction to Drinfeld modules* in *The arithmetic of function fields (Columbus, OH, 1991)*, Ohio State Univ. Math. Res. Inst. Publ. **2**, pp. 1–32.

[11] Perkins, R. (2013), *What is Anderson's log-algebraicity?*, notes from a seminar talk.

[12] Poonen, B. (1995), *Local height functions and the Mordell-Weil theorem for Drinfeld modules*, Comp. Math. **97**, no. 3, pp. 349–368.

[13] Taelman, L. (2010), *A Dirichlet unit theorem for Drinfeld modules*, Math. Ann. **348**, pp. 899–907.

[14] Taelman, L. (2011), *The Carlitz Shtuka*, J. Number Theory **131**, pp. 410-418.

[15] Taelman, L. (2012), *Special L-values of Drinfeld modules*, Ann. of Math. **175**, pp. 369–391.

[16] Taelman, L. (2012), *A Herbrand-Ribet theorem for function fields*, **188**, pp. 253–275.

[17] The Stacks Project Authors (2015), *Stack Project*, `http://stacks.math.columbia.edu`